Direct numerical simulation of transitional Stokes boundary layer

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**ABSTRACT:** The Stokes boundary layer, where the fluid oscillates parallel to an infinite fixed wall, is studied using direct numerical simulation of the Navier-Stokes equations in the turbulent regime. Compared to previous works reported in the literature, the methodology and numerical technique employed in this work do not require any artificial mechanism to trigger the transition to turbulence and to avoid relaminarization. The simulations presented here were performed at \( \text{Re}_w \approx 5 \times 10^5 \). They compare well with the measurements of Jensen (1989). In this article, the evolution of the different terms of turbulent kinetic energy budget over the oscillation cycle is particularly addressed.

1 INTRODUCTION

The study of Stokes (oscillatory) boundary layers is of obvious importance in the study of coastal and ocean hydrodynamics and sediment transport. In a more general perspective, however, the oscillatory boundary layer represents a canonical case for the study of unsteady boundary layers that are found in diverse areas of fluid mechanics, from aircraft design to biomedical fluid mechanics.

Oscillatory boundary layers in coastal and oceanic environments are large Reynolds number flows in the transitional and fully turbulent regimes. The detailed understanding of turbulence processes in oscillatory boundary layers is of great importance for the study of the transport of sediments and mixing processes in these environments. Experimental works have reported detailed measurements of the vertical structure of the mean flow, turbulent kinetic energy, dissipation of turbulent kinetic energy and Reynolds stresses profiles (see for example Jensen et al. 1989).

Direct numerical simulation is a complementary tool for the analysis of transport processes in detail and for the visualization of turbulent structures (see for example Spalart and Baldwin 1987), which are extremely difficult to measure experimentally. Previous works on direct numerical simulation of the Stokes boundary layer used artificial mechanisms to trigger the transition to turbulence. This was necessary to avoid relaminarization of the boundary layer during the phases of negligible mean velocity due to excessive dissipation of the small scales. For example, Akhavan (1991) added random noise at the nodes of the numerical domain to seed the turbulence during the simulation. A different approach was used by Vittori (1998) and Tuzi (2008) who introduced small perturbations at the wall nodes to keep the boundary layer turbulent.

During the flow reversal, when the outer oscillation velocity becomes zero, the turbulent kinetic energy reaches values very close to zero and the excessive numerical diffusion of some numerical schemes further reduce the turbulence level relaminarizing the flow. The spectral method used herein (Cantero et al. 2007a,b) presents both very low numerical dispersion and diffusion. This makes the seeding of turbulence unnecessary for the oscillatory boundary layer problem. This work focuses on the evolution, along the oscillation cycle, of the turbulent kinetic energy budget of a Stokes boundary layer in the vicinity of a smooth wall.

2 MATHEMATICAL AND NUMERICAL MODELING

In this article the flow in a horizontal channel, where the flow is driven by an oscillatory mean pressure gradient in the longitudinal direction, is considered. The mean pressure gradient \( \mathbf{G} = \rho U_0 \sin(\omega t) \mathbf{i} \), where \( \rho \) is the fluid density; is associated with the
maximum orbital velocity outside the boundary layer $U_0 \cos(\omega t) \hat{i}$. The dimensionless form of the Navier-Stokes equations are obtained by employing the inverse of the angular frequency of the oscillation $1/\omega$ as time scale, the modulus of the maximum orbital velocity outside the boundary layer $U_0$ as velocity scale, $\rho U_0^2$ as pressure scale and the amplitude of the fluid excursion outside the boundary layer $A=U_0/\omega$ as length scale. Therefore, the dimensionless set of equations governing the flow takes the form

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re_w} \nabla^2 \mathbf{u} + \sin(t) \mathbf{i}$$ (1)

$$\nabla \cdot \mathbf{u} = 0,$$ (2)

where $\mathbf{u} = u\hat{i} + v\hat{j} + w\hat{k}$ is the dimensionless velocity vector, $p$ is the dimensionless dynamic pressure and $\sin(t) \mathbf{i}$ is the dimensionless driving force. The dimensionless parameter $Re_w$ in Equation (1) is the so-called wave Reynolds number defined as $Re_w = U_0 A/\nu$, with $\nu$ the dynamic fluid viscosity. The simulations presented herein were performed at $Re_w = 4.95 \times 10^5$.

The governing equations are solved using a dealiased pseudospectral code (Canuto et al., 1988). Fourier expansions are employed for the flow variables in the horizontal directions $(x,y)$. In the inhomogeneous vertical direction ($z$) a Chebyshev expansion is used with Gauss-Lobatto quadrature points. An operator splitting method is used to solve the momentum equation along with the incompressibility condition (see for example Brown et al., 2001). First, an advection-diffusion equation is solved to compute an intermediate velocity field. After this intermediate velocity field is computed, a Poisson equation is solved to compute the pressure field. Finally, a pressure correction step is performed to obtain the final incompressible velocity field. A low-storage mixed third order Runge-Kutta and Crank-Nicolson scheme is used for the temporal discretization of the advection-diffusion terms. More details of the implementation of this numerical scheme can be found in Cortese and Balachandar (1995). Validation of the code can be found in Cantero et al. (2007a, b).

The length of the channel is $L_x = 0.05$, the width $L_y = 0.025$ and the height is $L_z = 0.04$. The grid resolution used is $N_x = 96$ by $N_y = 48$ by $N_z = 96$ and the non-linear terms are computed in a grid $3N_x/2 \times 3N_y/2 \times N_z$ in order to prevent aliasing errors. The bottom wall represents a smooth no-slip boundary to the flow and the top wall is a free slip wall. Then, the boundary conditions employed are

$$\mathbf{u} = 0 \text{ at } z = 0 \text{ and }$$ (3)

$$\frac{\partial u}{\partial z} = 0, \quad \frac{\partial v}{\partial z} = 0 \text{ and } w = 0 \text{ at } z = L_z.$$ (4)

3 COMPARISON WITH EXPERIMENTAL DATA

The simulations presented herein were performed at $Re_w = 4.95 \times 10^5$, which coincides with the experimental conditions of to Jensen (1989) and Jensen et al. (1989), Test No 6. These experimental measurements were obtained in a U-shaped oscillatory-flow water tunnel using a 2 component Laser Doppler Anemometer.

Figures 1 and 2 present the experimental data together with the results of the numerical simulations. Averaged values from the numerical simulation were computed after a statistically steady state was reached. The averaging process consisted on phase-averaging over 39 oscillation cycles the spatially averaged fields in the spanwise and streamwise directions. In this work the overbar is used to indicate phase averaged mean values and the primes to indicate fluctuations around the phase averaged means. Figure 1 shows the mean streamwise velocity, $\bar{u}$, and Figure 2 shows the streamwise component of the Reynolds stress, $\bar{u}'w'$, for several phases of the oscillation cycle. The agreement with the experimental data is very good. It is worth mentioning that the agreement is particularly good for the Reynolds stresses considering the difficulties associated with their experimental estimation in the vicinity of a surface and the experimental running time needed to produce statistically significant quantities by phase averaging.

In Figure 1 the velocity and the distance to the wall are made dimensionless using the usual near-wall scaling for stationary boundary layers. For the experimental data the shear velocity, $u_\ast$, was obtained by Jensen (1989) by fitting a logarithmic law to the data. Second order turbulent statistics values present similar agreements to the results in Figure 2; they are not reported in this work.

4 RESULTS AND DISCUSSION

This section focuses on the near-wall flow turbulent statistics. A logarithmic scaling of the vertical coordinate is used in the contours displayed in Figures 3 to 8 in order to observe the flow details in
the near-wall region. The contour representation allows for an easier comparative analysis over the oscillation cycle of the different quantities involved. Figure (3) shows the mean dimensionless streamwise velocity as function of the dimensionless distance from the bed and oscillation phase. Maximum mean velocities are observed before the outer velocity maximum in the region above $\log_{10}(z/A) \approx -2.5$. In this region, as the distance from the wall is reduced, the velocity maximum occurs earlier in the oscillation cycle. As it can be seen in Figure 1, a behavior close to the logarithmic law of the wall is observed in this region. The earlier occurrence of the velocity maximum makes the maximum shear velocity to present a phase lead respect to the outer velocity maximum. Near $\log_{10}(z/A) \approx -2.5$ the phase of the maximum velocity quickly transitions and the velocity maximum in this near-wall region occurs slightly after the outer velocity maximum.

The dimensionless turbulent kinetic energy is shown in Figure 4. For locations close to the wall (below $\log_{10}(z/A) \approx -2.75$), the maximum occurs almost in phase with the mean flow. At locations around $\log_{10}(z/A) \approx -2.75$ the turbulent kinetic energy maximum starts to be delayed respect to the maximum mean flow velocity and the delay increases with the distance to the wall.

Note that

$$\frac{z}{\delta_s} = \frac{z}{A} \sqrt{\frac{Re_w}{2}},$$

with $\delta_s$ the Stokes-layer thickness $\delta_s = \sqrt{2\nu/\omega}$. Therefore, $\log_{10}(z/A) = -2.75$ corresponds to $z/\delta_s = 0.9$, showing that the change in behavior is associated with viscous effects. Note that the maximum in the turbulent kinetic energy is associated with the change in slope of the mean velocity profile for each phase.
Figure 3: Contours showing the mean dimensionless velocity.

Figure 4: Contours showing the dimensionless turbulent kinetic energy.

Figure 5: Contours showing the dimensionless production of turbulent kinetic energy.

Figure 6: Contours showing the dimensionless dissipation of turbulent kinetic energy.

Figure 7: Contours showing the local derivative of the turbulent kinetic energy.

Figure 8: Contours showing the transport of turbulent kinetic energy.
The peak value of the turbulent kinetic energy is observed below this point at \( \log(z/A) = -3.25 \), \( z/\delta_s = 0.3 \), and in phase with the outside maximum velocity. Note that the turbulent kinetic energy reaches values very close to zero during the flow reversal, when the velocity outside the boundary layer crosses zero. Excessive numerical diffusion during this phase will dissipate the small fluctuations that remain in the flow and flow re-laminarization may occur.

The turbulent kinetic energy budget can be expressed as

\[
\frac{\partial k}{\partial t} + \nabla \cdot \mathbf{T}' = P - \varepsilon
\]

where \( k \) is the turbulent kinetic energy, \( \frac{\partial k}{\partial t} \) is the mean-flow convection, \( \nabla \cdot \mathbf{T}' \) is the transport, \( P \) is the production, and \( \varepsilon \) is the dissipation. Figures 5 and 6 show the dimensionless production and dissipation of turbulent kinetic energy, respectively. The peak production occurs almost in phase with the outer velocity maximum at \( z/\delta_s = 0.3 \). The maximum dissipation occurs at the wall and peaks also in phase with the outer velocity maximum.

The mean flow convection reduces in the present case to \( \partial k/\partial t \) since \( \nabla k \) is perpendicular to the mean flow. Figure 7 displays the dimensionless value of \( \partial k/\partial t \), where the difficulty to obtain smooth high order phase average statistics can be observed. Combining the data in figures 5, 6 and 7 and using Equation (6), the transport \( \nabla \cdot \mathbf{T}' \) can be obtained, which is shown in Figure 8. From the combined inspection of Figures 4 to 8 it can be observed the importance of the transport \( \nabla \cdot \mathbf{T}' \), compared to the much more moderate importance of the local variations of the turbulent kinetic energy \( \partial k/\partial t \). Note that the local derivative is one order of magnitude smaller than any of the other terms in Equation (6).

In Figure 8 it can be seen that the transport of turbulent kinetic energy, over a portion of the boundary layer, is close to zero for an important part of the cycle. Over this range, which coincides with the zone were the log-law is observed in Figure 1, the production and dissipation balanced each other, as is the case of a stationary boundary layers. On the other hand, when the outside velocity is close to zero this balance breaks down, dissipation dominates and a logarithmic layer is not observed (see Figure 1).

During the velocity maxima, production and dissipation also peak. At this point, the transport of turbulent kinetic energy away from the production zone into the fluid column is expected to be dominated by turbulent convection. For phases close to the velocity maxima intense dissipation is also observed in the wall vicinity. In this case the turbulent kinetic energy is transported from the production zone into the dissipation zone near the wall. This transport is expected to be dominated by viscous diffusion.

5 CONCLUSIONS AND FUTURE WORK

This article presented direct numerical simulation of an oscillatory boundary layer in the turbulent regime. The numerical results present very good agreement with experimental data reported in the literature. The phase evolution of the turbulent kinetic energy balance over the oscillation cycle has been analyzed for oscillatory boundary layers using the direct numerical simulation results. The pseudo-spectral method used in this work was able to model the oscillatory boundary layer over a smooth wall without the need for any artificial mechanisms to trigger the transition to turbulence.

In the near feature it is planed to work on the des-aggregation of the turbulent kinetic energy transport term, in order to study the different transport mechanism separately. Additional future work includes the study of the evolution of the coherent structures in this type of flows, particularly the effect of unsteadiness on the formation, interaction and destruction of the coherent structures. The analysis and findings of this article shed light on the understanding of turbulence behavior of accelerating flows. One of the main uses, and long term goal, of this work is the improvement of the turbulence closure schemes for unsteady boundary layers needed for Reynolds Averaged simulations.

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