Influence of particle inertia and settling on turbidity currents

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Abstract

In this work we address the effect of particle inertia on turbidity currents. We employ a two-fluid model based on the equilibrium Eulerian approach (Ferry and Balachandar, 2001) and present highly resolved two-dimensional (2D) simulations of the flow for a Reynolds number of Re = 3450 (this particular choice corresponds to a value of Grashof number of Gr = Re²/8 = 1.5 × 10⁶). The simulations capture physical aspects of two-phase flows, such as particle preferential concentration and particle migration down turbulence gradients (turbophoresis), which modify substantially the structure and dynamics of the flow. In the simulations we observe the migration of particles from the core of Kelvin-Helmholtz vortices shed from the front of the current as well as their accumulation in the current head. This redistribution of particles in the current affects the flow structure, deposition rate and deposition patterns.

1. Introduction

Turbidity currents are flows driven by the action of gravity on suspended sediment particles that induce an excess fractional density with respect to the surrounding ambient fluid. The dynamics of turbidity currents is relevant for dusty thunderstorm fronts, pyroclastic flows produced in volcano eruptions, aerosol releases in the environment, flows originated by the discharge of a sediment-laden river into the ocean and snow avalanches. Many more examples can be found in the book by Simpson (1997).

Turbidity currents have been studied extensively in the past. Depositional turbidity currents have been studied by García (1994) in the context of poorly-shorted sediment. Bonnecaze et al. (1993) have also studied depositional density currents for different geometrical settings. Shallow water equations models have been used to study the currents dynamics and deposition patterns (Bonnecaze et al., 1993; Choi and García, 1995). Recently, highly resolved simulations (Necker et al., 2002) have been performed for particle-driven gravity currents. In these works, the particles are moved with a velocity field that differs from the fluid velocity field by a constant settling velocity (in the direction of gravity).

Several geological phenomena involve the transport of coarse particles by turbidity currents. In these cases not only particle settling is important, but also particle inertia may play an important role. In this work, we study the effect of particle inertia on the dynamics of turbidity currents. We focus on the flow structure and deposition patterns. The analysis is carried out by two-dimensional (2D) highly resolved simulations of the Navier-Stokes equations with an inertia correction term for the particles velocity field based on the equilibrium Eulerian approach of Ferry and Balachandar (2001).
2. Mathematical and numerical model

We consider the lock-exchange configuration in a channel. The channel is filled at one end with a sediment-fluid mixture separated by a gate from the rest of the channel, which is filled with clear fluid. When the simulation begins the gate is lifted and the flow develops forming an underflow intrusion of the mixture into the clear fluid. Our interest is in simulating buoyant flows driven by the presence of solid particles of finite inertia. In this situation particles not only modify the bulk density but also move with their own velocity. In this work we use an Eulerian-Eulerian model based on an asymptotic expansion of the two-phase flows equations (Zhang and Prosperetti, 1997) in parameters describing the particle inertia, $\tau$, and the particle volumetric concentration, $\phi_d$. To approximate the particles velocity field we use the equilibrium Eulerian model (Ferry and Balachandar, 2001) which include settling and inertial effects. (Cantero et al., 2004).

Let the indices $c$ and $d$ denote the continuum and disperse phases, respectively. We denote the densities and velocities of each phase by $\rho_c$, $u_c$, and $\rho_d$, $u_d$, respectively. Let the height of the channel, $H$, be the length scale, $U = \sqrt{g \Phi H}$ be the velocity scale, and the initial particle volume fraction, $\Phi$, be the disperse phase volumetric concentration scale. Here $R = (\rho_d - \rho_c)/\rho_c$ is the submerged specific gravity of the particles and $g$ is the magnitude of the gravitational acceleration. Then, time and pressure scales are $H/U$ and $\rho_c U^2$, respectively. The dimensionless equations of the model are

\[
\frac{D_c \tilde{u}_c}{Dt} = \tilde{\phi}_d \mathbf{e}^g - \nabla p + \frac{1}{Re} \nabla^2 \tilde{u}_c
\]
\[
\nabla \tilde{u}_c = 0
\]
\[
\tilde{u}_d = \tilde{u}_c + \tilde{V}_s \mathbf{e}^g - \tau \frac{D_c \tilde{u}_c}{Dt}
\]
\[
\frac{\partial \tilde{\phi}_d}{\partial t} + \nabla \left( \tilde{\phi}_d \tilde{u}_d \right) = \frac{1}{Sc Re} \nabla^2 \tilde{\phi}_d.
\]

Here $D_c \cdot /Dt$ indicates material derivative following the continuous phase velocity, and $\mathbf{e}^g$ is a unit vector pointing in the direction of gravity. The dimensionless parameter that characterizes the strength of the current is the Reynolds number, defined as $Re = U H / \nu_c$, where $\nu_c$ is the continuous phase momentum diffusivity. Two controlling parameters define the individual suspended particles in terms of particle Stokes number, $\tilde{\tau} = \tau U/H$, and dimensionless settling velocity, $\tilde{V}_s = V_s/U$. These parameters characterize the inertial and settling effects of the particle, respectively. The particle response time is defined as $\tau = (d^2(\rho_d/\rho_c + 1/2))/(18\nu_c)$, where $d$ is the particle diameter. The settling velocity is the free-fall velocity in quiescent fluid. The Schmidt number is defined as $Sc = \kappa/\nu_c$, where $\kappa$ is the particle diffusivity. Particle diffusivity is a way to account for the departure in particle motion from equilibrium prediction which arise from close interaction of particles. On the other hand, solution of equation (4) with little or no diffusion is numerically unstable. Based on numerical considerations we simply chose $Sc$ to be order 1 (Härtel et al., 2000; Cantero et al., 2006a).

The dimensionless governing equations are solved using a de-aliased pseudospectral code. Fourier expansions are employed for the flow variables in the horizontal direction ($x$). In the inhomogeneous vertical direction ($z$) a Chebyshev expansion is used with Gauss-Lobatto quadrature points. A third order Runge-Kutta and Crank-Nicolson scheme is used for the temporal discretization of the advection-diffusion terms. More details on the
implementation of this numerical scheme can be found in Cantero et al. (2006a). The computational domain is a box of size \( L_x = 25 \times L_z = 1 \), which extends from \( \hat{x} = -12.5 \) to \( \hat{x} = 12.5 \) and from \( \hat{z} = 0 \) to \( \hat{z} = 1 \). The flow is initialized from rest with \( \hat{\phi}_d = 1 \) in \( \hat{x} \in (-1,1) \) for all \( \hat{z} \), and \( \hat{\phi}_d = 0 \) otherwise with a smooth transition. The solution was advanced in time until the front reached location of \( \hat{x} = 11.5 \) to avoid the influence of finite domain size (Härtel et al., 2000; Cantero et al., 2006a,b). The simulations were performed using a resolution of \( N_x = 1536 \times N_z = 150 \).

Periodic boundary conditions are enforced for all the variables in the horizontal direction. At the top and bottom walls no-slip conditions are enforced for the continuous phase velocity. Zero particle net flux is set at the top wall and zero particle resuspension is set at the bottom wall.

In this work, we present 2D direct numerical simulations for \( Re = 3450 \). This particular choice corresponds to a value of Grashof number \( Gr = g \Phi RH^3/\nu_c^2 = 1.5 \times 10^6 \), the dimensionless parameter used by Necker et al. (2002). We concentrate on the effect of particle inertia on the flow structure, deposition rate and deposition patterns.

3. Results and discussion

Figures 1 and 2 show the flow structure for the simulation with \( \tilde{V}_s = 0.005 \) for two values of \( \tau = 0 \) and 0.025, respectively. Two different types of contours are shown in this figure: solid line contours correspond to \( \hat{\phi}_d \leq 1 \), and dash line contours correspond to \( \hat{\phi}_d \geq 1 \). Two main differences are observed. The first one is the migration of particles away from the core of Kelvin-Helmholtz vortices, and the second one is the accumulation of particles in the front of the current, producing regions of particle concentration greater than the initial value (visualized in these figures by dash line contours). These two effects can be explained by noting that the divergence of the particles velocity field is

\[
\nabla \cdot \tilde{\mathbf{u}}_d = \tilde{\tau} \left( \| \Omega \|^2 - \| S \|^2 \right)
\]

where \( S \) and \( \Omega \) are the symmetric and skew-symmetric parts of the local fluid velocity gradient tensor. Note that \( \nabla \cdot \tilde{\mathbf{u}}_d > 0 \) when \( \| \Omega \| > \| S \| \), which means that particles migrate from regions of vorticity and accumulate in regions of high strain rate. For early times (\( \tilde{t} < 15 \)) the mass loss due to deposition is not significant. The effect of deposition is observed for later times when the total suspended mass has decreased enough to modify the flow structure.

Figure 3 shows the deposition rate, \( \dot{m}_s \), which is defined as

\[
\dot{m}_s(\tilde{t}) = \int_0^{\tilde{L}_x} \tilde{V}_s \hat{\phi}_d(\tilde{x}, \tilde{z} = 0, \tilde{t}) \, d\tilde{x}
\]

Here \( \tilde{z} = 0 \) is the bed location. Up to \( \tilde{t} \approx 15 \) the deposition rate increases. The increase is mainly due to the increase of area covered by the current, which increases over time. Then, the deposition rate decreases due to the loss of suspended mass. As can be seen from this picture, the net effect of inertia is to increase the deposition rate. The deposition rate is increased due to the larger accumulation of particles near the bottom. As explained
before, these particles have migrated from the high vorticity regions of the interface to the high shear regions near the bottom. The total deposition can be computes as

\[
\hat{D}(\hat{x}, \hat{t}) = \int_0^t \hat{V}_s \hat{q}_d(\hat{x}, \hat{z} = 0, \hat{t}) \, d\hat{t}.
\]

Figure 4 shows \(\hat{D}\) for three time instances \(\hat{t} = 10, 20\) and 45. Frame (a) shows the results for \(\hat{t} = 0\) and frame (b) for \(\hat{t} = 0.025\). As explained above, deposition is enhanced by inertia. It can be seen from figure 4 that not only the total deposition is increased but also the deposit pattern is changed. Preferential concentration of particles generate localized regions of increased deposition which explains the different peaks in frame (b) of this figure.

4. Conclusions

We have presented simulations of turbidity currents employing a two-phase flow model which includes not only settling effects but also particle inertia effects. The results presented in this work show that particle inertia has an important influence in the structure and deposition dynamics of turbidity currents. Particles migrate from the core of Kelvin-Helmholtz vortices and accumulate in the front and body of the current, which affects the deposition rate and the deposition pattern.

Acknowledgements

We gratefully acknowledge the support of the Coastal Geosciences Program of the Office of Naval Research, under its EuroStrataForm Program. Support from the National Center for Supercomputer Applications (NCSA) at the University of Illinois at Urbana-Champaign is also acknowledged. Mariano Cantero was supported by a Graduate Student Fellowship from the Computational Science and Engineering Program at the University of Illinois at Urbana-Champaign.

References


**Figure 1:** Contours of particles concentration. Solid line: $0.1 < \tilde{\phi}_d < 1$, dashed line: $1.0 < \tilde{\phi}_d$. Solution for $\tilde{\tau} = 0$ and $\tilde{V}_s = 0.005$.

**Figure 2:** Contours of particles concentration. Solid line: $0.1 < \tilde{\phi}_d < 1$, dashed line: $1.0 < \tilde{\phi}_d$. Solution for $\tilde{\tau} = 0.025$ and $\tilde{V}_s = 0.005.$
Figure 3: Sediment deposition rate as a function of time for $\tilde{V}_s = 0.005$. The net effect of inertia is to increase the deposition rate due to the larger accumulation of particles near the bottom.

Figure 4: A posteriori analysis of deposition (without resuspension included). The figure shows the influence of particle inertia on total deposition $\tilde{D}$. The deposit is visualized for three time instants: $\tilde{t} = 10, 20$ and $45$. Frame (a): $\tilde{V}_s = 0.005$, $\tilde{\tau} = 0$. Frame (b): $\tilde{V}_s = 0.005$, $\tilde{\tau} = 0.025$. 