Direct numerical simulation of sediment-laden turbulent channel flow

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**ABSTRACT:** Direct numerical simulation results are presented for sediment-laden turbulent channel flow. The focus of the work is on the stratification effects on velocity and concentration profiles. The results show that the inner linear law and logarithmic law are still present for flows stratified by suspended sediment as long as the flow does not relaminarizes. The inner linear law is not modified by the presence of suspended sediment. The logarithmic law, however, shows appreciable change of the slope with stratification. The change of slope is associated to a decrease of the apparent von Karman constant, which can be explained in terms of the variation of the Monin-Obukov scale in the near-wall region.

1 INTRODUCTION

Understanding sediment transport processes is of importance for many engineering and geophysical applications. For example, water quality and ecological health of rivers and lakes very much depend on turbidity levels and on the presence of sediment-carried contaminant. Other examples are turbidity currents, which are one of the main sediment transport manifestations in the ocean environment. These strong flows are responsible for carving canyon on the continental shelf and depositing submarine deltas.

The transport of sediment particles in the environment occurs mostly by means of turbulent flows. When the sediment transport occurs in suspension a concentration gradient develops in the vertical owing to the inherent characteristic of sediment particles to settle down. As a consequence of the vertical concentration gradient the flow stratifies and strongly modifies its turbulence structure, thus reducing, and sometimes suppressing, its own ability to carry particles in suspension.

The influence of stratification by contaminants or temperature on turbulent flows has been vastly studied (see for example Armenio and Sarkar (2002) and references therein). On the other hand, turbulence modulation by stratification effects associated to suspended sediment has been devoted less attention. Pioneering experimental results were presented by Niño and García (1996) on particle-turbulence interaction; however, their work focused mainly on sediment near-wall dynamics. Probably the first researchers to identify stratification effects on sediment-laden flows were Vanoni (1946) and Einstein and Chien (1952), which postulated a decrease of the von Karman constant with increasing suspended sediment load as the interpretation of their experimental results. More recently Winterwerp (2001) presented interesting results on the stratification effects of sediments on open channel flow with a one-dimensional k-ε numerical model.

Direct numerical simulation (DNS) is a useful tool for the analysis of basic phenomena of turbulence. The concept behind DNS is to solve for all relevant time and length scales present in the flow without the need of turbulence closure models. As such, DNS minimizes the empiricism in the analysis of turbulent flows, albeit, at the cost of reducing the analysis to moderate Reynolds numbers. Nevertheless, the Reynolds numbers flows that can be currently computed show a mature degree of turbulence. Moreover, DNS has not been fully exploited in the context of sediment-laden flows and very useful information still remains to be obtained, even at moderate Reynolds numbers flows. The present works presents DNS results on sediment transport in a closed conduit, focusing on the stratification effects of the sediment load on the flow velocity and concentration profiles.

The aim (and future goal) of the present work is to present a detailed description of basic phenom-
ena associated to sediment transport processes with the hope that they will shed some light on the understanding and development of closure strategies and empirical relations employed in larger scale modeling approaches such as Reynolds Averaged Navier-Stokes (RANS) and shallow water models.

2 MATHEMATICAL AND NUMERICAL MODELING

A horizontal channel is considered in which the flow is forced by a uniform pressure gradient in the $x$-direction. The flow is loaded with sediment particles of uniform size and negligible inertia, but with a settling velocity of magnitude $V$ in the direction of gravity ($z$-direction). The sediment particles are also assumed to be small enough for which Eulerian representations can be employed. The flow is also assumed to be dilute such that particle-particle interaction can be neglected, and that Boussinesq approximation can be employed. The dimensionless set of equations that describes the flow is thus (Cantero et al. 2008)

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \mathbf{G} - \nabla p + \frac{1}{\text{Re}_c} \nabla^2 \mathbf{u} + \text{Re}_b \left( c - c^b \right) \mathbf{e}_z, (1)$$

$$\nabla \cdot \mathbf{u} = 0,$$  

$$\frac{\partial c}{\partial t} + \left( \mathbf{u} + \mathbf{V} \right) \cdot \nabla c = \frac{1}{\text{Re}_c} \nabla^2 c \quad (3)$$

where $\mathbf{u}$=$(u_x,u_y,u_z)$ is the fluid velocity, $c$ is the volumetric concentration of sediment particles, $c^b$ is the horizontally-averaged concentration field, $\mathbf{e}_c$=$(0,0,-1)$ is a unit vector pointing in the direction of gravity, and $\mathbf{V}$=$(0,0,-V)$ is the settling velocity of the particles. Here $\mathbf{G}$=$(1,0,0)$ is the driving force of the flow, and $p$ is the pressure field that remains after removing the component that is hydrostatically balanced by the bulk density field.

The velocity scale is the average shear velocity, $u_{*,avg} = \left[ \left( \tau_x + \tau_y \right) / \left( 2 \rho_f \right) \right]^{1/2}$, where $\tau_x$ and $\tau_y$ are the top and bottom wall shear stresses, respectively, and $\rho_f$ is the fluid density. The channel half-height, $h$, is used as the length scale. The derived scale for time is $(h/u_{*,avg})^2$, and for pressure is $(\rho_f u_{*,avg}^2)$. The dimensionless numbers in (1)-(3) are the shear Reynolds number, the shear Richardson number and the Schmidt number defined as

$$\text{Re}_c = \frac{u_{*,avg} h}{v}, \quad \text{Re}_b = \frac{g R e^c c^b h}{u_{*,avg}^2}, \quad \text{Sc} = \frac{v}{\kappa} \quad (4)$$

respectively. Here $v$ is the kinematic viscosity of the fluid, $g$ is magnitude of gravity acceleration, $R = \rho_s / \rho_f - 1$ with $\rho_s$ the sediment particles density, $c^v$ the volume-averaged concentration, and $\kappa$ is a diffusivity for the sediment particles. The mathematical model includes also the dimensionless sediment settling velocity, $V$, as a parameter.

The mathematical framework employed in this work assumes smooth walls and does not provide an implicit mechanism for particles resuspension from the walls, i.e. a sediment particle at the wall does not feel any turbulence perturbation that could lead to its resuspension into the flow. The diffusion term in (3) is thus a surrogate for the details of this process and gives a mechanism to include particles resuspension from the walls in the mathematical model.

The dimensionless governing equations are solved using a de-aliased pseudospectral code (Canuto et al. 1988). Fourier expansions are employed for the flow variables in the horizontal directions ($x$-$y$). In the inhomogeneous vertical direction ($z$) a Chebyshev expansion is used with Gauss-Lobatto quadrature points. An operator splitting method is used to solve the momentum equation along with the incompressibility condition (see for example Brown et al. 2001). A low-storage mixed third order Runge-Kutta and Crank-Nicolson scheme is used for the temporal discretization of the advection-diffusion terms with pressure correction at the end of each stage. More details on the implementation of this numerical scheme can be found in Cortese and Balachandar (1995).

The dimensions of the channel have been selected based on the work of Moser et al. (1999). The length of the channel is $L = 4 \pi$, the width is $L_y = 3 \pi/4$, and the height is $L_z = 2$. The grid resolution used is $(N_x, N_y, N_z)$=$(128, 128, 129)$, and the non-linear terms are computed in a grid $(3N_x/2, 3N_y/2, N_z)$ in order to prevent aliasing errors. The top and bottom walls of the channel represent a smooth boundary to the flow. The sediment is assumed to be fine enough that the flow does not allow for net deposition, i.e. any particle that settles is instantly re-entrained into suspension. The conditions above are enforced by the following boundary conditions

$$\mathbf{u} = 0 \quad \text{at} \quad z = -1 \quad \text{and} \quad z = 1, \quad (5)$$

$$c V + \frac{1}{\text{Re}_c \text{Sc}} \frac{\partial c}{\partial z} = 0 \quad \text{at} \quad z = -1 \quad \text{and} \quad z = 1. \quad (6)$$

Along the horizontal directions periodic boundary conditions are applied for all variables. The integration of (3) in the volume with prescribed boundary conditions reduces to $dc/dt = 0$. Thus, as the flow evolves from the initial condition, the sediment particles are redistributed in the channel, but the total sediment load of the flow is maintained constant and equal to initial value $c^v$. 

Mean dimensionless variables are denoted by an overbar and are obtained by time-averaging instantaneous horizontally averaged variables. Perturbations from the mean are denoted by a prime. The dimensionless integration time employed in this work is $T=30$ for all the cases after the flow has achieved a statistically steady state. This time has been selected based on the work of Armenio and Sarkar (2002) and has been checked to be long enough for the accurate computation of first and second order statistics.

3 SELF-STRATIFICATION BY SUSPENDED SEDIMENT PARTICLES

In the problem studied in this work the stratification of the flow is established by the settling characteristic of the particles. The mean concentration profile develops as a consequence of the balance between the (downward) settling flux and the (upward) Reynolds flux of sediment particles. This work seeks to address the effect of settling velocity of the sediment particles on the flow for a given sediment load. The study is undertaken by varying systematically the sediment settling velocity while fixing the values of the remaining parameters. The shear Reynolds number is set to $Re_s=180$, the shear Richardson number to $Ri_s=18$ and the Schmidt number to $Sc=1$. This particular choice of parameter corresponds to a set used by Armenio and Sarkar (2002) in the study of stratification effects by scalars. Table 1 presents detailed information on the cases studied.

Table 1: cases studied in this work.

<table>
<thead>
<tr>
<th>Case</th>
<th>$V$</th>
<th>$z_{a,max}$</th>
<th>$z_{pyc}$</th>
<th>$\bar{c}_s$</th>
<th>$\bar{c}_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>SS1</td>
<td>$5 \times 10^{-3}$</td>
<td>0</td>
<td>0</td>
<td>0.874</td>
<td>1.143</td>
</tr>
<tr>
<td>SS2</td>
<td>$10^{-2}$</td>
<td>0</td>
<td>-0.025</td>
<td>0.727</td>
<td>1.370</td>
</tr>
<tr>
<td>SS3</td>
<td>$1.5 \times 10^{-2}$</td>
<td>-0.074</td>
<td>-0.098</td>
<td>0.577</td>
<td>1.803</td>
</tr>
<tr>
<td>SS4</td>
<td>$1.75 \times 10^{-2}$</td>
<td>-0.122</td>
<td>-0.195</td>
<td>0.516</td>
<td>2.228</td>
</tr>
<tr>
<td>SS5</td>
<td>$2 \times 10^{-2}$</td>
<td>-0.383</td>
<td>-</td>
<td>0.416</td>
<td>4.521</td>
</tr>
</tbody>
</table>

The settling velocity induces a vertical gradient of concentration and produces a stable stratification of the flow. The main effect of stable stratification is to suppress vertical momentum and mass transports which lead to less mixed mean profiles. The effect of stratification on the mean streamwise velocity can be seen in figure 1(a). Two main features can be observed in this figure with increasing stratification. The first one is the loss of symmetry by a gradual deviation of the velocity maximum toward the bottom wall with increasing values of $V$. The change is negligible for cases SS1 and SS2 corresponding to $V=5 \times 10^{-3}$ and $10^{-2}$, but clearly observed for cases SS3, SS4 and SS5 corresponding to $V=1.5 \times 10^{-2}$, $1.75 \times 10^{-2}$ and $2 \times 10^{-2}$, respectively. The locations of the velocity maxima are listed in table 1 as $z_{a,max}$. The second effect is the deviation from a flat, well-mixed velocity profile for case CF1 (the unstratified channel flow) to more curved, less-mixed velocity profiles with increasing values of $V$. It can also be observed in figure 1(a) that with increasing values of $V$ the velocity profiles near the bottom wall deviate from the sharper turbulent profile to a more rounded laminar-like profile. This is better seen in the inset frame in figure 1(a), which shows a zoom up of the profiles near the bottom wall. Cases SS1, SS2 do not show appreciable deviation from case CF1 while cases SS3, SS4 and SS5 clearly show less sharp gradients with increasing values of $V$. The spanwise and vertical mean velocities are nominally zero and are not shown here.

The effects of stratification are also apparent in the mean concentration profiles, shown in figure 1(b) for the same cases of figure 1(a). Case SS1 presents a well-mixed, nearly uniform concentration profile. Stratification manifests clearer for cases SS2, SS3, SS4 and SS5, which present larger concentration gradients. In all cases concentration increases towards the bottom wall as expected, and less-mixed profiles occur with increasing $V$. Table 1 presents the top and bottom wall mean concentrations for all cases as $\bar{c}_s$ and $\bar{c}_b$, respectively. Figure 1(b) shows clearly that the concentration profiles present a strong change of curvature with increasing $V$. The mixing induced by wall turbulence is inhibited by the flow stratification preferentially in the bottom half of the channel, which explains the less mixed profiles in this region. There is also an increase of the gradient of concentration at the bottom wall, which is consistent with the imposed boundary condition. In the top half, stratification plays a minor role and turbulence production at the wall is not inhibited by stratification as much as in the bottom half. For the cases SS1, SS2, SS3 and SS4 the concentration profiles develop a pycnocline in the central region of the channel. The pycnocline separates two well-defined layers in the flow that are mixed by wall-induced turbulence. The location of the pycnocline is reported in table 1 as $z_{pyc}$ and is marked in figure 1(c) by black dots. The result of sharpened concentration profiles between rather well-mixed layers is consistent with previous experimental observations (Moore and Long 1971, Crapper and Linden 1974) and numerical results.
Figure 1: frame (a) shows the mean streamwise velocity profiles, frame (b) shows the mean concentration profiles, and frame (c) shows the mean concentration profiles normalized by the bottom mean concentration.

With increasing values of \( V \) the pycnocline is displaced towards the bottom wall, and for case SS4 the bottom layer is barely defined. For case SS5 the pycnocline disappears and the concentration profile is well-mixed in the top region and strongly stratified in the bottom region.

Figure 2 shows the streamwise velocity profiles in wall units for cases CF1 and SS1 to SS5. Figure 2(a) shows the bottom half and figure 2(b) the top half. The corresponding shear velocities have been used in defining the wall variables in each half, that is \( u^{*}_{b} \) for the bottom half and \( u^{*}_{t} \) for the top half. The corresponding wall units are defined as \( u^{+} = u/u^{*}_{b} \) and \( z^{+} = z u^{*}_{b}/\nu \). The inner linear law is present in all the cases for both halves of the channel and does not show any appreciable modification with stratification. A logarithmic region exists for the case of CF1 from \( z^{+} \approx 30 \) to \( z^{+} \approx 150 \) for both halves of the channel as expected. The logarithmic region is also present for cases SS1 to SS5 in the top half of the channel and does not present any major change respect to the non-stratified case CF1. On the other hand, for the bottom half, the logarithmic region is present only for cases SS1 to SS4, and presents some variations with increasing stratification. The extent of the logarithmic region in the bottom half of the channel diminishes with increasing stratification as can be observed in figure 2(a). This owes mainly to the lowering of the velocity maximum. The slope of the velocity profile increases with stratification in the region of validity of the logarithmic law. The logarithmic law for smooth walls reads

\[
    u^{+} = \frac{1}{K_{b}} \ln(z_{b}^{+}) + B_{b}
\]  

(7)

where \( K_{b} \) and \( B_{b} \) are apparent parameters for the bottom half that are affected by stratification effects (\( K_{b} \) is the apparent von Karman constant). For unstratified flows these parameters take the classical values \( K_{b} = K = 0.41 \) and \( B_{b} = B = 5.5 \). Figure 2(a) shows that the validity of the logarithmic law holds for cases SS1 to SS4 by changing \( K_{b} \) and \( B_{b} \). As an example, the best fit corresponding to case SS4 is included in this figure (\( K_{b} = 0.21 \) and \( B_{b} = 1.25 \)). The increase of the slope is associated to a decrease of the apparent von Karman parameter \( K_{b} \), which can be defined as \( K_{b} = (du^{+}_{b}/dz_{b}^{+})^{-1} \) (Wright and Parker 2004). When stratification effects are important, the velocity profile in the logarithmic law region can be written as (Turner 1973)

\[
    u^{+} = \frac{1}{K} \ln(z_{b}^{+}) + B + \frac{\alpha z_{b}^{+}}{K L_{b}^{+}}
\]  

(8)
where \( L_b = u_+^3/\left(KgRe^2w^+\right) \) is the Monin-Obukov length scale and \( L_b^+ = L_b u_+/\nu \). The Monin-Obukov length scale is positive for stably stratified flows and \( L_b \rightarrow \infty \) as stratification becomes negligible. From (8) the apparent von Karman parameter can be calculated as

\[
K_b = \frac{K}{1 + \alpha \frac{z^+_b}{L_b^+}}. \tag{9}
\]

According to (9), a decrease of \( L_b \) predicts a decrease in \( K_b \). This trend is clearly seen in figure 3. Figure 3 shows the Monin-Obukov scale for the cases SS1 to SS4. The values of \( L_b^+ \) decrease very rapidly away from the wall and reach a uniform value which decreases with increasing values of \( V \), i.e. with increasing stratification effects.

4 CONCLUSIONS

This work presents the application of DNS to sediment-laden flows. The analysis is performed by systematically varying the settling velocity of the particles and focuses on the stratification effects on the velocity and concentration profiles. As a consequence of the settling of particles strong concentration (and thus mixture density) gradients develops, which damp turbulence preferentially near the bottom wall of the channel. The main effect observed is the modification of the logarithmic law of the wall, which still exists provided the flow remains turbulent. The logarithmic law presents increased slope which is associated to a decrease of the apparent von Karman constant. This effect is explained in terms of the variation of the Monin-Obukov scale in the region near the bottom wall. The DNS results are in agreement with previous experimental observations.

Figure 3: Monin-Obukov length for cases SS1 to SS4.

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