DIRECT NUMERICAL SIMULATION OF THERMOHALINE GRAVITY CURRENTS

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ABSTRACT

This work presents highly-resolved simulations of planar and cylindrical gravity currents. As the currents develop the interface between light and heavy fluids rolls up forming Kelvin-Helmholtz (KH) vortices. At the same time, incipient lobes and clefts start to form at the lower frontal region, which grow in size and extend to the upper part of the front destabilizing the KH rolls. Lobes and clefts continuously merge and split resulting in a complex pattern that evolves dynamically. The decay of the KH rolls produce regions of high turbulence in the flow. In the case of the planar flow the body of the current is populated by hairpin vortices while for the cylindrical flow this happens only in the near front region. Due to the high resolution of the simulations, we have been able to link the flow structures to local flow patterns and vortical entities.

1. INTRODUCTION

Density or gravity currents are flows that are driven by horizontal pressure gradients generated due to the action of gravity over two different fluids with density difference (Allen, 1985; Simpson, 1997). In many real applications and laboratory experiments the current is canalized and is confined to flow between parallel lateral walls. In such situations, if the viscous effects of the lateral walls can be ignored, the current moves as a statistically two-dimensional (2D) flow with a nominally planar front (planar current). There are a number of other applications, such as the release of heavy gas into an open space, the collapse of an axisymmetric volcanic plume, or a point discharge into a lake or ocean, in which the gravity current is not canalized and is allowed to spread out over the entire horizontal plane. In such situations, the current moves as a statistically axisymmetric flow with a nominally cylindrical front (cylindrical current). Many more examples of engineering, environmental and geological applications can be found in the books by Allen (1985) and Simpson (1997).

Planar and cylindrical gravity currents are two canonical configurations that have been studied in the past. The lock-exchange problem in a rectangular channel is a well-studied

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example of a planar gravity current, where the heavy fluid moves away from the lock with a nominally straight front. On the other hand, the release of a finite column of heavy fluid into a surrounding ambient of light fluid results in a cylindrical gravity current (Penney et al., 1952; Spicer and Havens, 1987). In a planar current the planform increases linearly with front location, while in a cylindrical current the planform increases quadratically. This fundamental difference dramatically decays the intensity of the cylindrical current as it evolves. On the other hand, in a cylindrical current the concentrated vorticity at the head of the current initially intensifies as the current flows out due to intense vortex stretching (Patterson et al., 2006). These differences contribute to a distinct dynamics for the cylindrical currents. There are instances of directed release, for example heavier fluid released from a finite opening down a flat slope, where the current does not spread all around, but forms a conical planform. Density currents in such sector-shaped geometric tanks are often studied in the laboratory, instead of a full cylindrical current, due to their simplicity.

Several studies have focused mainly on the spreading rate of the currents. Huppert and Simpson (1980) described the spreading of a gravity current in three phases: an initial *slumping phase* of nearly constant speed, followed by an *inertial phase* in which buoyancy balances inertial forces, and a final *viscous phase* where viscous effects dominate. More recently Shin et al. (2004) and Marino et al. (2005) have carefully described the dynamics of spreading for the lock-exchange problem of planar currents. For the case of cylindrical currents, the earliest work was reported by Martin and Moyce (1952). More recent works on cylindrical currents include those by Bonnecaze et al. (1995), Alahyari and Longmire (1996) and Patterson et al. (2006). Also, three-dimensional (3D) simulations have been reported lately for moderate Reynolds numbers \( Re \) (Härtel et al., 2000b; Cantero et al., 2006) for both planar and cylindrical currents.

In this work we concentrate on the structures present on the flow, such as the lobe and cleft instability at the front, Kelvin-Helmholtz (KH) billows present at the interface between light and heavy fluid and hairpin vortical structures present in the body of the current. We present direct numerical simulations of planar and cylindrical currents for a Reynolds number \( Re = 8950 \) which corresponds to a Grashof number of \( Gr = Re^2/8 = 10^7 \), the dimensionless parameter used by (Härtel et al., 2000b).

2. MATHEMATICAL AND NUMERICAL MODEL

The physical configuration of the gravity currents to be considered here is shown in figure 1. At the start of the computation the region with heavy fluid of density \( \rho_1 \) (shown in figure 1 as the shaded region) is separated from the light fluid of density \( \rho_0 \). In the planar case, the heavy fluid is a slab of half width \( x_0 \) along the flow direction. In the present simulations the slab of heavy fluid extends over the entire height \( H \) of the channel (full-depth release) and infinitely along the spanwise \((y)\) direction. In the cylindrical case, the region of heavy fluid is a cylinder of radius \( r_0 \) and height \( H \).

We consider flows in which the density difference is small enough that the Boussinesq approximation is valid. The dimensionless equations read

\[
\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_i}{\partial x_k} = \frac{\partial \tilde{p}}{\partial \tilde{x}_i} - \rho \tilde{\alpha}_i - \frac{1}{Re} \frac{\partial^2 \tilde{u}_i}{\partial \tilde{x}_k \partial \tilde{x}_k}, \tag{1}
\]

\[
\frac{\partial \tilde{u}_k}{\partial \tilde{x}_k} = 0, \tag{2}
\]

\[
\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial}{\partial \tilde{x}_k} (\tilde{\rho} \tilde{u}_k) = \frac{1}{ScRe} \frac{\partial^2 \tilde{\rho}}{\partial \tilde{x}_k \partial \tilde{x}_k}. \tag{3}
\]
Figure 1: Sketch of a gravity current and nomenclature used in this work. The flow is started from the initial condition shown by the shaded region between dashed lines. As the flow evolves, the intruding front develops the structure of a head followed by a body.

Here \( \tilde{u}_i \) is the dimensionless velocity vector, and \( \mathbf{e}_i^g \) is a unit vector pointing in the gravity direction. Any variable with a tilde on top is to be understood as dimensionless.

The channel height \( H \) is the length scale and we adopt \( U = \sqrt{g' H} \) as the velocity scale, where \( g' = (\rho_1 - \rho_0)/\rho_0 \). Consequently, the time scale is \( H/U \). The dimensionless density and pressure are given by

\[
\tilde{\rho} = \frac{\rho - \rho_0}{\rho_1 - \rho_0}, \quad \text{and} \quad \tilde{p} = \frac{p}{\rho_0 U^2}.
\]

The two dimensionless parameters in equations (1)–(3) are the Reynolds and Schmidt numbers defined as

\[
Re = \frac{U H}{\nu} = \sqrt{\frac{g' H^3}{\nu^2}} \quad \text{and} \quad Sc = \frac{\nu}{\kappa},
\]

respectively, where \( \nu \) is the kinematic viscosity and \( \kappa \) is the diffusivity of temperature or chemical species producing the density difference. The ratios \( x_0/H \) (planar case) or \( r_0/H \) (cylindrical case) are additional geometric parameters that control the volume of initial release. In this work we consider the cases of \( \tilde{x}_0 = 1 \) and \( \tilde{r}_0 = 1 \).

The governing equations are solved in a rectangular box of size \( \bar{L}_x \times \bar{L}_y \times \bar{L}_z \). Periodic boundary conditions are employed along the streamwise (\( \bar{x} \)) and spanwise (\( \bar{y} \)) directions. The box is typically taken to be very long along the streamwise direction (25 channel heights for the planar case and 15 channel heights for the cylindrical case) in order to allow free unhindered development of the current for a long time as it flows out from the center of the computational box (see figure 1). Based on simulation results we observe that the interaction of the front with the adjacent currents across the periodic boundaries can be neglected till the front reaches about 1 dimensionless unit from the boundaries. For the planar case the width of the periodic domain is chosen to be 1.5 dimensionless units along the spanwise direction, which is adequate to include several spanwise lobe and cleft structures. These choices for the computational domain are consistent with that of HärTEL et al. (2000b). For the cylindrical case \( \bar{L}_x = \bar{L}_y \).

Fourier expansions are employed for the flow variables along the horizontal directions (\( \bar{x} \) and \( \bar{y} \)). In the inhomogeneous vertical direction (\( \bar{z} \)) a Chebyshev expansion is used with Gauss-Lobatto quadrature points (Canuto et al., 1988). The flow field is time advanced using a Crank-Nicolson scheme for the diffusion terms. The advection terms are handled with the Arakawa method (Durran, 1999) and advanced with a third-order Runge-Kutta scheme. The buoyancy term is also advanced with a third-order Runge-Kutta scheme. More details on the implementation of this numerical scheme can be found in Cortese and Balachandar (1995). In the simulations, periodic boundary conditions are enforced along the horizontal directions for all variables. At the top and bottom walls no-slip and zero-gradient conditions are enforced for
Figure 2: Flow evolution during the early stage of development for the planar current. The dashed line shows the interface between the heavy and light fluids visualized by the span-averaged density contour $\tilde{\rho} = 0.5$. Solid lines represent contours of span-averaged spanwise vorticity. The front accelerates from rest and the roll-up of the interface begins and continues to develop till $\tilde{t} = 2.5$ ($x_F - x_0 \approx 1$).

velocity and density, respectively.

In both configurations the initial density was smoothly varied from 0 to 1 over a very thin region located at the interface. The details of the initial conditions used in this work can be found in Cantero et al. (2006). The flow was started from rest with a minute random disturbance prescribed in the density field.

In this work a value of $Re = 8950$ and $Sc = 1$ are set for the simulations. This $Re$ corresponds to $Gr = 10^7$, the dimensionless parameter used by (Härtel et al., 2000b). The grid resolution employed for the simulations are $1536 \times 112 \times 180$ and $880 \times 880 \times 180$ and thus they involve approximately 31 and 140 million grid points for the planar and cylindrical simulations, respectively. The numerical resolution for each simulation was selected to have between 6 and 8 decades of decay in the energy spectrum for all the variables. The time step was selected to produce a Courant number smaller than 0.5. The cylindrical simulation required about 1 month of continuous run on 64 processors of the new SGI Altix 3000 (put to production on April 2005 at NCSA, University of Illinois at Urbana-Champaign), about 70Gb of RAM memory to run, 600Gb of storage for raw data, and 18Tb (18000 Gb) of storage for visualization postprocessing made by NCSA scientists.
Figure 3: Flow structure of the planar current at $\tilde{t} = 21.2$. Frame (a): isosurface of $\tilde{\rho} = 0.05$, and frame (b): isosurface of $\tilde{\chi}_{ci} = 2.12$. At this time the current shows a rather complex state of three-dimensionality. In frame (b) several hairpin vortices are pointed with arrows to help their visualization.

3. RESULTS AND DISCUSSION

3.1. Planar current

After the release of the denser fluid, an intrusive front forms in both configurations geometrical settings. Initially, the flow evolves in an two-dimensional or axisymmetric fashion in which Kelvin-Helmholtz rolls develop and form the front and the nose. Figure 2 shows the front at several time instances marked by the contour of span-averaged $\tilde{\rho} = 0.5$ plotted along with span-averaged spanwise vorticity contours for the planar case. Below the nose, which is raised from the bottom, an unstable stratified region forms as a consequence of the no-slip condition. In this region, three-dimensional instabilities develop and evolve into a lobe and cleft pattern in the foremost part of the current. This feature has been observed in experiments for both planar Simpson (1972) and cylindrical currents Spicer and Havens (1987). Behind the front, the flow develops into a very intense three-dimensional structure where the KH billows shed from the front deform, bend, and break up. This region is the so-called body of the current.

The structure of the advancing planar front can be seen in figure 3(a), which shows the flow at $\tilde{t} = 21.2$ visualized by a surface of $\tilde{\rho} = 0.05$. This figure shows clearly the structure of head and body described above. As it is explained below, the head and body of the current are much more complex and 3D than what it is shown in figure 3(a). The undulations provide clear indication of KH instability and the presence of periodic train of rolled-up vortices, which appear to be bent, stretched, and eventually broken up into smaller scale structures. These small structures can be observed in the body of the current behind the leading front giving the appearance of a turbulent wake that eventually dissipates toward the tail of the current. The
Figure 4: Structure the the front of a particulate gravity current from a laboratory experiment. The flow presents strong vortex shedding and clearly shows channel-size structures populated by smaller scale vortices.

turbulent 3D structure of the body of the current have been observed and well documented in numerous experiments (see for example Simpson and Britter, 1979; García and Parker, 1989; Alahyari and Longmire, 1996; García and Parsons, 1996; Simpson, 1997; Parsons and García, 1998; Thomas et al., 2003). For example, figure 4 shows the front of a particulate gravity current from a laboratory experiment, which presents strong vortex shedding and clearly shows channel-size structures populated by smaller scale vortices.

The complex 3D vortical structure of the wake is not entirely apparent in the density isosurface presented in figure 3(a). The corresponding isosurface of swirling strength is shown in figure 3(b). Here the swirling strength, $\lambda_{ci}$, is defined as the absolute value of the imaginary portion of the complex eigenvalues of the local velocity gradient tensor.\(^4\) As discussed in Zhou et al. (1999) and Chakraborty et al. (2005) the swirling strength provides a clean measure of the compact vortical structures of the flow. Swirling strength picks out regions of intense vorticity, but discriminates against planar shear layers, where vorticity is balanced by strain-rate. Thus, as can be seen from figure 3(b), the 3D vortical structure of the planar current is well extracted by $\lambda_{ci}$. At the time instance shown the mean and rms values of dimensionless $\lambda_{ci}$ are 0.16 and 0.57, respectively. In figure 3(b) the isosurface of $\lambda_{ci} = 2.12$ is plotted and thus it captures intense vortical regions. The flow is dominated by inclined vortical structures and several hairpin vortices can be observed (indicated by arrows). These structures are similar to those observed in a turbulent wall layer, where the vortical structures are tilted from the wall in the flow direction. The gravity current shown in figure 3 flows to the right. In a frame of reference moving with the front, the flow within the current can be from right to left which explain the observed orientation of the vortical structures within the current. The net effect of the vortical structures on the concentration (density) field is seen in figure 3. The isosurface of swirling strength confirms what was observed earlier in the density isosurface, that is the body of the current is far more 3D than the head.

\(^4\)The local velocity gradient tensor has 3 eigenvalues. If all three eigenvalues are real then swirling strength is zero. If only one eigenvalue is real, then the other two are complex conjugates and there is local swirling motion.
Figure 5: Frame (a): near-bed flow pattern at the front of the planar gravity current for $\tilde{t} = 21.2$ when the front is located at $\tilde{x} \approx 9.5$. Vectors show the horizontal flow at $\tilde{z} = 0.03$. Thin line contours show vertical flow velocity at the same height, solid line for positive vertical velocity and dash line for negative vertical velocity. The thick solid line indicate the front location visualized by a bottom density contour of $\tilde{\rho} = 0.01$. Frame (b): time evolution of lobe and cleft pattern in the planar current visualized by contours of constant density ($\tilde{\rho} = 0.01$). The time separation between contours is $\Delta \tilde{t} = 0.014$. Dotted lines mark the transition locations between phases of spreading. Frame (c): lobe size as a function of local Reynolds number $Re_F = Re \bar{\nu}_F \bar{h}_H$. The figure includes our results from the planar current for $Re = 3450$ (closed diamonds), $Re = 8950$ (closed circles), and from experimental data by Simpson (1972) (open squares). The line is the empirical prediction by Simpson (1972): $\tilde{\lambda}/\bar{h}_H = 7.4 Re_F^{0.39}$. 
The front of the current does not advance forward as one fixed entity. The propagation of the front varies along the span (or along the circumferential direction) and this variation initially leads to the formation of lobes and clefts (Simpson, 1972). Figure 5(a) shows the flow at the front of the planar current at $t = 21.2$. In this figure, the location of the front is visualized by a thick solid line contour of $\bar{\rho} = 0.01$, the horizontal flow is visualized by vectors and the vertical flow is visualized by thin line contours (solid line contours correspond to positive vertical velocity and dash line contours to negative vertical velocity). The horizontal flow tends to diverge from the center of the lobes and to concentrate in the clefts. Also the near-bed flow moves upward in the clefts and downward in the lobes. The spanwise variation in front propagation continues after the initial formation of lobes and clefts and, as a result, the number and location of lobes and clefts constantly rearrange along the front. Figure 5(b) shows a composite picture of the front plotted on the $\tilde{x} - \tilde{y}$ plane (top view) with several equispaced time intervals superposed for the planar current. At the beginning (toward the left end of the plot) the front is nearly flat, but small random disturbances introduced in the initial condition quickly develop into well formed lobe and cleft structures. This figure illustrates the footprint of clefts on the horizontal $\tilde{x} - \tilde{y}$ plane as they advance over time. A complex pattern is etched by the clefts as the front advances, with repeated merger of the clefts and splitting of the lobes. With increasing time the instantaneous Reynolds number of the flow, $Re_F = u_F H/h = \Re \bar{u}_F \bar{h}_H$, decreases and this decrease in $Re_F$ has the dominant influence on the increase in the length scale of the lobe and cleft pattern. Figure 5(c) shows the normalized lobe size, $\lambda/h$, as a function of $Re_F$ for planar currents at $Re = 3450$ and $Re = 8950$. Also in the figure are the experimental data presented by Simpson (1972) and his empirical prediction $\lambda/h = 7.4 Re_F^{-0.39}$. The numerical results present very good agreement with the experimental data. This $Re$ effect on the wavelength of the lobe and cleft pattern is in agreement with the results on the most unstable mode from the linear stability analysis of HärTEL et al. (2000a).

3.2. Cylindrical current

Figure 6 shows the time development of the flow structure for the cylindrical current. Initially, the flow evolves nearly axisymmetric in which KH rolls develop and form along the front and body of the current (Cantero et al., 2006). In addition to the initial vortex ring A1, two additional anti-clockwise vortex rings A2 and A3 can be observed upstream. Also, as a consequence of the bottom no-slip boundary condition, the strong anti-clockwise vortex ring A1, results in the formation of a clockwise rotating vortex ring C1 closer to the bottom wall. The presence of counter-rotating vortices has been observed in cylindrical gravity currents in the experiments of Alahyari and Longmire (1996). As explained below, the net effect of C1 is the formation of a new anti-clockwise vortex ring A5 downstream of A1.

The roll up process of the interface can be better observed from circumferentially-averaged variable plots. In figure 7 circumferentially-averaged velocity vector plots are presented at several time instances. To allow better visualization of the rolled up vortices, the velocity field is plotted in a frame of reference moving with the front of the current together with contours of constant density. The interface between the heavy and light fluids is taken to be marked by $\bar{\rho} = 0.15$. At the earliest time shown ($\tilde{t} = 1.77$) only the incipient roll up of the anti-clockwise vortex ring A1 can be seen. By $\tilde{t} = 2.7$ the back propagating front has almost reached the axis and in addition to the initial vortex ring A1, two additional anti-clockwise vortex rings A2 and A3 can be observed upstream (see also figure 6). Also, as a consequence of the bottom no-slip boundary condition, the strong anti-clockwise vortex ring A1, results in the formation of a clockwise rotating vortex ring C1 closer to the bottom wall. At $\tilde{t} = 3.54$ the pair of vortex
Figure 6: Flow visualized for the cylindrical current by isosurface of $\tilde{\rho} = 0.15$. The figure shows only one quadrant of the computational domain. Initially, the flow evolves axisymmetrically. KH vortex rings develop forming the front and body of the current marked A1, A2, A3 and A4 on the $\tilde{x} - \tilde{z}$ plane. Later, three-dimensionality develops and destabilize the vortex rings in the azimuthal direction which decay to smaller scale turbulence. Eventually, the KH vortex decay and the body of the current becomes a calm region where most of the flow manifest as interface waves. The front instabilities grow very rapidly with time and form a pattern of lobes and clefts. The insets show a front view of the flow.
Figure 7: Flow field visualized by circumferentially-averaged velocity vectors. To allow for better vortex visualization, the nose velocity has been subtracted in each frame. The current interface is visualized by the contour of $\tilde{\rho} = 0.15$. Density contours are also shown to help visualize the current structure.
rings A1 and C1 form the head of the current. Vortex rings A2 and A3 have merged and have resulted in the formation of the clockwise vortex ring C2. A new rolled up anti-clockwise vortex A4 and the associated clockwise vortex ring C3 can be seen as well. The resulting vortex ring structure and the interface can be verified in figure 6.

By $\tilde{t} = 5.31$ an important development has occurred. The effect of C1 is to lift the dominant vortex ring A1 further away from the bottom wall. This lift up lowers the outward propagation velocity of the vortex ring. The front thus advances faster than the vortex ring A1 and results in the formation of a new anti-clockwise vortex ring A5, downstream of A1. For this Re the formation of A5 occurs at $\tilde{t} \approx 4.25$ when the current is located at $\tilde{r} \approx 2.6$. The coherence of the KH vortices is maintained until $\tilde{t} \approx 4.5$ when the current is located at $\tilde{r} \approx 2.7$. At this time instabilities grow and destabilize the Kelvin-Helmholtz rings A1, A2+A3 and A4 which suffer strong stretching and bending in the azimuthal direction. By $\tilde{t} = 7$ in figure 6 the vortex rings have lost the azimuthal coherence and present a wide range of scales.

By $\tilde{t} = 7$ a fully developed pattern of lobes and clefts can be observed at the front in figure 6, but the vortex rings have decayed to smaller and weaker vortices. Despite the decay, the lobe and cleft pattern is still present. The level of turbulence has decayed substantially and only the front of the current presents some vortical structures. The body of the current has become a relatively calm region, where most of the flow features manifest as interface waves. The complex 3D vortical structure of the current is not entirely apparent in the density isosurface presented in figure 6. The corresponding isosurface of *swirling strength* at the later two times is shown in figure 8. As can be seen from this figure, the 3D vortical structure is well extracted by $\tilde{\lambda}_{ci}$. The root mean square swirling strength within the vortical region at the two different times are 1.05 and 0.93, respectively. The isosurface of $\tilde{\lambda}_{ci} = 2.12$ is plotted in figure 8, thus the
figure captures only the intense vortical regions of the flow for the times displayed. At the first instance shown in figure 8, the strong vortical structures extracted by $\lambda_{ci}$ near the head of the current are associated with the KH vortex rings A1 and A5. The effect of lobes and clefts on the vortical structure can be clearly observed. Two other rings of intense turbulence can be seen. The one centered around $\tilde{r} \approx 1.75$ is associated with the merged vortex ring A2+A3. The final weaker ring of turbulent structure is associated with vortex ring A4. By $t = 14$, all except the vortex ring associated with the front of the current have already decayed and any turbulence associated with them are no longer captured by $\lambda_{ci} = 2.12$.

Apart from the vortex rings and the lobe and cleft structure the turbulent region of the flow behind the head is dominated by inclined vortical structures and several hairpin vortices can be observed (see for example inset for $\tilde{t} = 14$). These structures are similar to those observer in a turbulent wall layer, where the vortical structures are tilted from the wall in the flow direction. In case of cylindrical gravity current the flow is directed radially out. However, in a frame of reference moving with the front, the flow within the current is radially inward, which explain the observed orientation of the vortical structures within the current. Similar train of inclined vortical structures were observed for the case of planar gravity currents as well. The difference, however, is that at the present $Re$ in the planar current the region of turbulence extended over a large portion of the body of the current, while in the cylindrical case the turbulence is limited to only vortex rings and the head of the current. The net effect of the vortical structures on the concentration (density) field is seen in figure 6.

Figure 9(a) shows in details the lobe and cleft pattern observed in experiments performed at $Re = 8950$ visualized by potassium permanganate. The figure shows the front of the current when it is located at $\tilde{r} \approx 6.5$. At this late time the concentration of potassium permanganate at the front has decreased enough to permit clear visualization of the lobes and clefts in the flow. In the photographs, long streaks of fresh clear water trapped between the bottom of the tank and the current can be clearly identified. These streaks mark the path traversed by the clefts and has been demarcated in the figure with dashed lines to help their visualization. The photograph not only provides information on the instantaneous structure of the front, but also captures the path etched by the clefts in the recent past.

The 3D lobe and cleft structure of the advancing front can be seen in figure 6(a). The circumferential variation in front propagation continues after the initial formation of lobes and clefts and, as a result, the number and location of lobes and clefts constantly rearrange along the front. The front of the current identified by contour of $\tilde{\rho} = 0.015$ at the bottom boundary is plotted on the $\tilde{x} - \tilde{y}$ plane (top view) in figure 9(b). The front location at several equispaced time intervals of $\Delta \tilde{t} = 0.354$ are superposed. The composite picture provides a clear view of the formation of lobes and lefts and the footprint the clefts leave on the horizontal $\tilde{x} - \tilde{y}$ plane as the front advances over time. At the beginning (toward the center of the figure) the front is nearly axisymmetric, but small random disturbances introduced in the initial condition quickly develop into well formed lobe-and-cleft structures. Different instability mechanisms for the formation of lobe and cleft structure of the front have been proposed in the context of planar currents (Allen, 1971; Simpson, 1972; Härte, et al., 2000a). These mechanisms are likely to be active and responsible for the lobe and cleft structure of the cylindrical currents as well. Nevertheless, the formation of lobes and clefts is due to circumferential variation in the speed of the current. Even after they are fully formed, the speed of the current continues to vary along the circumference of the front, thus resulting in repeated splitting and merging of existing lobes and clefts. A complex pattern is etched by the clefts as the front advances, with repeated formation of new ones and merger between older ones, which is well captured in figure 9. The simulations results can be compared with the experimental photograph presented in figure 9(a) and the qualitative
Figure 9: Frame (a): dye visualization of lobes and cleft from an experiment with $Re = 8950$ and $Sc = 700$. At this time the dye concentration at the front has decreased enough to allow the visualization of cleft by streaks of fresh clear water trapped between the bottom boundary and the current in the near-front region. These streaks has been demarcated in the figure with dashed lines to help their visualization. Two merger of clefts (marked with arrows) can be observed in the figure. Frame (b): composite picture of the front location over time. Front visualized by bottom contours of $\tilde{\rho} = 0.015$ from the numerical simulation for $Re = 8950$ and $Sc = 1$. The time separation between contour is $\Delta \tilde{t} = 0.354$. The details show several lobe splitting and merger.
features are in agreement. For example, what appears to be initiation of new clefts and merges between existing clefts (marked with arrows) can be observed in the figure.

4. CONCLUSIONS

We have presented direct numerical simulations of planar and cylindrical gravity currents at $Re = 8950$. The particular choice of $Re = 8950$ corresponds to a Grashof number $Gr = 10^7$. This highly accurate numerical formulation allows the capture of all relevant length scales present in the flow.

The objective of the study is to examine the structure of planar and cylindrical gravity currents. For both geometrical settings, as the front spreads, a shear layer forms between the heavy forward advancing and the light backward retreating fronts. As the interface develops between the two fronts, it rolls up forming KH vortices.

Regions of high turbulence in the cylindrical current are associated to KH vortices decay. These regions are populated with trains of hairpin vortices tilted toward the axis. Similarly, the body of the planar current is populated with hairpin vortices, however, in this case the hairpin vortices are distributed evenly all over the head and body of the current.

For the two geometrical settings a clear pattern of lobes and clefts develops. The minute perturbations in the initial condition grow very fast, originally at the lower part of the leading front, but very rapidly extend to the upper and rear part of the front. After lobes and clefts are formed, they evolve very dynamically presenting merging of clefts and splitting of lobes into new ones. Also for both geometrical settings, the wavelength of the lobes grows with time as the front spreads and the local Reynolds number of the flow decreases. This is consistent with previous studies on planar currents that show that the most unstable wavelength decreases with the Reynolds of the flow (Härtel et al., 2000a). However, for cylindrical currents the number of lobes is maintained over time and the increase of wavelength is associated to the increase of circumferential length of the current.

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