MODELING AND LARGE SCALE SIMULATIONS OF
THERMOHALINE AND PARTICULATE DENSITY CURRENTS

BY

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Abstract

Density or gravity currents are flows driven by horizontal pressure gradients generated due to the action of gravity over fluids with different density. The density difference may be caused either by a scalar field that moves with the flow such as salinity or temperature, or by particles in suspension. The applications of such flows is very wide in many areas of engineering and geology. For example, density currents are flows well known to be one of the main sediment transport mechanisms into deep sea, whose deposits become oil reservoirs over geological time scales. Other examples of density currents are thunderstorm fronts, collapsing volcano ash plumes, contaminant releases in the environment, oil spills in the ocean, dust flows due to the collapse of buildings, flows originated by the discharge of a sediment-laden flow into oceans or lakes, and snow avalanches.

This work presents highly resolved simulations (on the order of $\sim 140$ million grid points) of planar and cylindrical density currents for Reynolds numbers ranging from about $10^3$ to about $10^4$. A rigorous formulation of the problem that systematically incorporates the inertial, settling and coupling effects of the particles is also presented.

The simulations show in great detail the dynamics of the flow and the rich interaction between turbulent structures with a wide range of scales. Front spreading velocity, Kelvin-Helmholtz vortex dynamics, lobe and cleft formation and evolution, flow patterns, and bed shear stress patterns, are visualized and explored in detail. The accurate numerical technique employed to solve the equations of the mathematical model captures all the relevant time and length scales present in the flow without the need of any
sub-grid scale model. In particular, the simulations permit to study the influence of fine structures on macroscale features of the flow and the new phenomena incorporated by the effects of particle inertia in isolation of semi-empirical turbulence closure relations. The simulation results are compared throughout the work to previously published experimental data and to laboratory experiments performed for this study and present very good agreement.
To Facundo and his coming siblings.
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List of Abbreviations

SP  Slumping phase
IP  Inertial phase
VP  Viscous phase
2D  Two-dimensional
3D  Three-dimensional
DNS Direct numerical simulation
LES Large eddy simulation
RANS Reynolds averaged Navier-Stokes
Re  Reynolds number
Sc  Schmidt number
Fr  Froude number
List of Symbols

\( u \) Velocity
\( p \) Pressure
\( \rho \) Density
\( \phi \) Volumetric concentration
\( \omega \) Voticity
\( t \) Time
\( Gr \) Grashof number
\( Re \) Reynolds number
\( Sc \) Schmidt number
\( Fr, F \) Froude number
\( D/Dt \) Material derivative
\( g \) Gravity
\( H \) Channel height
\( L_x, L_y \) Channel horizontal lengths
\( h_0 \) Height of initial release
\( x_0, r_0 \) Length of initial release
\( h \) Current height
\( x_F, r_F, l_F \) Front location
\( u_F \) Front velocity
\( h_H \) Head height
$U_0, U$  Velocity scale
$g'$  Submerged gravity
$\nu$  Kinematic viscosity
$\kappa$  Diffusivity
$t_{SI}$  Transition time from the slumping phase to the inertial phase
$t_{SV}$  Transition time from the slumping phase to the viscous phase
$t_{IV}$  Transition time from the inertial phase to the viscous phase
$\lambda_{ci}$  Swirling strength
$\Gamma$  Interface circulation
$d$  Particle diameter
$\tau$  Particle response time
$\Phi$  Scale for volumetric concentration
$V_s$  Settling velocity
Chapter 1

Introduction

Density or gravity currents are flows driven by horizontal pressure gradients generated by the action of gravity over fluids with different density. These flows manifest themselves either as horizontal currents of heavy fluid running below light fluid, or as currents of light fluid above heavy fluid. In some cases the currents manifest as a combination of these two, and in this case, they are also called intrusions.

When the density difference is due to a scalar field that moves with the fluid such as chemical species, temperature or salinity, density currents are called scalar or thermohaline density currents. Since the excess density is conserved over the bulk of the fluid in thermohaline density currents, they are also called conservative density currents. On the other hand, when the density difference is due to particles in suspension, density currents are called particulate density currents or turbidity currents. In contrast to a scalar field, particles may settle or be re-entrained into the flow and particulate density currents are sometimes referred to as non-conservative density currents.

Examples of density currents are thunderstorm fronts, collapsing volcano ash plumes, contaminant releases in the environment, oil spills in the ocean, dust flows due to the collapse of buildings, flows originated by the discharge of a sediment-laden flow into oceans or lakes, and snow avalanches. Many more examples can be found in the literature (see for example Allen, 1985; Simpson, 1997).

Density currents are stratified flows that present the structure of an intruding front or head followed by a body. This structure is shown schematically in figure 1.1(a). If the current is strong enough, intense vortex shedding and high mixing can be expected
in the flow. In this case the current is turbulent and presents a wide range of time and length scales. Figure 1.1(b) shows the front of a particulate density current from a laboratory experiment that clearly shows channel-size structures populated by smaller scale vortices. Another important characteristic that can be observed in this picture is the presence of sharp unstable interfaces.

Density currents may be generated by minute density differences, but still constitute flows with large transport capacity of mass and energy. This makes the application of density currents in fields such as environmental engineering, hydraulic engineering, mechanical engineering, atmospheric sciences, petroleum engineering and geology, to mention a few, very broad.

1.1 Motivation

As mentioned above, density currents are flows common to many fields. Several studies on the accidental release of liquefied gas have been done modeling the flow as a non-conservative density current that losses mass due to evaporation (Spicer & Havens, 1996). For example, the release of contaminant in a city spreads as channelized density currents through urban canyons. In the atmosphere, most of the severe squalls associated with thunderstorms are caused by the arrival of an enormous density current of cold air (Simpson, 1997). The front of the current produces large variations in horizontal and vertical winds, and strong recirculation and turbulence that influences aircraft security. In the ocean, sediment slumps can trigger turbidity currents capable of traveling several kilometers. These swift flows can carve submarine canyons (Fukushima et al., 1985) and mold the seabed producing different bed forms patterns as ripples, dunes, antidunes and gullies. The sediment deposits generated by turbidity currents constitute off-shore oil reservoirs over geological time scales.

In many applications the current is channelized and confined to flow between lateral
walls. In such situations, the current moves as a statistically two-dimensional flow with a nominally planar front (planar current). There are a number of other applications, such as the release of heavy gas into an open space, the collapse of an axisymmetric volcanic plume, or a point discharge into a lake or ocean, in which the gravity current is not confined and is allowed to spread out over the entire space. In such situations, the current moves as a statistically axisymmetric flow with a nominally cylindrical front (cylindrical current).

Planar and cylindrical currents are two extreme geometrical setting that present distinct features. In a planar current the planform increases linearly with front location, while in a cylindrical current the planform increases quadratically. This fundamental difference dramatically decays the intensity of the cylindrical current as it evolves. On the other hand, in a cylindrical current the concentrated vorticity at the head of the current initially intensifies as the current flows out due to intense vortex stretching (Patterson et al., 2006; Cantero et al., 2007b).

As a planar current spreads, it passes through different phases, namely slumping, inertial and viscous (Huppert & Simpson, 1980). In the slumping phase the planar current moves at a near constant speed. The duration of the slumping phase depends on the initial volume and Reynolds number of the release. Provided the Reynolds number of the current is sufficiently large, the flow first enters an inertial self-similar phase (Rottman & Simpson, 1983), in which the current moves under the balance of buoyancy and inertial forces. In this phase the current’s spreading rate (front velocity) scales as $t^{-1/3}$ (see for example Hoult, 1972). At later times, when viscous effects become important the current transitions to the viscous phase. In this phase the current’s spreading rate scales as $t^{-5/8}$ if the dominant viscous effect arises from interfacial friction between the heavy and light fluids (Hoult, 1972) or as $t^{-4/5}$ if the viscous effect is primarily from bottom friction (Huppert, 1982).

Cylindrical currents are also thought to pass through a similar sequence of different
phases of spreading. For a cylindrical current in the initial slumping phase the velocity of the front does not appear to remain at a near constant value. After the slumping phase, the current enters the self-similar inertial phase (for high enough Reynolds numbers) and the spreading rate scales as $t^{-1/2}$ (see for example Hoult, 1972). During the final viscous phase, the front velocity of the cylindrical current scales as $t^{-3/4}$ if the dominant viscous effect arises from interfacial friction (Hoult, 1972) or as $t^{-7/8}$ if the viscous effect is primarily from bottom friction (Huppert, 1982).

As the front spreads, a shear layer forms between the heavy forward advancing front and the ambient fluid. As the interface develops, it rolls up forming Kelvin-Helmholtz vortices. The dynamic of the interface is highly dependent on the Reynolds number of the flow. Also, at the front, a clear pattern of lobes and clefts develops, which produces high local flow variation.

A wide range of models, ranging from very simplified drag-based models to detailed simulations of the flow, have been developed to study different features of density currents such as front speed, local Froude number conditions, deposition and erosion patterns. It has been reported that the three-dimensionality of the flow may play an important role. For example, Cantero (2002) and Cantero et al. (2003) reported numerical simulations of planar gravity currents developing over a favorable slope and compared the results with experimental data. Figure 1.2 taken from Cantero (2002) shows the time evolution of the front along the centerline of the channel. It is clearly seen in this figure that the three-dimensional simulation better captures the experimental observations and the two-dimensional model lags behind.

Although density current flows have been widely studied in the last 30 years, a literature perusal shows the existence of several open questions which still remain unanswered:

- What are the main differences between planar and cylindrical currents?
- How important is the effect of three-dimensionality of the flow?
• Which features of the flow can be predicted by two-dimensional models and which cannot?

• What type of turbulent structures are observed in density currents?

• How is the dynamics of the lobe and cleft pattern observed at the front of density currents?

• How are lobes and cleft related to local flow features for planar and cylindrical currents?

• What is the effect of particle inertia and particle settling on turbidity currents?

• How are sediment deposits affected by particle inertia?

There are, basically, three ways to attack a problem: analytically, experimentally (laboratory and field) and numerically. As mentioned above, density currents are unsteady highly non-linear stratified flows. Under these circumstances, the insight that can be gained about the flow through analytical and experimental tools is limited. On the other hand, with the increase of computational power in the last two-decades, direct numerical simulation has become a very useful tool for the study of complex flows. Although highly resolved direct numerical simulations can be performed only for moderate Reynolds numbers (≈ 10000, i.e. laboratory scale turbulent flow experiments), they provide very detailed information of the finest structures of the flow that are, otherwise, not available. Highly resolved simulations constitute a detailed database of the flow that can be used to explore the physics in detail and to link the macroscale features that are easily observed in the field and in experiments to the fine structure and dynamics of the flow.
1.2 Scope and objectives of the thesis

The scope of the present work is to address those questions presented above through highly resolved simulations for planar and cylindrical density current for moderate Reynolds numbers (< 10000). More precisely, the objectives of this work are: 1) to develop a sound theoretical and computational model for scalar and particulate density currents, including both particle inertia and settling effects; 2) to assess the hydrodynamics of density currents, specifically oriented to the effect of three-dimensionality and turbulent structures on the macroscale features of the flow for planar and cylindrical settings; and 3) to assess the effect of particle inertia and settling on macroscale features of turbidity currents.

These objectives are reached in three stages. The first stage of this research consists on assessing the hydrodynamics and fine coherent structures of scalar density currents. The attention is focused on macroscale features such as front velocity, heavy-light fluids interface dynamics, turbulent structures, lobe and cleft instabilities at the front of the current, and local flow patterns. The second stage of this research focuses on the development of a sound mathematical model for particulate density currents which includes inertial and settling effects. The third and final stage consists on assessing the hydrodynamics and characterizing the fine coherent structures of particulate density currents. Here the main focus is on front velocity, bottom shear stress and deposition.

1.3 Structure of the thesis

This thesis is organized in six chapters: chapter 1 is the introduction, chapter 2 to 5 document the main findings of the work, and chapter 6 presents the conclusions. Chapters 2 to 5 are self-contained, each one presents a literature review relevant to the topic treated in that chapter and partial conclusions.

In chapter 2 the work methodology is validated. In this section, the collapse of a
heavy fluid column in a lighter environment is studied by direct numerical simulation of the Navier-Stokes equations using the Boussinesq approximation for small density difference for planar and cylindrical geometries. The findings of this chapter have been published in Cantero et al. (2006).

The primary objective of chapter 3 is to systematically study the propagation of gravity currents and to investigate the influence of key parameters on front velocity. Results from highly resolved simulations of planar and cylindrical gravity currents for varying $Re$ are presented in this section. For the planar case, small and large release volumes are considered. A detailed comparison of the results will illustrate the role of planar vs. cylindrical nature of the current and also the effect of volume of release on the mean velocity of the front and the transition between the different phases. In each of the cases considered, both fully resolved three-dimensional simulations and corresponding two-dimensional or axisymmetric simulations are presented. These results permit to firmly establish the role of three-dimensionality as well as the mechanisms behind the faster propagation of the three-dimensional front. The three-dimensional structure of the propagating front is also explored in both, the planar and cylindrical configurations. The findings of this chapter have been published in Cantero et al. (2007c).

In chapter 4 the attention is centered on the release of a fixed cylindrical volume of homogeneous fluid in a slightly less dense environment and the time evolution of the resulting cylindrical current. Highly resolved simulations for three-dimensional cylindrical currents at three different Reynolds numbers are presented and the results documented. The simulation results are compared to laboratory experiments performed in scale 1 to 1 for the largest Reynolds number investigated numerically. The findings of this chapter have been published in Cantero et al. (2007b).

Chapter 5 concentrates on the effect of particle inertia on the flow structure and dynamics of particulate density currents. It is important to recognize that particles move with a velocity field which is different from the fluid. The fact that particles with
finite size cannot follow exactly the fluid velocity plays a very important role and modifies substantially the structure and dynamics of the flow, as well as deposition and erosion patterns. In this section a novel Eulerian-Eulerian mathematical model for simulating particulate density currents based on the well-accepted formalism of two-phase flow and the equilibrium Eulerian approach is presented, which captures particle inertia and settling effects. This model is used to perform direct numerical simulations to assess the effect of finite inertia on the current structure, front velocity, erosion and deposition of planar currents. The findings of this chapter have been published in Cantero et al. (2007a).

Finally, chapter 6 summarizes the main conclusion of this work.
Figure 1.1: Density current structure. Frame (a) show schematically the structure of a density current. Frame (b) shows the front of a particulate density current from a laboratory experiment. Courtesy of Marcelo H. García.
Figure 1.2: Evolution of the front location of a finite-volume release planar current in favorable slope. The 3D simulation predicts better the experimental results than the 2D simulation, Cantero (2002).
Chapter 2

Direct numerical simulations of planar and cylindrical density currents

2.1 Introduction

Two fluids having different densities that are initially separated by a physical boundary and are suddenly allowed to mix, interact freely forming a density current. Depending on the situation, the density current can be in the form of a denser intrusion penetrating horizontally into the lighter fluid along the bottom boundary, in the form of a lighter intrusion spreading into the heavier fluid along the top boundary, or as a combination of both conditions. Examples are snow avalanches, thunderstorm fronts, volcano eruptions, oil spills in the ocean, the release of contaminants in the environment and flows generated by the collapse of a building. Many more examples can be found in the books by Simpson (1997) and Allen (1985). In most environmental and industrial flows of this type, the density difference is only a few percent and it is caused either by scalar fields, such as temperature, salinity and a chemical species, or by particles in suspension leading to the development of turbidity currents (García & Parker, 1989; García, 1994).

Consider the case of a denser fluid released into a lighter environment. Soon after the release a density current develops, which presents a front, a body, and a tail. The front is a discontinuity in density that penetrates into the lighter fluid. The denser fluid rides over a thin layer of light fluid that remains attached to the bottom boundary as a consequence of the no-slip condition. This results in a front whose nose is somewhat lifted above the bottom boundary. The front of the current is a complex, dynamic region where most of the mixing occurs. This mixing, driven by Kelvin-Helmholtz instabilities
and vortex shedding, plays an important role regulating the flow since it modifies the
driving force by entraining ambient fluid into the current, and thus, diminishing density
differences. Behind the front, the body and the tail of the current follow, and their length
depends on the amount of dense fluid initially released. In this region, the vortices shed
from the front pair, stretch, and eventually break down.

The earliest theoretical attempts to describe the spreading rate of these types of flows
were made by von Kármán (1940) and Benjamin (1968). Benjamin (1968) proposed that
in a lock-exchange configuration the front should move at a speed of \( \sqrt{1/2 g (\rho_1 - \rho_0)/\rho_0 h_0} \),
where \( \rho_0 \) and \( \rho_1 \) are the densities of the lighter and heavier fluids, respectively, \( h_0 \) is the
channel half height and \( g \) is the acceleration of gravity. Later works used shallow water
theory, along with an empirical Froude condition to close the model, in order to describe
the propagation of the front (Rottman & Simpson, 1983; Bonnecaze et al., 1993; Choi
& García, 1995; Hallworth et al., 1996, 2001). Most of these analyses do not account
for the mixing with the ambient fluid.

Several experiments have also been performed to study the front dynamics. Huppert
& Simpson (1980) have studied experimentally the release of a fixed volume of denser
fluid in a lighter ambient. They found that initially a given current spreads at an
approximately constant speed and then continuously decelerates until it is dissipated
by viscous effects, calling them the slumping, inertial, and viscous phases, respectively.
If the Reynolds number of the flow is large enough the deceleration starts with the
beginning of the inertial phase. However, for low Reynolds numbers the inertial phase
is not present and the deceleration of the flow occurs dominated by viscous effect during
the final viscous phase. They also proposed an empirical Froude condition that has
been used in box models and to close integral shallow water models of density currents.
The propagating front undergoes three-dimensional instability in the form of lobes and
cLEFTs. Allen (1971) and Simpson (1972) devoted a great deal of effort to studying the
lobe and cleft patterns. Simpson (1972) proposed that the lobe and cleft instability
forms only in no-slip surfaces and it is caused by denser fluid overrunning on top of less dense fluid. However, the exact origin of this instability is still not well known and recent work (McElwaine & Patterson, 2004) has brought Simpson’s theory back into discussion. García & Parsons (1996) and Parsons & García (1998) studied the similarity of density currents fronts finding that the Reynolds number of the current front plays an important role in the mixing with the ambient fluid. Very recent observations of density current activity in the Chicago River, Illinois, by García et al. (2005) supports the observation that the front dynamics is affected by scale (i.e., Reynolds number effects).

Recently, high-resolution numerical computations have been performed in both two and three dimensions to explore the dynamics of density currents (Droegemeier & Wilhelmson, 1987; Terez & Knio, 1998\textit{b,a}; Hārtel et al., 2000\textit{b}; Necker et al., 2002). These works have provided a detailed description of the flow topology at the foremost portion of the current. The simulations have concentrated on planar and axisymmetric configurations and, to date, no such effort has been attempted for the corresponding cylindrical three-dimensional configuration. In this work, the release of a fixed volume of a homogeneous fluid into a slightly less dense environment in a cylindrical configuration is considered, and the results are compared with the planar case.

The planar lock-exchange configuration studied by Hārtel et al. (2000\textit{b}) is considered first. We present results from a three-dimensional simulation with the same conditions as reported by them and compare the results qualitatively as well as quantitatively. Then we consider the release of a cylindrical region of denser fluid into a less dense ambient and compare our results qualitatively with previously reported experiments (Alahyari & Longmire, 1996).
2.2 Numerical formulation

We consider the case of an initial cylindrical volume of heavier fluid surrounded by an infinite extent of lighter fluid. The released volume is a cylinder of radius $r_0$ and height $2h_0$ (see figure 2.1) and the lighter fluid extends between top and bottom boundaries separated vertically by a distance $H$. Here we consider both the top and bottom boundaries to be rigid and no-slip. Attention will be restricted to the case where the density difference is due to a scalar field (e.g., salinity or temperature).

The density difference is assumed to be small enough so that the Boussinesq approximation can be adopted. With this approximation density variations are important only in the buoyancy term. The dimensionless equations of motion read (Härtel et al., 2000b)

\[
\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_i}{\partial \tilde{x}_k} = \tilde{\rho} e_i - \frac{\partial \tilde{p}}{\partial \tilde{x}_i} + \frac{1}{\sqrt{Gr}} \frac{\partial^2 \tilde{u}_i}{\partial \tilde{x}_k \partial \tilde{x}_k},
\]

(2.1)

\[
\frac{\partial \tilde{u}_k}{\partial \tilde{x}_k} = 0,
\]

(2.2)

\[
\frac{\partial \tilde{p}}{\partial t} + \frac{\partial}{\partial \tilde{x}_k} (\tilde{\rho} \tilde{u}_k) = \frac{1}{Sc \sqrt{Gr}} \frac{\partial^2 \tilde{\rho}}{\partial \tilde{x}_k \partial \tilde{x}_k}.
\]

(2.3)

Here $\tilde{u}_i$ is the velocity vector, $\tilde{p}$ is the dynamic pressure, $\tilde{\rho}$ is the density, $Gr$ is the Grashof number, $Sc$ is the Schmidt number and $e_i$ is a unit vector pointing in the gravity direction. We have adopted the initial condition half height, $h_0$, as the length...
scale. Since there is no externally imposed velocity scale for the flow, the following velocity scale is defined

\[ U_0 = \sqrt{\frac{g (\rho_1 - \rho_0)}{\rho_0} h_0}. \]  

(2.4)

Consequently, the time scale is \(h_0/U_0\). Here \(\rho_1\) is the density of the denser fluid and \(\rho_0\) is the density of the ambient fluid. The dimensionless density and dynamic pressure are given by

\[ \tilde{\rho} = \frac{\rho - \rho_0}{\rho_1 - \rho_0}, \quad \tilde{p} = \frac{p}{\rho_0 U_0^2}. \]  

(2.5)

The two dimensionless numbers in equations (2.1)–(2.3) are given by

\[ Gr = \left(\frac{U_0 h_0}{\nu}\right)^2 \quad \text{and} \quad Sc = \frac{\nu}{\kappa}, \]  

(2.6)

(2.7)

where \(\nu\) is the kinematic viscosity and \(\kappa\) is the diffusivity of temperature or salinity responsible for the density difference. The definitions are similar to those employed by HärTEL et al. (2000b) in their study of planar case. Note that the Grashof number is essentially the square of the Reynolds number. The ratios \(r_0/h_0\) and \(h_0/H\) are additional geometric parameters introduced by the initial condition. In this work we will concentrate on the condition \(H = 2h_0\), where the denser fluid initially extends vertically over the entire height of the layer.

The governing equations are solved using a de-aliased pseudospectral code (Canuto et al., 1988). Fourier expansions are employed for the flow variables along the horizontal directions (\(x\) and \(y\)). In the non-homogeneous vertical direction (\(z\)) a Chebyshev expansion is used with Gauss-Lobatto quadrature points. The flow field is time advanced using a Crank-Nicolson scheme for the viscous and scalar diffusion terms. The advection term in the momentum equation is handled with the Arakawa scheme, where the
nonlinear term is alternately considered in its convective form (as written in equation (2.1)) followed by the conservative form. A third-order Runge-Kutta scheme is used to advance the nonlinear terms. The buoyancy term is also advanced with a third-order Runge-Kutta scheme. More details on the implementation of this numerical scheme can be found in the work by Cortese & Balachandar (1995).

For the planar lock-exchange configuration the computational domain is a box of size $\tilde{L}_x = 30 \times \tilde{L}_y = 3 \times \tilde{L}_z = 2$, where the spanwise width of the domain ($\tilde{L}_y = 3$) has been shown to be more than adequate to capture the lobe and cleft instability (Härtel et al., 2000b). For the cylindrical configuration the computational domain is also a box of size $\tilde{L}_x = 30 \times \tilde{L}_y = 30 \times \tilde{L}_z = 2$. Periodic boundary conditions are enforced in the horizontal directions for all variables. At the top and bottom walls no-slip and zero-gradient conditions are enforced for velocity and density, respectively. The use of a rectangular grid to solve a cylindrical problem may seem odd. However, a Cartesian grid with equi-spaced grid points provides uniform resolution along the horizontal directions over the entire domain. This allows adequate resolution as the cylindrical front propagates radially out and we are able to better resolve the fine structures of the flow at the front (lobes and clefts). With a cylindrical grid, the circumferential resolution will be far more than what is needed as the center is approached. Furthermore, a cylindrical computational domain requires to carefully address the singularity presented by the pole at the origin as well as the outflow boundary condition at the outer extent of the computational domain. Herein, by adopting a rectangular domain and periodic boundary conditions these difficulties are clearly avoided.

For the case of the cylindrical current, periodic boundary conditions along the horizontal directions strictly imply an infinite layer of lighter fluid with a doubly periodic array of cylindrical regions of heavier fluid released into it. Here we consider the initial non-dimensional radius of the cylindrical region to be $\tilde{r}_0 = 2$ and thus the released volume is of unit aspect ratio. Owing to the periodic boundary conditions, the lateral
spacing between the cylindrical releases is 30 along both the \(x\) and \(y\) directions. Only when the head of the gravity current approaches the lateral boundaries of the computational domain, it begins to interact with the front of the adjacent currents. Based on simulation results we observe that this interaction effect can be neglected till the front reaches about 2 non-dimensional units from the lateral boundaries. This behavior is similar to that observed by Härte et al. (2000b) for the planar case along the \(x\) direction.

As the cylindrical gravity current expands from the initial radius of 2 to about 13, its evolution is not influenced by the periodic boundary condition and can, thus, be taken as an isolated cylindrical density current spreading into an infinite lighter medium.

In this work, two different Grashof numbers will be considered: \(Gr = 10^5\) and \(Gr = 1.5 \times 10^6\). As will be discussed below, with increasing \(Gr\) the complexity of the flow increases and thus the simulation at the higher Grashof number requires increased resolution. In the cylindrical configuration the flow was solved using approximately 12 millions grid points for \(Gr = 10^5\) \((N_x = 420 \times N_y = 420 \times N_z = 72)\) and 28 millions for \(Gr = 1.5 \times 10^6\) \((N_x = 512 \times N_y = 512 \times N_z = 110)\). For the lock-exchange configuration approximately 2 million grid points were used for \(Gr = 1.5 \times 10^6\) \((N_x = 560 \times N_y = 48 \times N_z = 64)\). The numerical resolution for each simulation was selected to have between 6 and 8 decade decay in the energy spectrum for all the variables, i.e. the three velocity components and density. The time step was selected to produce a Courant number smaller than 0.5 for all time steps.

The flow was started from rest and a small random disturbance superposed on the density field to accelerate the three-dimensional development. The following initial condition was used in all the cylindrical simulations to be reported:

\[
\tilde{u}_i = 0 \quad \forall (\tilde{x}, \tilde{y}, \tilde{z}),
\]

\[
\tilde{\rho} = \frac{1 + \gamma_1}{2} \text{erf}\left\{ \sqrt{Gr \, Sc^2} \, [\tilde{r} - (\tilde{r}_0 + \gamma_2)] \right\}.
\]
Here $\tilde{r} = \sqrt{\tilde{x}^2 + \tilde{y}^2}$, $\gamma_1 \in (-0.05, 0)$ and $\gamma_2 \in (-\Delta x/2, \Delta x/2)$. These two last parameters are random numbers chosen from a uniform distribution. For the planar case we use the same initial condition as that of Härter et al. (1997), where they justify the use of an error function to prescribe the initial density profile based on the solution to the pure diffusion equation for early times when the flow has not yet developed. The values of $\gamma_1$ and $\gamma_2$ were selected to produce a decorrelated interface with a white noise energy spectrum. In this way we assure that there is no artificially selected wavelength that could evolve in an artificial lobe and cleft pattern. The solution was advanced in time until the front reached the radial location of $\tilde{r} = 13$ to avoid the influence of the lateral boundaries (Härter et al., 1997). The above initial condition ensures that the rectangular planform of the domain and the grid do not introduce any bias in the evolution of a cylindrical front.

2.3 Results

2.3.1 Planar lock-exchange configuration

In order to validate the code, we have performed a planar lock-exchange simulation under the same conditions reported by Härter et al. (2000b), i.e. $Gr = 1.5 \times 10^6$ and $Sc = 0.71$. This configuration can also be seen as the limiting case of a cylindrical configuration with the condition of $\tilde{r}_0 \to \infty$.

Figure 2.2 shows three-dimensional views of the flow time evolution visualized by a surface of constant density ($\tilde{\rho} = 0.5$). The flow starts as two dimensional ($\tilde{t} = 5$) forming the head of the current and the nose. Kelvin-Helmholtz instabilities are also observed in the interface between light and heavy fluid. The flow turns into a three-dimensional state ($\tilde{t} = 10$ and 15) starting with instabilities at the bottom foremost part of the current that grow very rapidly forming a pattern of lobes and clefts. Then, the whole flow becomes three dimensional ($\tilde{t} = 20$) presenting vortex pairing at the rear end of the
head. These results are in complete agreement with the findings of Härzel et al. (2000b) and with laboratory observations (García & Parker, 1989).

Figure 2.3 shows the mean flow visualized by density contours. Mean variables are computed as spanwise averages of the three-dimensional results, i.e.

\[
\bar{f}(\tilde{x}, \tilde{z}) = \frac{1}{L_y} \int_0^{L_y} f(\tilde{x}, \tilde{y}, \tilde{z}) \, d\tilde{y}.
\] (2.10)

In this figure the dynamics of the Kelvin-Helmholtz instabilities can be more clearly appreciated. Observe that the flow is initially symmetric, but as it becomes three dimensional the symmetry is lost. The last snapshot \((\tilde{t} = 15)\) also shows the beginning of vortex pairing on the lower-advancing front.

The front velocity has also been computed from two-dimensional simulations of the lock-exchange configuration with \(Gr = 10^5\) and \(Gr = 10^7\). The results are presented in figure 2.4. For comparison, the result of Härzel et al. (2000b) is also shown. Observe that the agreement is not only qualitative as mentioned above, but also quantitative. The trend of the front velocity with the \(Gr\) number is also correct.

We will not expand any further on the planar lock-exchange problem, since Härzel et al. (2000b,a) have presented a very fine and detailed analysis of the flow for this configuration. Now we turn into the cylindrical configuration.

### 2.3.2 Cylindrical configuration

**Flow structure**

To study the structure of the flow, simulations for \(Gr = 10^5\) and \(Gr = 1.5 \times 10^6\) were performed. The \(Sc\) number was set to 1. As addressed by Härzel et al. (2000b) its influence on the flow is weak as long as it is kept order 1.

Figure 2.5 shows the time development of the flow structure for the higher \(Gr = \)
1.5 × 10^6. In this figure the flow is visualized by a surface of constant density ˜ρ = 0.25. After the release of the denser fluid, an intrusive front forms. Initially, the flow evolves in an axisymmetric fashion in which Kelvin-Helmholtz rolls develop and form the front and the nose. Below the nose, which is raised from the bottom, an unstable stratified region forms as a consequence of the no-slip condition. In this region, three-dimensional instabilities develop and evolve into a lobe and cleft pattern in the foremost part of the current. This feature has been observed in experiments for both planar (Simpson, 1972) and cylindrical currents (Spicer & Havens, 1987). Behind the front, the flow develops into a very intense three-dimensional structure where the Kelvin-Helmholtz billows shed from the front deform, bend, and break up. This behavior is similar to the planar case (Härtel et al., 2000b; Cantero et al., 2003).

One of the main differences between the cylindrical (finite volume release) and the planar lock-exchange configurations is the maximum density value inside the current. Figure 2.6 shows the time evolution of the maximum density, ˜ρ_{max}, with time for planar lock-exchange and cylindrical currents with Gr = 1.5 × 10^6. In a truly planar lock-exchange configuration, which corresponds to infinite volume release, at all finite times the maximum and minimum concentration levels remain at 1.0 and 0.0, and, respectively, correspond to unmixed heavy and light fluids. In the present periodic finite volume planar lock-exchange, over the time interval computed and shown in figure 2, the maximum and minimum concentrations remain 1.0 and 0.0, respectively. In the cylindrical configuration the small finite volume release of heavy fluid quickly mixes with the surrounding light fluid as it flows out. Thus the maximum concentration remains equal to 1.0 only for a short duration after which the concentration decreases. In the present periodic case the final well mixed concentration will be 2.8% and it depends on the ratio of volume released to the volume of the periodic box. From figure 2.6 it is clear that the released heavy fluid is everywhere diluted by entrainment of lighter fluid, but the mixing process is far from complete. While ˜ρ_{max} remains equal to the initial
value ($\rho_{\text{max}} = 1$) the cylindrical current moves at approximately constant velocity. This phase of spreading is called slumping phase (Huppert & Simpson, 1980). In the case of the planar lock-exchange, $\rho_{\text{max}} = 1$ for all the computation time and the front spreads at constant speed.

The lobe and cleft structure of the front is shown in detail in figure 2.7. Figure 2.7a is a visualization of the front in a laboratory experiment for $Gr \sim 10^8$ and $Sc = 700$ using the same geometrical configuration of the numerical simulations. Figure 2.7b is a close view of the numerical results for $Gr = 1.5 \times 10^6$ and $Sc = 1$. We can observe in this figure the similitude between the experimental and numerical results despite the difference in the $Sc$ ($Sc$ in the experiment is two order of magnitude larger than in the numerical simulation). This is in agreement with the findings of H"{a}rtel et al. (2000b) who state that the $Sc$ does not influence the flow as long as it is kept order 1 or larger. In contrast to the planar case (H"{a}rtel et al., 2000b), the number of lobes in the front stays almost constant as the front evolves. However, since the current is spreading radially, the size of the lobes grows as the current spreads out until it is dissipated by mixing of light fluid. The origin and dynamics of this instability are still not well understood. Simpson (1972) states that the lobe and cleft instability forms only in no-slip surfaces and it is caused by denser fluid overrunning less dense fluid. However, recent work by McElwaine & Patterson (2004) suggests that this is not necessarily the case and that lobes and clefts may still form in free-slip surfaces provided the $Gr$ number of the flow is large enough. Our simulations show that the formation of lobes and clefts is highly $Gr$-dependent even in the case of no-slip surfaces. For example, the solution for $Gr = 10^5$ does not present this feature. The solution is completely axisymmetric for all time (see figure 2.8).

The structure of the mean flow is also dependent on the $Gr$ number. In the cylindrical
configuration the mean flow is computed as

\[
\bar{f}(\tilde{r}, \tilde{z}) = \frac{1}{2\pi} \int_0^{2\pi} f(\tilde{r}, \tilde{\theta}, \tilde{z}) \, d\tilde{\theta}.
\]  

(2.11)

Figures 2.9 and 2.10 show the mean flow visualized by contours of constant density for \(Gr = 1.5 \times 10^6\) and \(Gr = 10^5\), respectively. The main structures of the flow, namely head, nose, and body are present for both \(Gr\) investigated, however, there are substantial differences. The head of the current for \(Gr = 10^5\) features a single vortex that evolves in time to become a rounded structure. On the other hand, the current for \(Gr = 1.5 \times 10^6\) features two vortices in the head that eventually pair and form a triangular structure.

It is also clearly seen from these figures that the nose location (\(h_N\) in figure 2.1) of the current for \(Gr = 10^5\) is always higher compared to the current for \(Gr = 1.5 \times 10^6\). This feature is in agreement with experimental observations (Simpson & Britter, 1979). In contrast to the behavior of the nose location, the height of the head (\(h_F\) in figure 2.1) is approximately the same for both \(Gr\). However, \(h_F\) diminishes over time transforming the potential energy of the head into kinetic energy of the flow, which is subsequently expended in mixing light fluid into the current and dissipated by viscous effects. Another clear difference between the two \(Gr\) solutions is the structure and height of the body (\(h_B\) in figure 2.1) of the current. The lower \(Gr\) current presents a higher body with a regular structure while the higher \(Gr\) current presents a lower body with vortical structures in it.

**Mean flow dynamics**

When viscous effects are not important (high \(Gr\)) the current that develops from the release of a fixed volume of heavy fluid passes through three different phases (Huppert & Simpson, 1980), provided that the volume of released fluid is large enough. Soon after the release, the current enters the slumping phase, which is characterized by a
nearly constant front velocity. Huppert & Simpson (1980) proposed that this phase lasts until $h_B = 0.075H$. Then, the current enters a self-similarity phase called the inertial phase. During this phase the front decelerates and the front velocity evolves as $t^{-1/2}$ (cylindrical configuration). This self-similar phase lasts until viscous effects take over, and the current enters the viscous phase. Depending on the initial configuration of the flow, the inertial phase may or may not be present. In the following we will describe the dynamics of the mean flow during the slumping and inertial/viscous phases.

An idea of how the flow evolves can be gained from figures 2.9 and 2.10 that show the time evolution of the mean flow for $Gr = 1.5 \times 10^6$ and $Gr = 10^5$, respectively. The development of the mean flow starts with a short acceleration phase. In this phase, the nose is formed and the front reaches the slumping phase velocity. After the initial acceleration phase, the flow enters the slumping phase and moves at approximately constant speed, which depends on $Gr$. In this phase the flow presents a very interesting behavior. First, a large billow is formed in the front (B1 in figure 2.9), which gives the current the characteristic structure of front (or head) and body. Then, two more billows are formed. One counter-rotating billow is formed in the lower region of the front (B2), which has been interpreted as boundary layer separation by Alahyari & Longmire (1996) caused by the adverse pressure gradient produced by the first billow (B1). The other billow (B3) is formed in the body of the current and rotates in the same direction as the first billow (B1). Finally, the first billow formed in the front (B1) retards the upper part of the front, which gives place to the formation of another billow at the front (B4). At the same time, billows B2 and B3 lose their identity.

After the slumping phase, the flow enters into the inertial/viscous phases. During these phases the third billow formed at the front (B4) becomes more prominent and undergoes the same dynamics as the first billow (B1) in the slumping phase. Billow B4 retards the front and pair with billow B1 to form a triangular wedge that eventually dissipates.
The dynamics of the flow described here is in good agreement with the findings of Alahyari & Longmire (1996) based on their laboratory experiments. It is worth noticing that the $Gr$ of their experiment is larger ($Gr \approx 10^7$) than our simulation, however, the dynamics and structure of the flow are quite similar.

**Front velocity**

The planar lock-exchange configuration (infinite volume release) from the previous section can be seen as the limiting case of the cylindrical configuration with infinite radius. Thus, the planar lock-exchange density current will stay in the slumping phase and will never reach the inertial phase, while the corresponding cylindrical current started from a finite volume release will transition from the initial slumping phase to an inertial phase and finally to a viscous phase. The time at which these transitions take place depend on the amount of fluid being released and on the Reynolds number of the flow. The slumping phase is characterized by a constant front velocity, which based on theory takes a value of $1/\sqrt{2}$ for the case of $2h_0 = H$ (von Kármán, 1940; Benjamin, 1968).

Based on a best fit to experimental data, Huppert & Simpson (1980) proposed the same dimensionless front velocity for both, planar and cylindrical currents. In this section we present front velocity results in the slumping phase obtained from our simulations of planar lock-exchange and cylindrical currents for two values of $Gr$.

The front velocity is computed by tracking the front location over time. If $\tilde{r}_F$ denotes the front location, the front velocity is computed as

$$\tilde{u}_F = \frac{d\tilde{r}_F}{dt}. \quad (2.12)$$

The front location is defined as the largest radial location where the mean flow density equals a preset density value (for example, $\tilde{\rho} = 0.01$).

Figure 2.4 shows the front velocity for the planar lock-exchange configuration and
for the slumping phase of the cylindrical configuration. The figure also shows the value reported by Härtel et al. (2000b) (open square) in good agreement with our results. There is a clear dependency of the front velocity on $Gr$ number, which was originally observed by Simpson & Britter (1979). This $Gr$-dependency is less strong for larger $Gr$ (see Härtel et al., 2000b) and it is likely to be negligible for large enough values of $Gr$, reaching an asymptotic state close to the theoretical value.

There is also a well defined dependency on the geometrical configuration of the current. The cylindrical current is slower than the planar current. This is in contradiction with the findings of Huppert & Simpson (1980), who reported the front velocity to be independent of the geometrical configuration. It can be argued that the numerical simulations are at lower $Gr$ and over a narrow range compared to the experimental results, and that for larger $Gr$ the cylindrical currents could reach the same asymptotic state as the planar lock-exchange configuration. The answer to this question will be addressed in a forthcoming work.

### 2.4 Concluding remarks

In the present work, we have presented and discussed the results of three-dimensional direct numerical simulations of density currents in planar lock-exchange and cylindrical configurations. There were two main objectives in this paper. The first one was to validate the present computational methodology by comparing our results with previously published experimental and numerical works (Alahyari & Longmire, 1996; Härtel et al., 2000b) and with experimental visualizations produced for this work. The second one was to present a detailed analysis and visualization of three-dimensional density currents in cylindrical configuration. The simulations were performed employing a de-aliased pseudospectral code, which allows accurate representation of all length scales.

We have presented three-dimensional results for $Gr = 1.5 \times 10^6$ and $Sc = 0.71$ and
two-dimensional results for $Gr = 10^5$ and $Gr = 10^7$ with $Sc = 1$ in the planar lock-exchange configuration. The flow starts as two dimensional and preserves the initial symmetry for early times. For later times the flow becomes three dimensional, presents a pattern of lobes and clefts at the front, and the symmetry of the flow is lost. These results are in complete agreement with the results reported by Härtel et al. (2000b), qualitatively as well as quantitatively. We have also presented three-dimensional simulations in cylindrical configuration for $Gr = 10^5$ and $Gr = 1.5 \times 10^6$, and $Sc = 1$. These highly resolved simulations allowed for a detailed analysis and visualization of the flow structures (2D and 3D structures) and dynamics. The simulation for $Gr = 1.5 \times 10^6$ exhibits the main features observed in laboratory experiments (Simpson, 1972; Spicer & Havens, 1987) (see also figure 2.7), and the dynamics of the flow computed in this simulations is in agreement with experimental observations (Alahyari & Longmire, 1996) at higher $Gr (\sim 10^7)$.

The results for front velocity indicate dependencies on both, the $Gr$ and on geometrical configuration. However, it is possible that with increasing $Gr$ the front velocity will reach an asymptotic state that is independent of $Gr$ and geometrical configuration. Simulations for larger $Gr$ are under way and the answer to this question will be presented in a forthcoming paper.
Figure 2.2: Three-dimensional planar current in lock-exchange configuration for $Gr = 1.5 \times 10^6$ and $Sc = 0.71$. Flow visualized by an isosurface of density $\tilde{\rho} = 0.5$. At $\tilde{t} = 0$ the left half of the domain has $\tilde{\rho} = 1$ and the right half $\tilde{\rho} = 0$. The flow starts as two-dimensional forming the head of the current ($\tilde{t} = 5$), then the flow turns three dimensional ($\tilde{t} = 10, 15$ and $20$) developing the lobes and clefts observed in experiments.
Figure 2.3: Mean flow of planar current in lock-exchange configuration for $Gr = 1.5 \times 10^6$ and $Sc = 0.71$. Flow visualized by density contours. The flow starts symmetrically, but this symmetry is lost as the flow becomes three dimensional. Observe also the beginning of vortex pairing at $\tilde{t} = 15$ in the front advancing to the right.
Figure 2.4: Front velocity in the slumping phase as a function of \(Gr\) number. Planar refers to the planar lock-exchange configuration and Cylindrical to the finite volume release in cylindrical configuration. The open square is the outcome of the simulation by Härtel et al. (2000b) in good agreement with our results.
Figure 2.5: Three-dimensional cylindrical current for $Gr = 1.5 \times 10^6$ and $Sc = 1$. Flow visualized by an isosurface of density $\tilde{\rho} = 0.25$. The figure shows only one quarter of the simulation domain. At $\tilde{t} = 0$ the cylindrical region has $\tilde{\rho} = 1$ and everywhere outside it $\tilde{\rho} = 0$. The flow starts as two dimensional, but soon after it develops three-dimensional instabilities at the front. The flow becomes completely three dimensional eventually.
Figure 2.6: Maximum value of $\tilde{\rho}$ ($\tilde{\rho}_{\text{max}}$) over time. Planar refers to the planar lock-exchange configuration and Cylindrical to the finite volume release in cylindrical configuration, both for $Gr = 1.5 \times 10^6$. The value of $\tilde{\rho}_{\text{max}}$ in the current is related to the front velocity. While $\tilde{\rho}_{\text{max}} = 1$ the current moves at approximately constant velocity.
Figure 2.7: Lobe and cleft instability in a cylindrical current. Frame (a): visualization of the front in a laboratory experiment for $Gr \sim 10^8$ and $Sc = 700$ using the same geometrical configuration of the numerical simulations. Frame (b): numerical result for $Gr = 1.5 \times 10^6$ and $Sc = 1$. 
Figure 2.8: Three-dimensional cylindrical current for $Gr = 10^5$ and $Sc = 1$. Flow visualized by surface of density $\tilde{\rho} = 0.25$. The figure shows only one quarter of the simulation domain. For this case, the pattern of lobes and clefts is not observed.
Figure 2.9: Mean flow of a cylindrical current for $Gr = 1.5 \times 10^6$ and $Sc = 1$. Flow visualized by density contours. The main vortex structures are indicated in the figure. The dynamic of the vortical structures is in complete agreement with the experimental results of Alahyari & Longmire (1996) for $Gr \sim 10^7$. 
Figure 2.10: Mean flow of a cylindrical current for $Gr = 10^5$ and $Sc = 1$. Flow visualized by density contours. For this case the flow presents weak vortex structures.
Chapter 3

On the front velocity of gravity currents

3.1 Introduction

Gravity currents (also called density currents) are buoyancy-driven flows which manifest either as a horizontal current of heavy fluid running below light fluid, or as a current of light fluid above heavy fluid. In some applications the gravity current manifests as a combination of these two, and in this case, they are also called intrusions. Gravity currents can be produced with very small density differences (of a few percent), yet they can still travel for very long distances (García, 1992). Examples of these flows are thunderstorm fronts, volcanic eruptions, oil spills on the ocean, and snow avalanches (Simpson, 1997; Allen, 1985).

The need to predict the arrival time of a gravity current’s front and the maximum spreading distance has motivated the development of relatively simple models (Allen, 1985) all the way to detailed simulations (Lee & Wilhelmson, 1997a,b; Härtel et al., 2000b; Necker et al., 2002; Özgökmen et al., 2004; Necker et al., 2005; Cantero et al., 2006). The first theoretical attempt to describe the spreading rate of a gravity current using potential flow theory was made by von Kármán (1940). He showed that a deeply submerged heavy fluid current of density \( \rho_1 \) will move into a semi-infinite lighter environment of density \( \rho_0 \) with a mean front velocity of \( u_F = \sqrt{2g' h_F} \), where \( g' = g(\rho_1 - \rho_0)/\rho_0 \) is the reduced gravity and \( h_F \) is the height of the current. Later, Benjamin (1968) arrived at the same conclusion with a more precise analysis using the theory of hydraulic jumps. More recent work by Shin et al. (2004) has shown the importance of including
both the front of the gravity current and the backward propagating disturbance in the analysis.

Based on careful experimental observations, Huppert & Simpson (1980) described the spreading of a gravity current in three phases: an initial *slumping phase* where the current moves at nearly constant speed, followed by an *inertial phase* in which the current moves under the balance of buoyancy and inertial forces, and finally a *viscous phase* where viscous effects dominate and balance buoyancy. Power law expressions for the self-similar evolution of the front have been obtained for both the inertial and viscous regimes (Fay, 1969). Fannelop & Waldman (1971) and Hoult (1972) have later shown that the power law expressions result from a similarity solution of the shallow-water equations and therefore are valid only sufficiently long after the initial release (see also Rottman & Simpson, 1983; Bonnecaze *et al.*, 1993; Choi & García, 1995; Huppert, 1998; Bonnecaze & Lister, 1999; Ungarish & Zemach, 2003).

Planar and cylindrical gravity currents are two canonical configurations that have been studied in the past. The lock-exchange problem in a rectangular channel is a well-studied example of a planar gravity current, where the heavy fluid moves away from the lock with a nominally straight front. On the other hand, the release of a finite column of heavy fluid into a surrounding ambient of light fluid results in a cylindrical gravity current (Penney *et al.*, 1952; Spicer & Havens, 1987). In the cylindrical configuration the front is nominally circular and propagates radially outward. In a planar current, as the front propagates, the planform area of the released heavy fluid increases linearly with front location, whereas in a cylindrical current, the planform increases quadratically. This difference changes the spreading rate of the cylindrical current compared to the planar case.

In the lock-exchange problem, as the front of heavy fluid moves away from the lock, a disturbance is also formed which propagates in the opposite direction into the lock. In the case of a finite volume release, this backward propagating disturbance reflects off
the back wall, or the symmetry plane, and begins to propagate forward. The reflected disturbance travels faster than the front of the gravity current, eventually catching up with the front. The near constant velocity of the current (the slumping phase) continues up to this point and, as observed by Rottman & Simpson (1983), the arrival of the reflected wave at the front initiates the transition to the inertial phase. The volume of heavy fluid (per unit width of the channel) released behind the lock plays an important role as to when this transition to the inertial phase occurs. In the limit of an infinite release the slumping phase persists indefinitely for high Reynolds numbers ($Re$). At lower $Re$ viscous effects will eventually reduce the front velocity, and in the case of finite releases the inertial phase may not exist at all, and the current may directly transition from the slumping phase to the viscous phase (Rottman & Simpson, 1983; Amy et al., 2005). Although the physical mechanism of this transition has not been nearly as well understood, it can be expected that the volume of release will play also a role as to when such direct transition from the slumping to the viscous phase will occur.

If the effect of the sidewalls can be neglected, then the planar current can be considered statistically two-dimensional (2D) and homogeneous along the spanwise direction. Similarly, a cylindrical current is statistically axisymmetric and homogeneous along the circumferential direction. The two-dimensionality of the planar current and the axisymmetry of the cylindrical current have been exploited both in theoretical formulations and in computations (Daly & Pracht, 1968; Droegemeier & Wilhelmson, 1986, 1987; Terez & Knio, 1998a,b; Hallworth et al., 2001; Özgökmen & Chassignet, 2002; Ungarish & Zemach, 2003; Birman et al., 2005; Patterson et al., 2006). At high $Re$, gravity currents are strongly three-dimensional (3D) and fully turbulent. In such situations the two-dimensionality or the axisymmetry of the current is only in a statistical sense.

The effect of three-dimensionality on the speed of the current can be expected to be significant. For example, Cantero (2002) and Cantero et al. (2003) reported numerical simulations of planar gravity currents developing over a favorable slope and compared
the results with experimental data. They found that a 3D simulation captures the experimental observations more accurately and that the 2D model lags behind. Similar results have been reported by Necker et al. (2002) in the context of particulate gravity currents. Considering the front of the heavy fluid as a bluff body intruding into the light fluid, a crude analogy with the drag force on a cylindrical body subjected to cross flow can be drawn for which 2D model significantly overpredicts the drag force (Mittal & Balachandar, 1995). Thus, a weaker resistance to the flow can be expected in the case of a 3D front than for the case of a coherent 2D front, which could explain the faster spreading of 3D currents.

The primary objective of the present work is to study systematically the propagation of gravity currents and to investigate the influence of key parameters on the front velocity as well as the transition between phases. In this work we present results from highly resolved simulations of planar and cylindrical gravity currents for varying Re. For the planar case, we consider both small and large release volumes. A detailed comparison of the results illustrates the role of the planar vs. cylindrical nature of the current as well as the effect of the volume of release on the mean velocity of the front and the transition between the different phases. In each of these cases we consider both fully resolved 3D simulations and corresponding 2D or axisymmetric simulations for planar and cylindrical configurations, respectively. The 3D structure of the propagating front is also explored for both the planar and cylindrical configurations.

### 3.2 Numerical formulation

The physical configuration of the gravity currents is shown in figure 3.1. At the start of the computation the region with heavy fluid of density $\rho_1$ (shown in figure 3.1 as the shaded region) is separated from the light fluid of density $\rho_0$. In the planar case, the heavy fluid is a slab of half width $x_0$ along the flow direction. In the present
Figure 3.1: Sketch of a gravity current and nomenclature used in this work. The flow is started from the initial condition shown by the shaded region between dashed lines. As the flow evolves, the intruding front develops the structure of a head followed by a body.

simulations the slab of heavy fluid extends over the entire height \( H \) of the channel (full-depth release) and infinitely along the spanwise (\( y \)) direction. In the cylindrical case, the region of heavy fluid is a cylinder of radius \( r_0 \) and height \( H \).

We consider flows in which the density difference is small enough that the Boussinesq approximation is valid. The dimensionless equations read

\[
\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_i}{\partial \tilde{x}_k} = \tilde{\rho} \varepsilon_i^g - \frac{\partial \tilde{p}}{\partial \tilde{x}_i} + \frac{1}{Re} \frac{\partial^2 \tilde{u}_i}{\partial \tilde{x}_k \partial \tilde{x}_k},
\]

(3.1)

\[
\frac{\partial \tilde{u}_k}{\partial \tilde{x}_k} = 0,
\]

(3.2)

\[
\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial}{\partial \tilde{x}_k} (\tilde{\rho} \tilde{u}_k) = \frac{1}{Sc} \frac{\partial^2 \tilde{\rho}}{\partial \tilde{x}_k \partial \tilde{x}_k}.
\]

(3.3)

Here \( \tilde{u}_i \) is the dimensionless velocity vector, and \( \varepsilon_i^g \) is a unit vector pointing in the gravity direction. Any variable with a tilde on top is to be understood as dimensionless.

The channel height \( H \) is the length scale and we adopt \( U = \sqrt{g'H} \) as the velocity scale. Consequently, the time scale is \( H/U \). The dimensionless density and pressure are given by

\[
\tilde{\rho} = \frac{\rho - \rho_0}{\rho_1 - \rho_0}, \quad \text{and} \quad \tilde{p} = \frac{p}{\rho_0 U^2}.
\]

(3.4)

The two dimensionless parameters in equations (3.1)–(3.3) are the Reynolds and Schmidt
numbers defined as

\[ Re = \frac{U H}{\nu} = \sqrt{\frac{g' H^3}{\nu^2}} \quad \text{and} \quad Sc = \frac{\nu}{\kappa}, \]

(3.5)

respectively, where \( \nu \) is the kinematic viscosity and \( \kappa \) is the diffusivity of temperature or chemical species producing the density difference. The ratios \( x_0/H \) (planar case) or \( r_0/H \) (cylindrical case) are additional geometric parameters that control the volume of initial release. In the planar configuration, we consider the cases of both a small volume of release with \( \tilde{x}_0 = 1 \) (small-release case) and a large volume of release with \( \tilde{x}_0 = \tilde{L}_x/4 \) (large-release case), where \( \tilde{L}_x \) is the length of the computational domain in the spreading direction. Thus in the large-release simulations half the computational domain is filled with the heavy fluid and serves to approximate the infinite-release case.

In the 3D planar simulations the governing equations are solved in a rectangular box of size \( \tilde{L}_x \times \tilde{L}_y \times \tilde{L}_z \). Periodic boundary conditions are employed along the streamwise (\( \tilde{x} \)) and spanwise (\( \tilde{y} \)) directions. Periodicity along the streamwise direction implies that a periodic array of planar gravity currents, each initially separated by a distance \( \tilde{L}_x \), is being simulated. The box is typically taken to be very long along the streamwise direction (25 or more channel heights) in order to allow free unhindered development of the current for a long time (see figure 3.1). Based on simulation results we observe that the interaction of the front with the adjacent currents across the periodic boundaries can be neglected till the front reaches about 1 dimensionless unit from the boundaries. Along the spanwise direction the width of the periodic domain is chosen to be 1.5 dimensionless units, which is adequate to include several spanwise lobe and cleft structures. These choices for the computational domain are consistent with that of Härtel et al. (2000b).

The 3D cylindrical simulations are also in a rectangular box of size \( \tilde{L}_x \times \tilde{L}_y \times \tilde{L}_z \), however, since the current spreads radially outward along the entire \( \tilde{x} – \tilde{y} \) plane we choose \( \tilde{L}_x = \tilde{L}_y \). Periodic boundary conditions are employed along the (\( \tilde{x} \)) and (\( \tilde{y} \))
directions and thus here we approximate a doubly periodic array of cylindrical gravity currents with a lateral spacing of $\tilde{L}_x = \tilde{L}_y$ along the horizontal directions. The planform of the periodic box is typically taken to be very large (15 channel heights) in order to allow unhindered development of the current for a long time. As in the planar case, the interaction of the radially advancing front with the adjacent currents across the periodic boundaries becomes significant only as the front reaches within 1 dimensionless unit from the boundaries.

The 2D planar simulations are in a rectangular domain of size $\tilde{L}_x \times \tilde{L}_z$ and the flow is taken to be invariant along the spanwise ($\tilde{y}$) direction. The axisymmetric cylindrical simulations are on the $\tilde{r} - \tilde{z}$ (radial-axial) plane in a rectangular computational domain of size $\tilde{L}_r \times \tilde{L}_z$, and the flow is invariant along the circuferential ($\theta$) direction.

In this work we have employed two different numerical techniques: a spectral multi-domain code (Deville et al., 2002) and a fully de-aliased pseudo-spectral code (Canuto et al., 1988). The spectral multi-domain code was used to simulate axisymmetric cylindrical currents while the spectral code was used in all other simulations. Table 3.1 lists all the simulations to be discussed in this paper.

In the spectral code, Fourier expansions are employed for the flow variables along the horizontal directions ($\tilde{x}$ and $\tilde{y}$). In the inhomogeneous vertical direction ($\tilde{z}$) a Chebyshev expansion is used with Gauss-Lobatto quadrature points (Canuto et al., 1988). The flow field is time advanced using a Crank-Nicolson scheme for the diffusion terms. The advection terms are handled with the Arakawa method (Durran, 1999) and advanced with a third-order Runge-Kutta scheme. The buoyancy term is also advanced with a third-order Runge-Kutta scheme. More details on the implementation of this numerical scheme can be found in Cortese & Balachandar (1995). In the simulations, periodic boundary conditions are enforced along the horizontal directions for all variables. At the top and bottom walls no-slip and zero-gradient conditions are enforced for velocity and density, respectively.
Table 3.1: Numerical simulations performed for this study. The table specifies size of the domain, numerical method and resolution for each geometrical setting. Size of domain specified as \(\tilde{L}_x \times \tilde{L}_y \times \tilde{L}_z\) in 3D configuration, as \(\tilde{L}_x \times \tilde{L}_z\) in 2D planar configuration, and as \(\tilde{L}_r \times \tilde{L}_z\) in axisymmetric configuration. Resolution specified as \(N_x \times N_y \times N_z\) in 3D configuration, as \(N_x \times N_z\) in 2D planar configuration, and as \(N_r \times N_z\) in axisymmetric configuration. *The domain size for this run is \((10 \times 10 \times 1)\).
The spectral multi-domain code employs a domain decomposition methodology along the radial direction ($\tilde{r}$) (Balachandar & Parker, 2002). The entire computational domain of length $\tilde{L}_r$ is divided into $N_e$ subdomains. Within each subdomain Chebyshev expansions are used along both, radial ($\tilde{r}$) and vertical ($\tilde{z}$) directions. The non-linear and buoyancy terms are treated explicitly using the third-order Adams-Bashforth scheme. The diffusion terms are treated implicitly with the Crank-Nicolson scheme. Appropriate symmetry boundary conditions are employed for all variables along the centerline ($\tilde{r} = 0$). At the outer radial boundary ($\tilde{r} = \tilde{L}_r$) a non-reflecting outflow boundary is used (Mittal & Balachandar, 1996). The two different codes were verified to yield identical results for the 2D planar problem.

In all the configurations the initial density was smoothly varied from 0 to 1 over a very thin region located at the interface. The details of the initial conditions used in this work can be found in Cantero et al. (2006). The flow was started from rest with a minute random disturbance prescribed in the density field.

In this work three different $Re$ are considered: $Re = 895, 3450$ and $8950$. These correspond to Grashof numbers of $10^5, 1.5 \times 10^6$ and $10^7$, respectively, and the intermediate case can be directly compared with that of Härtel et al. (2000b). As will be discussed below, with increasing $Re$ the complexity of the flow increases and thus, the simulation at the higher $Re$ requires increased resolution. The numerical resolution for each simulation was selected to have between 6 and 8 decades of decay in the energy spectrum for all the variables. The time step was selected to produce a Courant number smaller than 0.5.

### 3.3 Theoretical background

Several theoretical and empirical models have been proposed to predict the front velocity during the slumping, inertial and viscous phases of the current. In this section, we
will briefly describe some of the models that are of immediate relevance to subsequent discussion.

3.3.1 Slumping phase

Benjamin (1968) analyzed the flow in a planar, 2D emptying cavity using the hydraulic theory in a frame of reference moving with the front. The only free parameter is the ratio of the depth of the current \( (h_F) \) to the depth of the ambient fluid \( (H) \). The theory does not distinguish between head height and body height and in relation to figure 3.1 \( h_B = h_H = h_F \). Benjamin derived the following expression for the Froude number of the front

\[
F_B = \frac{u_F}{\sqrt{g' H}} = \left[ \tilde{h}_F \left( 2 - \tilde{h}_F \right) \left( \frac{1 - \tilde{h}_F}{1 + \tilde{h}_F} \right) \right]^{1/2},
\]

where \( \tilde{h}_F = h_F/H \) is the dimensionless current height and \( u_F \) is the dimensional front velocity. Benjamin argued that without external energy input the largest possible current height can only be \( h_F = H/2 \). In this limit of \( \tilde{h}_F \to 1/2 \), \( F_B \to 1/2 \), and energy losses are associated with currents of smaller depth. Benjamin further argued that dissipation is an essential ingredient of gravity current flow and therefore \( h_F/H \leq 0.347 \). With \( h_F/H = 0.347 \), a maximum dissipation is achieved within Benjamin’s framework.

Shin et al. (2004) revisited and extended Benjamin’s theory and stressed the need to take into account the exchange between the advancing front of the gravity current and the backward propagating disturbance. They showed that for both the full-depth and partial-depth lock-exchange releases, the height of a energy-conserving current is half the initial depth of the release. The corresponding speed of the current depends only on the dimensionless initial depth of the release \( (\tilde{D}) \)

\[
F_S = \frac{1}{2} \sqrt{\tilde{D} \left( 2 - \tilde{D} \right)}.
\]
The Froude number definition in the above equation is the same as in (3.6) and the subscripts "B" and "S" indicate relations proposed by Benjamin (1968) and Shin et al. (2004). In the limit of full-depth release (i.e., $\tilde{D} \to 1$) the above expression yields the same Froude number as Benjamin’s expression.

Based on experimental measurements Huppert & Simpson (1980) proposed the following empirical expression for the Froude number (defined as in equation (3.6))

$$F_{HS} = \begin{cases} 
\frac{1}{2} \tilde{h}_F^{1/6} & \text{if } 0.075 \leq \tilde{h}_F \leq 1, \\
1.19 \tilde{h}_F^{1/2} & \text{if } 0 \leq \tilde{h}_F \leq 0.075 
\end{cases} \quad (3.8)$$

where $\tilde{h}_F$ is to be interpreted as the dimensionless height of the body of the current. The above empirical expression predicts the Froude number to monotonically decrease to zero as the height of the current decreases to zero. The above expression yields $F_{HS} = (1/2)^{7/6}$ in the limit $\tilde{h}_F \to 1/2$, which is somewhat lower than the value $1/2$ predicted by the hydraulic theories of both Benjamin (1968) and Shin et al. (2004). In the limit of deeply-submerged current ($\tilde{h}_F \to 0$), the above empirical expression yields $F_{HS} \to 1.19 \sqrt{\tilde{h}_F}$, which is higher than $F_S \to \sqrt{\tilde{h}_F}$ predicted by Shin et al. (2004), but lower than $F_B \to \sqrt{2\tilde{h}_F}$ predicted by Benjamin (1968).

### 3.3.2 Inertial phase

Transition from the slumping to the inertial phase occurs when the reflected back propagating wave catches up with the front (Rottman & Simpson, 1983). It is well-accepted that for a planar current the transition happens after the front has traveled between 5 to 9 lock lengths (Rottman & Simpson, 1983; Metha et al., 2002; Marino et al., 2005). The asymptotic behavior of the current in the inertial phase has been established to be (Fay, 1969; Fannelop & Waldman, 1971; Hoult, 1972; Huppert & Simpson, 1980; Rottman &
for planar currents, and

\[ \tilde{r}_F = \pi^{1/4} \xi_c \tilde{h}_0^{1/4} (\tilde{r}_0 \tilde{t})^{1/2} \quad \text{and} \quad \tilde{u}_F = \frac{1}{2}\pi^{1/4} \xi_c \tilde{h}_0^{1/4} \tilde{r}_0^{1/2} \tilde{t}^{-1/2} \]  

(3.10)

for cylindrical currents. Here \( \tilde{x}_F \) and \( \tilde{r}_F \) are the dimensionless streamwise and radial location of the planar and cylindrical currents, respectively. The initial size of the release is characterized by its height \( \tilde{h}_0 \), and length \( \tilde{x}_0 \) or radius \( \tilde{r}_0 \). The difference between the theories is in the constants \( \xi_p \) and \( \xi_c \). For the planar current, \( \xi_p = 1.6 \) and 1.47 have been proposed by Hoult (1972) and Huppert & Simpson (1980), respectively, while Marino et al. (2005) suggests a range from 1.35 to 1.8. For the cylindrical current, Hoult (1972) and Huppert & Simpson (1980) have proposed \( \xi_c = 1.3 \) and 1.16, respectively.

### 3.3.3 Viscous phase

By balancing the buoyancy and viscous forces in a boundary layer approximation, Hoult (1972) obtained the following self similar solution for a planar gravity current

\[ \bar{x}_F = \xi_{pHt} \tilde{h}_0^{1/2} \tilde{x}_0^{1/2} Re^{1/8} \tilde{t}^{3/8} \quad \text{and} \quad \bar{u}_F = \frac{3}{8} \xi_{pHt} \tilde{h}_0^{1/2} \tilde{x}_0^{1/2} Re^{1/8} \tilde{t}^{-5/8}, \]

(3.11)

where \( \xi_{pHt} = 1.5 \). The corresponding results for a cylindrical current are

\[ \bar{r}_F = \xi_{cHt} \tilde{h}_0^{1/3} \tilde{r}_0^{2/3} Re^{1/12} \tilde{t}^{1/4} \quad \text{and} \quad \bar{u}_F = \frac{1}{4} \xi_{cHt} \tilde{h}_0^{1/3} \tilde{r}_0^{2/3} Re^{1/12} \tilde{t}^{-3/4}, \]

(3.12)

where \( \xi_{cHt} = 1.38 \). The viscous force considered in the analysis of Hoult (1972) is from the interface between the heavy and light fluids. A revised analysis that accounts for
the viscous effect over a rigid horizontal surface was performed by Huppert (1982). The resulting self similar solutions for the viscous phase are different from those given in equation (3.11) and (3.12), and are given by

\[
\tilde{x}_F = \xi_{pHp} \tilde{h}^{3/5} \tilde{x}_0^{3/5} \tilde{t}^{1/5} \quad \text{and} \quad \tilde{u}_F = \frac{1}{5} \xi_{pHp} \tilde{h}^{3/5} \tilde{x}_0^{3/5} \tilde{t}^{-4/5},
\]

(3.13)

for planar currents where \( \xi_{pHp} = 1.133 \), and by

\[
\tilde{r}_F = \xi_{cHp} \tilde{h}^{3/8} \tilde{r}_0^{3/4} \tilde{t}^{1/8} \quad \text{and} \quad \tilde{u}_F = \frac{1}{8} \xi_{cHp} \tilde{h}^{3/8} \tilde{r}_0^{3/4} \tilde{t}^{-7/8}
\]

(3.14)

for a cylindrical current where \( \xi_{cHp} = 1.197 \).

### 3.4 Results and discussion

#### 3.4.1 Current height

The height of the current can be defined in a few different ways. Shin et al. (2004) and Marino et al. (2005) define a local equivalent height in an unambiguous way as

\[
\tilde{h}(\tilde{x}, \tilde{y}, \tilde{t}) = \int_0^1 \rho \, d\tilde{z},
\]

(3.15)

and herein this definition is adopted. Thus, at locations where the entire layer is occupied by the heavy fluid the dimensionless height is unity, while at locations where the light fluid fills the entire layer the height is zero. The local current height can then be averaged over the span in the case of a 3D planar current, or along the circumferential direction in the case of a 3D cylindrical current, to define the span-averaged current height\(^1\) as

\[
\overline{h}(\tilde{x}, \tilde{t}) = \frac{1}{L_y} \int_0^{L_y} \tilde{h} \, d\tilde{y} \quad \text{or} \quad \overline{h}(\tilde{r}, \tilde{t}) = \frac{1}{2\pi} \int_0^{2\pi} \tilde{h} \, d\theta.
\]

(3.16)

\(^1\)Any variable with an overbar is to be understood as dimensionless span-averaged quantity.
The above definition for the planar current is equivalent to the width-averaged dye concentration obtained in experiments (Shin et al., 2004; Marino et al., 2005).

Figure 3.2 shows the time evolution of the large-release planar gravity current at Re 895 and 8950 plotted in terms of the span-averaged equivalent current height. The corresponding results for the small-release planar current and cylindrical current are shown in figures 3.3 and 3.4, respectively. At Re = 895 both, the planar and cylindrical currents are laminar and remain 2D and axisymmetric, respectively. At the higher Re, the currents are highly 3D and appear turbulent at both the head and over the entire body of the current. Here, the results for both the 2D (or axisymmetric in case of cylindrical current) and 3D simulations, are plotted.

At the lower Re, the current appears smooth with a well defined raised head and an extended body. In the large-release cases (figure 3.2) the height of the raised head can be observed to be less than half the initial depth of release and can be observed to slowly decrease with time. After a short dip behind the head, the height of the current gently increases to half the channel height (by symmetry $\tilde{h} = 0.5$ at $\tilde{x} = 8.5$). Birman et al. (2005) reported similar results for large-release currents with slip boundary conditions from 2D highly resolved simulations. Here we extend these results to two more cases (planar small-release and cylindrical) for 2D and 3D simulations with no-slip boundary conditions. In the small-release cases the current has a well defined raised head and an extended body of near constant height, which can be well discerned at later times. Also at early times a backward-traveling disturbance wave can be seen as a second peak, which reflects back off the symmetry plane ($\tilde{x} = 0$ or $\tilde{r} = 0$). At later times it is difficult to identify this reflected disturbance unambiguously. Nevertheless, it is likely such disturbance catches up with the front and influence the propagation speed (see Simpson, 1982; Rottman & Simpson, 1983).

At the higher Re, very large undulations can be seen in the height of the current at early times. These undulations are the result of strong roll-up of the current due
to Kelvin-Helmholtz instability of the shear layer. At much later times, the strong effect of roll-up observed during the early stages of the current is somewhat mitigated and the undulations in the current height are reduced. The effect of roll-up is much stronger in the 2D simulations, since three-dimensionality helps break up the spanwise or circumferential coherence. The averaged structure and evolution of the large- and small-release currents seen in figures 3.2 and 3.3 are qualitatively similar to the width-averaged experimental results of Shin et al. (2004) and Marino et al. (2005), respectively. At higher Re, when the flow is highly turbulent, the arrival time of the 3D small-release front is earlier than the corresponding 2D or axisymmetric approximation (see figures 3.3 and 3.4). This observation suggests that the 3D currents are faster than the 2D (or axisymmetric) approximations.

3.4.2 Mean front location and velocity

The mean front location, $\bar{x}_F(t)$ (or $\bar{r}_F(t)$), can be unambiguously defined as the location where the span-averaged equivalent height, $\bar{h}$, becomes smaller than a small threshold $\delta$. Diffusion and numerical noise prevents $\bar{h}$ from being exactly zero. The above definition is insensitive to the exact value of $\delta$ as long as it is smaller than 0.05 and here we use $\delta = 0.01$. The distance the front has advanced, $\bar{x}_F - \bar{x}_0$ (or $\bar{r}_F - \bar{r}_0$), is shown as a function of time in figure 3.5. The result for all the 2D and 3D large-release planar cases at the three different $Re$ are shown in frame (a). The corresponding results for the small-release planar and the cylindrical currents are shown in frames (b) and (c), respectively. In the log-log plots (main frames) the dashed straight line corresponds to a slope of constant front velocity. It can be seen that in all the planar cases, after a brief initial period of acceleration, a period of near constant velocity is realized. The constant velocity however seems to depend on $Re$, at least over the limited range under consideration. Although subtle, departure from the constant velocity can be observed at later times for the small-release cases. The large-release cases continue to exhibit
constant velocity over the entire duration of simulation. For the cylindrical cases a period of constant velocity is not clearly identified. There is, however, a brief period of rather slower variation before the more pronounced decay.

The mean front velocity is computed as

\[ u_F = \frac{dF}{dt} \]  

for planar cases and in an analogous way for the cylindrical configurations. The front velocity as a function of \( \tilde{t} \) for the large-release planar cases is shown in the inset of figure 3.5(a). The front velocity of the small-release planar and cylindrical currents are shown in the inset in frames (b) and (c), respectively. An acceleration phase where the velocity sharply increases, a slumping phase where the velocity is nearly constant (planar currents) or varies rather slowly (cylindrical currents), and an inertial and/or viscous phase where the front velocity decays are clearly identifiable. In what follows, these different phases will be discussed in greater detail.

**Acceleration phase**

In the acceleration phase, the front velocity rapidly increases from zero, reaches a maximum and subsequently slightly falls before approaching a constant value. This early stage of the flow was also observed in experiments by Martin & Moyce (1952a,b) and in 2D simulations by Härtel et al. (1999). Here we extend the analysis to 3D simulations and link the process to the current interface roll-up dynamics. With our simulations we study the idealized case of an instantaneous release with the gate lifted infinitely rapidly. Nevertheless, the study of this phase is important because it puts in evidence the effect of interface friction on the front speed. During this phase of spreading, 3D disturbances introduced into the current in the initial condition have not grown to sufficient amplitude and the currents are therefore predominantly 2D (or axisymmetric). Thus, the
velocity of the current predicted by 3D and 2D simulations are nearly identical in all cases considered.

Figure 3.6 shows the front velocity as a function of the distance traveled by the front during the acceleration phase. The maximum front velocity increases with increasing $Re$. In the large-release planar cases the peak values of the front velocity are 0.411, 0.465 and 0.489 at $Re = 895, 3450, \text{ and 8950}$, respectively. Based on this trend, the peak front velocity can be expected to level off and become $Re$-independent at even higher $Re$. Furthermore, in all the planar and cylindrical cases the peak front velocity occurs at a distance of about $0.3H$ from the lock-location (see figure 3.6). This result is independent of the volume released or the strength of the current (i.e. $Re$ of the current) for the conditions of our simulations. In terms of time, at $Re = 8950$ the peak occurs at about one dimensionless time unit after the release, but at lower $Re$ the peak occurs slightly later, due to the lower current speed.

The cylindrical currents also go through a peak in the front velocity, however, these peaks are in general lower than the corresponding planar values. The $Re$ for the different simulations presented in table 3.1 are based on initial conditions. As the current advances from the lock, the effective depth of the heavy fluid decreases and the instantaneous $Re$, defined based on this effective depth, also decreases. At the instance of peak velocity, using the definition of $Re$ (equation (3.5)) and the mass balances $\tau_0 \bar{H}_0 = \tau_F \bar{H}_F$ and $\tau_0^2 \bar{r}_0 = \tau_F^2 \bar{r}_F$, an instantaneous $Re$ can be defined as

$$Re_{peak} = Re\left(\frac{\tau_0}{\tau_{peak}}\right)^{3/2} \quad \text{and} \quad Re_{peak} = Re\left(\frac{\tau_0}{\tau_{peak}}\right)^{3}$$

(3.18)

for the planar and the cylindrical cases, respectively. If we use the fact that $\tau_{peak} - \tau_0 \approx \tau_{peak} - \tau_0 \approx 0.3$, then the instantaneous $Re$ at the time of peak velocity are about 1.5 and 2.2 times lower than the initial $Re$ for the small-release planar and cylindrical currents, respectively. Due to the quadratic increase in planform area as it advances,
the strength of the cylindrical current measured in terms of instantaneous $Re$ falls off more rapidly. The observed lower peak velocity for the cylindrical current is consistent with its faster decay in strength. In the large-release planar cases, due to the large initial release volume ($\bar{x}_0 = 8.5$), we now have $\bar{x}_{\text{peak}}/\bar{x}_0 \approx 1.04$. As a result, at the time of peak velocity, the instantaneous $Re$ has fallen by only 6%. The inset in figure 3.6 shows a log-log plot of the peak velocity, $\bar{u}_{F,\text{peak}}$, as a function of the $Re_{\text{peak}}$. There is better collapse of $\bar{u}_{F,\text{peak}}$ as a function of $Re_{\text{peak}}$. Admittedly the above estimates are crude, but they provide qualitative support for the observed differences in peak velocity between the large- and small-release planar, and cylindrical currents.

The rapid increase in front velocity is to be expected, but the slight decrease before reaching a near constant value needs further investigation. A close look at the front during the acceleration phase is shown in figure 3.7, where at several time instances the front marked by the contour of span-averaged $\bar{\rho} = 0.5$ is plotted along with span-averaged spanwise vorticity contours for the 3D small-release planar case at $Re = 8950$. The results for all other cases are qualitatively similar. At around $\tilde{t} = 0.8$ the head of the current as indicated by the $\bar{\rho} = 0.5$ contour begins to lift up. Correspondingly a local peak in spanwise vorticity begins to develop (not yet visible in the plot) indicating an incipient roll up process. The roll up process intensifies till $\tilde{t} \approx 2.5$, which corresponds to about $\bar{x}_F - \bar{x}_0 \approx 1$ when the front velocity levels off to a constant value (see figure 3.6). The decrease in front velocity following the peak clearly occurs alongside the roll up of the interface between the advancing heavy and the retreating light fluid. Similar findings were reported by H"artel et al. (1999) for the case of slip surfaces indicating that the bottom boundary layer plays no role in this process.

Simple arguments based on gravitational free fall starting from rest can be made to obtain a scale estimate for the dimensional acceleration time to peak velocity as $t_{\text{peak}} \approx u_{F,\text{peak}}/g'$. The scale for the front location at peak velocity can correspondingly be estimated as $x_{\text{peak}} - x_0 \propto H$. The computational results are in agreement with these
simple scale estimates.

Slumping phase

From the insets in figure 3.5 it can be observed that both the large- and small-release planar cases approach a near constant velocity after the acceleration phase. In the cylindrical currents a period of near constant velocity is not observed. The velocity, however, shows a brief period of rather slower variation before exhibiting a more pronounced decay of $\bar{t}^{-1/2}$ in the inertial phase (this can be better seen in figure 8). Here we take the duration of slow variation to be the approximate slumping phase and obtain a mean value for the average sluming phase velocity.

The near constant velocity of the slumping phase for the different cases considered herein is shown in table 3.2 along with previously reported results from experiments and numerical simulations. We observe from our simulations that the front velocity in the slumping phase remains the same for both the large- and small-release planar cases, i.e. the constant velocity is independent of the released volume. It is, however, dependent on the planar vs. cylindrical nature of the current and also shows a Reynolds number dependence, over the range considered. Furthermore, from the insets in figure 3.5, it can be observed that front velocity in the slumping phase is well predicted by 2D approximations. Necker et al. (2002) reported similar results in the context of small-release planar particulate gravity currents. Here we extend this analysis to large-release planar currents and cylindrical currents.

The large versus small release planar cases examine two basic effects of the aspect ratio (length over height) of the release, which varied from 8.5 for the large release to 1 for the small release, on the front velocity. The first is the effect on the value of the front velocity. The second is the duration of the constant velocity or slumping phase. Based on the similarity in the behavior of the current front velocity during the acceleration and the slumping phases, it can be concluded that the aspect ratio does not
affect significantly the value of the front velocity during these two phases of spreading despite the fact that the currents look structurally very different due to the nature of the release.

The aspect ratio has an effect, however, on the duration of the slumping phase. In the large-release cases the constant velocity phase persists till the end of the simulation for the larger two $Re$. For the lower $Re$, viscous effects reduce the front velocity for $\tilde{t} > 10$. In the small-release cases, however, the constant velocity slumping phase extends over only a finite period for all the $Re$. Figure 3.8 presents the front velocity for the 3D finite-volume release simulations together with the scaling laws presented in the theory section. Frame (a) presents the results for the small-release planar case for the three $Re$ considered, and frame (b) presents the results for the cylindrical case for the three $Re$ considered. For comparison, this figure also includes experimental data from Marino et al. (2005) for the planar case at $Re = 6360$ and $8620$ with $\overline{x}_0 = 1$ and $\overline{h}_0 = 1$, and from Hallworth et al. (2001) for the cylindrical case at $Re = 1.52 \times 10^5$ and $\bar{r}_0 = 1.25$ and $\overline{h}_0 = 0.965$. In this last case the comparison is not totally fair because the initial condition geometric parameters $\overline{r}_0$ and $\overline{h}_0$ are not exactly 1. Nevertheless, the agreement of our numerical results with the experimental data is good for both cases. Further comparison with more experiments is presented in the following sections.

From figure 3.8(a) it can be estimated that in the small-release planar case at $Re = 8950$ the constant velocity phase is observed over $3 < \tilde{t} < 12$. The corresponding front location interval is $1 < \overline{x}_F - \overline{x}_0 < 5$ (5 lock lengths). During this period the dimensionless height of the current at the head remains nearly constant around 0.4 (see figure 3.3), which is in very good agreement with the experimental results reported in Marino et al. (2005). This indeed is also the maximum height of the current and it continues to decrease to lower values upstream of the head. For the cylindrical case it can be estimated from figure 3.8(b) that for $Re = 8950$ the nearly constant velocity phase is observed over $2 < \tilde{t} < 4.5$. This corresponds to the front location interval
<table>
<thead>
<tr>
<th>Reference</th>
<th>$Re$</th>
<th>Planar large</th>
<th>Planar small</th>
<th>Cylindrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present work 3D</td>
<td>895</td>
<td>0.361</td>
<td>0.361</td>
<td>0.318</td>
</tr>
<tr>
<td>Present work 3D</td>
<td>3450</td>
<td>0.407</td>
<td>0.407</td>
<td>0.368</td>
</tr>
<tr>
<td>Present work 3D</td>
<td>8950</td>
<td>0.421</td>
<td>0.421</td>
<td>0.408</td>
</tr>
<tr>
<td>Huppert &amp; Simpson (1980)†</td>
<td></td>
<td></td>
<td></td>
<td>0.445</td>
</tr>
<tr>
<td>Huppert &amp; Simpson (1980) Exp 5</td>
<td>25900</td>
<td></td>
<td></td>
<td>0.402</td>
</tr>
<tr>
<td>Rottman &amp; Simpson (1983)</td>
<td>9900 – 56000</td>
<td></td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>Bonnecaze et al. (1995)</td>
<td>33950</td>
<td></td>
<td></td>
<td>0.42</td>
</tr>
<tr>
<td>Härte et al. (2000b) 3D</td>
<td>3450</td>
<td>0.402</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Härte et al. (2000b) 2D</td>
<td>~ 10900</td>
<td>0.429</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Necker et al. (2002) 3D‡</td>
<td>6325</td>
<td></td>
<td>0.409</td>
<td></td>
</tr>
<tr>
<td>Shin et al. (2004)</td>
<td>1000 &lt;</td>
<td>0.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marino et al. (2005)</td>
<td>2790 – 133000</td>
<td></td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>Hallworth et al. (2001)</td>
<td>152000</td>
<td></td>
<td></td>
<td>0.42</td>
</tr>
</tbody>
</table>

Table 3.2: Mean front velocity, $\bar{u}_F$, in the slumping phase. The table shows the results for the present work from 3D simulations. For comparison, previously published data is also presented. Data by Härtel et al. (2000b) and Necker et al. (2002) are from highly-resolved numerical simulations, and data by Huppert & Simpson (1980), Rottman & Simpson (1983), Shin et al. (2004), Marino et al. (2005) and Hallworth et al. (2001) are from laboratory experiments. †Based on equation (3.8) with $\dot{h}_F = 1/2$. ‡Necker et al. (2002) report the front location for particulate gravity currents and also a case with zero particle settling velocity, which was used here. For the cylindrical currents average values during the period of slow variation are reported, which are representative of the mean velocity during the slumping phase.
0.4 < \bar{r}_F - \bar{r}_\theta < 1.7 \) (about 1.5 lock radii). The predicted value of the front velocity for the larger \( Re \) simulation is 0.408, which is only about 3\% lower than the value reported by Hallworth \textit{et al.} (2001) at \( Re = 152000 \) (see table 3.2).

The dimensionless front velocity (or the Froude number) of the planar currents is substantially lower than \( 1/2 \), the value predicted by the hydraulic theories for full-depth release. This result is consistent with the numerical predictions of Härtel \textit{et al.} (2000\textit{b}) and the experimental observation of Shin \textit{et al.} (2004) that the currents in their constant velocity phase are consistently slower than the theoretical prediction by as much as 10\% (see table 3.2). In particular, for the full-depth release, Shin \textit{et al.} (2004) experimental result yielded a Froude number of about 0.45 (see their figure 14 for \( D/H = 1 \)). Similar results were reported by Rottman & Simpson (1983) and Marino \textit{et al.} (2005) for the constant velocity phase of finite volume releases (see table 3.2). Shin \textit{et al.} (2004) conjectured that the deviation was primarily due to drag at the bottom wall. The effect of bottom friction was investigated earlier by Härtel \textit{et al.} (2000\textit{b}).

**Inertial and viscous phases**

From figure 3.8(a) it can be observed that the small-release planar currents for the lower two \( Re \) depart from the constant velocity phase at \( \tilde{t} \approx 12 \), after they have traveled about 4 lock lengths (\( \bar{x}_F \approx 5 \)) for \( Re = 895 \) and about 5 lock lengths (\( \bar{x}_F \approx 6 \)) for \( Re = 3450 \). After the transition, the front velocity follows a decaying law with a slope in good agreement to the viscous phase predictions presented in the theory section. For the case of the small-release planar current with \( Re = 8950 \), the front velocity departs from the constant velocity phase at \( \tilde{t} \approx 12 \) after the current has traveled about 5 lock lengths (\( \bar{x}_F \approx 6 \)). The front velocity follows the inertial phase scaling by Huppert & Simpson (1980) with good agreement until \( \tilde{t} \approx 17.3 \) (\( \bar{x}_F \approx 8 \)), and after this time it departs from the inertial phase scaling and falls off more rapidly following the viscous phase scaling laws. It can also be observed from this figure that the quantitative prediction of the
viscous phase scaling laws is poor. Figure 3.8(a) includes the viscous phase predictions for \( Re = 8950 \) and it can be clearly observed that both predictions by Hoult (1972) and Huppert (1982) underestimate the corresponding numerical result.

For the cylindrical current the situation is similar. From figure 3.8(b) it can be observed that the current for the lower \( Re = 895 \) leaves the nearly constant velocity phase at \( \tilde{t} \approx 5 \) after it has traveled about 1.6 lock radii (\( \tilde{r}_F \approx 2.6 \)). After this time the front velocity falls off rapidly with a slope in agreement with the viscous phase theoretical predictions. For the cases of the larger two \( Re \), the front velocity leaves the nearly constant velocity phase at \( \tilde{t} \approx 4.5 \) (\( \tilde{r}_F \approx 2.7 \)) and follows the inertial phase prediction by Huppert & Simpson (1980) with good agreement up to \( \tilde{t} \approx 17 \) (\( \tilde{r}_F \approx 5.7 \)). At this time the front velocity leaves the inertial phase and enters a phase of faster decaying. Included in 3.8(b) are the viscous scaling laws by Hoult (1972) and Huppert (1982) for \( Re = 8950 \). Again, the viscous scaling laws underpredict the simulations results.

### 3.4.3 Transition between phases of spreading

The transition point between the different phases of spreading can be computed by matching front velocity from the corresponding scaling laws at the time of transition. Before matching, the first step is to obtain the prefactors for the different scaling laws to be applied in the slumping, inertial and viscous phases. A large collection of experimental data, covering a range of width and depth of release (\( \tilde{x}_0 \) or \( \tilde{r}_0 \) and \( \tilde{h}_0 \)) and \( Re \), will be used. The experimental data used to this end is reported in tables 3.3 and 3.4 for planar and cylindrical currents, respectively.

Figure 3.9 presents the front velocity as a function of time for the experimental data in tables 3.3 and 3.4 with empty symbols. The geometric and \( Re \) effects have been scaled out appropriately in the different phases and the normalized front velocities are plotted in figure 3.9. Frames (a) and (e) present the velocity data scaled for the slumping
phase for planar and cylindrical settings, respectively, frames (b) and (f) correspond to the inertial phase for planar and cylindrical settings, respectively, frames (c) and (g) correspond to the viscous phase based on the theory of Hoult (1972) for planar and cylindrical settings, respectively, and frames (d) and (h) correspond to the viscous phase based on the theory of Huppert (1982) for planar and cylindrical settings, respectively. Included in these figures are also the best fit to the data, where the theoretical power law exponents are preserved and only the prefactor is optimized. In the slumping phase from the best fit we obtain $F_{p,sl} = 0.45 \bar{h}_0^{1/2}$ and $F_{c,sl} = 0.42 \bar{h}_0^{1/2}$ for planar and cylindrical currents, respectively. Here, $F_{p,sl}$ and $F_{c,sl}$ denote the constant numerical value of $\bar{u}_p$ during the slumping phase for planar and cylindrical currents, respectively. In the inertial phase the best fits yield $\xi_p = 1.47$ and $\xi_c = 1.23$ for planar and cylindrical currents, respectively (see equations (3.9) and (3.10)). In the viscous phase the best fits yield $\xi_{pHt} = 1.87$ and $\xi_{cHt} = 2.6$ for the theory of Hoult (1972) (see equations (3.11) and (3.12)), and $\xi_{pHp} = 3.2$ and $\xi_{cHp} = 4.64$ for the theory of Huppert (1982) (see equations (3.13) and (3.14)). In figure 3.9 the data from our simulations is also included with filled symbols. Three things can be observed in these figures: 1) the agreement between numerical and experimental results is good, 2) as $Re$ increases the numerical results approach the best fit, and 3) the larger $Re$ simulations seem to capture appropriately every phase of spreading for both geometrical configurations.

**Transition times for planar currents**

Matching the constant front velocity of the slumping phase, $F_{p,sl}$, with the front velocity of the inertial phase given in equation (3.9), the transition time from slumping to inertial phase, $\tilde{t}_{SI}$, of a planar current can be obtained as

$$\tilde{t}_{SI} = \left( \frac{2}{3} \xi_p \right)^3 \frac{\bar{x}_0 \bar{h}_0}{F_{p,sl}^3}.$$  (3.19)
Table 3.3: Experimental data used to revisit the scaling laws during the phases of spreading for planar currents with finite volume release. Here $Re_{h_0} = \sqrt{g' h_0^3 / \nu^2}$.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$Re_{h_0}$</th>
<th>$\tilde{h}_0$</th>
<th>$\tilde{x}_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amy <em>et al.</em> (2005) Exp A0-1</td>
<td>2220</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bonnecaze <em>et al.</em> (1993)</td>
<td>71500</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Marino <em>et al.</em> (2005) Exp 1</td>
<td>2790</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Marino <em>et al.</em> (2005) Exp 2</td>
<td>6360</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Marino <em>et al.</em> (2005) Exp 3</td>
<td>8620</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Marino <em>et al.</em> (2005) Exp 4</td>
<td>15500</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Huppert &amp; Simpson (1980) Exp 1</td>
<td>15700</td>
<td>1</td>
<td>2.62</td>
</tr>
<tr>
<td>Huppert &amp; Simpson (1980) Exp 2</td>
<td>28000</td>
<td>1</td>
<td>2.66</td>
</tr>
<tr>
<td>Huppert &amp; Simpson (1980) Exp 7</td>
<td>16200</td>
<td>0.34</td>
<td>0.89</td>
</tr>
<tr>
<td>Huppert &amp; Simpson (1980) Exp 9</td>
<td>42500</td>
<td>0.33</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Table 3.4: Experimental data used to revisit the scaling laws during the phases of spreading for cylindrical currents. Here $Re_{h_0} = \sqrt{g' h_0^3 / \nu^2}$.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$Re_{h_0}$</th>
<th>$\tilde{h}_0$</th>
<th>$\tilde{x}_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonnecaze <em>et al.</em> (1995)</td>
<td>34000</td>
<td>1</td>
<td>1.89</td>
</tr>
<tr>
<td>Hallworth <em>et al.</em> (2001) Exp S1</td>
<td>59000</td>
<td>0.82</td>
<td>2</td>
</tr>
<tr>
<td>Hallworth <em>et al.</em> (2001) Exp S2</td>
<td>152000</td>
<td>0.965</td>
<td>1.25</td>
</tr>
<tr>
<td>Hallworth <em>et al.</em> (2001) Exp S3</td>
<td>136000</td>
<td>0.574</td>
<td>1.25</td>
</tr>
<tr>
<td>Hallworth <em>et al.</em> (2001) Exp S7</td>
<td>201000</td>
<td>0.565</td>
<td>1.25</td>
</tr>
<tr>
<td>Huppert &amp; Simpson (1980) Exp 1</td>
<td>18400</td>
<td>1</td>
<td>4.67</td>
</tr>
<tr>
<td>Huppert &amp; Simpson (1980) Exp 5</td>
<td>26000</td>
<td>1</td>
<td>1.47</td>
</tr>
<tr>
<td>Martin &amp; Moyce (1952b)</td>
<td>4300</td>
<td>0.27</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Rescaling the transition time we obtain

\[
\frac{t_{SI} \sqrt{g' h_0}}{x_0} = \frac{\bar{t}_{SI} \bar{h}_0^{1/2}}{\bar{x}_0} = \left( \frac{2}{3} \xi_p \right)^3 \left( \frac{F_{p,sl}}{\bar{h}_0^{1/2}} \right)^{-3}
\]

(3.20)

and using the best fit results presented above we obtain \( \bar{t}_{SI} \bar{h}_0^{1/2} / \bar{x}_0 = 10.33 \). This predicted transition time is demarcated in figure 3.9(a) and captures very well the time of departure from the constant velocity phase for the experimental data.

The computational data presents some Reynolds number dependence as can be confirmed in figure 3.8(a). As can be observed in table 3.2 the values of \( F_{p,sl}/\bar{h}_0^{1/2} \) for the present simulations at \( Re = 895, 3450, \) and 8950 are 0.361, 0.407, and 0.421, respectively. Based on these values the slumping to inertial phase transition times can be estimated from equation (3.19) as \( \bar{t}_{SI} = 20.0, 14.0 \) and 12.6, for the three different \( Re \) cases. The slumping to inertial phase transition time identified in figure 3.8(a) for the higher \( Re = 8950 \) simulation is in good agreement with the prediction. With further increase in \( Re \) the transition time can be expected to approach the asymptotic value of about 10.33. At the lower two Reynolds numbers departure from the constant velocity slumping phase occurs earlier and as we will discuss below this is due to direct transition from slumping to viscous phase.

The location at which transition between the slumping and inertial phases occurs can now be estimated as \( x_{SI}/x_0 \approx u_F t_{SI}/x_0 = (2/3 \xi_p)^3(F_{p,sl}/\bar{h}_0^{1/2})^{-2} \). Again, using the best fit values we can obtain the following estimate: \( x_{SI}/x_0 = 4.65 \), which is lower than the well-accepted predicted value of \( x_{SI}/x_0 \) between 6 and 10 (Rottman & Simpson, 1983; Metha et al., 2002; Marino et al., 2005). Note that the actual current velocity over substantial portion of the acceleration phase is higher than the constant slumping phase velocity. This difference can partly explain the underestimation of transition location by the best fit slumping phase velocity.

We now examine the possibility of direct transition from the slumping to the viscous
phase by matching the constant velocity of the front in the slumping phase, $F_{p,sl}$, with the front velocity of the viscous phase given in equation (3.11) or (3.13). Depending on the viscous phase scaling employed, two different estimates for this transition time, $\tilde{t}_{SV}$, can be obtained as

$$\tilde{t}_{SVHt} = \left(\frac{3}{8} \xi_{pHt}\right)^{8/5} \frac{(\bar{\tau}_0 \bar{h}_0)^{4/5}}{F_{p,sl}^{8/5}} Re^{1/5}, \quad \text{and}$$

$$\tilde{t}_{SVHp} = \left(\frac{1}{5} \xi_{pHp}\right)^{8/4} \frac{(\bar{\tau}_0 \bar{h}_0)^{3/4}}{F_{p,sl}^{5/4}} Re^{1/4}. \quad (3.21)$$

For the present full-depth small-release planar currents ($\bar{h}_0 = \bar{\tau}_0 = 1$), using the best estimates for $\xi_{pHt}$ and $\xi_{pHp}$ and the values of $F_{p,sl}$ from table 3.2, we obtain $\tilde{t}_{SVHt} = 11.3, 12.2, \text{and} 14.0, \text{and} \tilde{t}_{SVHp} = 11.2, 12.5, \text{and} 16.4, \text{at} \ Re = 895, 3450, 8950, \text{respectively.}$

At $Re = 895$ the predicted values for $\tilde{t}_{SV}$ by both theories are lower than the prediction for $\tilde{t}_{SI}$, which indicates that in this case the flow transitions directly from the slumping to the viscous phase without entering the inertial phase. From figure 3.8(a) it can be estimated that the current leaves the constant velocity phase at $\tilde{t} \approx 12$, which is in good agreement with the predicted values of $\tilde{t}_{SV}$. The situation is similar for the intermediate $Re = 3450$ case, for which the predicted values of $\tilde{t}_{SV}$ are lower than $\tilde{t}_{SI}$, and as a result this current can also be expected to transition directly from the slumping to the viscous phase. The proximity of predicted $\tilde{t}_{SI}$ and $\tilde{t}_{SV}$ for this case suggests that this current is in the critical range of $Re$ for an inertial phase to develop. For the case of $Re = 8950$, $\tilde{t}_{SV}$ is predicted to be larger than $\tilde{t}_{SI}$, which indicates the flow to enter the inertial phase. From figure 3.8(a) the transition time from slumping to inertial phase for this $Re$ can be estimated to occur at $\tilde{t} \approx 12$, which is in good agreement with the predicted $\tilde{t}_{SI} = 12.6$.

The transition time from inertial to viscous phase can be estimated by matching the
front velocity from equation (3.9) with equation (3.11) or (3.13) as

\[
\tilde{t}_{IVHt} = \left( \frac{3\xi_{pHt}/8}{2\xi_p/3} \right)^{24/7} \left( \frac{\bar{x}_0 \bar{h}_0}{8} \right)^{4/7} Re^{3/7}, \quad \text{and} \quad (3.23)
\]

\[
\tilde{t}_{IVHp} = \left( \frac{\xi_{pHp}/5}{2\xi_p/3} \right)^{15/7} \left( \frac{\bar{x}_0 \bar{h}_0}{5} \right)^{4/7} Re^{3/7}. \quad (3.24)
\]

Using the best estimates of \(\xi_p\), \(\xi_{pHt}\) and \(\xi_{pHp}\) the prefactors in the above equations can be obtained as 0.32 and 0.4, respectively. Interestingly, except for this difference in prefactor both the theories predict the same power law behavior. The predicted transition times for the \(Re = 8950\) case are \(\tilde{t}_{IVHt} = 15.7\) and \(\tilde{t}_{IVHp} = 19.8\). From figure 3.8(a) it can be observed that the current leaves the inertial phase scaling at \(\tilde{t} \approx 17.3\) (\(\bar{x}_F \approx 8.1\)) which is in good agreement with these predictions.

The slumping to inertial phase transition time, \(\tilde{t}_{SI}\) in equation (3.19), has a weak Reynolds number dependence through \(F_{p,sl}\) and becomes Reynolds number independent only at large \(Re\). In contrast, \(\tilde{t}_{SV}\) will continue to increase with \(Re\). Thus, at lower \(Re\) a direct transition from slumping to viscous phase occurs. However, at sufficiently large \(Re\), \(\tilde{t}_{SV}\) will become larger than \(\tilde{t}_{SI}\) and a slumping to inertial phase transition will occur before eventual transition to the viscous phase, which is what we observe at \(Re = 8950\). Matching equation (3.19) with (3.21) or (3.22) it can be estimated that the critical \(Re\) for the inertial phase to exist is

\[
Re_{crHt} = \left( \frac{2\xi_p/3}{F_{p,sl}} \right)^{15} \left( \frac{F_{p,sl}}{3\xi_{pHt}/8} \right)^8 \bar{x}_0 \bar{h}_0 \quad \text{for Hoult (1972), and} \quad (3.25)
\]

\[
Re_{crHp} = \left( \frac{2\xi_p/3}{F_{p,sl}} \right)^{12} \left( \frac{F_{p,sl}}{\xi_{pHp}/5} \right)^5 \bar{x}_0 \bar{h}_0 \quad \text{for Huppert (1982).} \quad (3.26)
\]

Assuming the best fit values, we obtain \(Re_{crHt} = 3400 \bar{x}_0 \bar{h}_0\) and \(Re_{crHp} = 2000 \bar{x}_0 \bar{h}_0\).

For \(Re > Re_{cr}\) there will be an inertial phase, while for lower \(Re\) the current will transition directly from slumping to viscous phase without an inertial phase. The present numerical results are in reasonable agreement with this prediction of \(Re_{cr}\).
Transition times for cylindrical currents

The transition times between the different phases in a cylindrical current can be estimated in an analogous way. The transition time from slumping to inertial phase can be predicted by matching the constant velocity during the slumping phase, $F_{c,sl}$, with the inertial phase velocity scaling given in equation (3.10) as

$$\tilde{t}_{SI} = \left(\frac{\pi^{1/4}}{2} \xi_c \right)^2 \frac{r_0 \bar{h}_0^{1/2}}{F_{c,sl}^2}. \quad (3.27)$$

Rescaling the transition time we obtain

$$\frac{t_{SI}}{r_0} \sqrt{g' h_0} = \frac{\tilde{t}_{SI} \bar{h}_0^{1/2}}{\bar{r}_0} = \left(\frac{\pi^{1/4}}{2} \xi_c \right)^2 \left(\frac{F_{c,sl}}{\bar{h}_0^{1/2}}\right)^{-2} \quad (3.28)$$

and using the best fit results presented above we obtain $\tilde{t}_{SI} \bar{r}_0^{1/2} / \bar{r}_0 = 3.8$. This predicted transition time is demarcated in figure 3.9(e) and it is in good agreement with the experimental data.

As with the planar case Reynolds number independence can be expected only at large $Re$. As can be observed in table 3.2 the values of $F_{p,sl} / \bar{h}_0^{1/2}$ for the present simulations at $Re = 895, 3450$, and 8950 are 0.318, 0.368, and 0.408, respectively, giving slumping to inertial phase transition times from equation (3.27) as $\tilde{t}_{SI} = 6.6, 4.9$ and 4.0, respectively. The slumping to inertial phase transition times identified in figure 3.8(b) for the higher two $Re$ cases are in good agreement with the above prediction. At the lower $Re$ departure from the constant velocity slumping phase occurs earlier due to direct transition from slumping to viscous phase.

The location at which transition between the slumping and inertial phases occurs can now be estimated as $r_{SI}/r_0 \approx u_F t_{SI}/r_0 = (\pi^{1/4} \xi_c/2)^2 (F_{c,sl} / \bar{h}_0^{1/2})^{-1}$. Again, using the best fit values we can obtain the following estimate: $r_{SI}/r_0 = 1.6$, which is lower than the corresponding experimental observation of $r_{SI}/r_0 \approx 3.0$ for $h_0/H = 1$ (Hallworth
et al., 1996).

Slumping to viscous phase transition for the cylindrical current can be investigated by matching the constant front velocity of the slumping phase with the velocity scaling in the viscous phase given in equation (3.12) or (3.14). The resulting expressions for the transition time are

\[
\tilde{t}_{SVHt} = \left(\frac{1}{4} \xi_{cHt}\right)^{4/3} \left(\frac{\tau_0^2 \bar{\tau}_o}{F_{c,sl}^{4/3}}\right)^{4/9} Re^{1/9}, \quad \text{and} \quad \quad (3.29)
\]

\[
\tilde{t}_{SVHp} = \left(\frac{1}{8} \xi_{cHp}\right)^{8/7} \left(\frac{\tau_0^2 \bar{\tau}_o}{F_{c,sl}^{8/7}}\right)^{3/7} Re^{1/7}. \quad (3.30)
\]

For the present full-depth currents \((\bar{\tau}_o = \tau_0 = 1)\), using the best estimates of \(\xi_{cHt}\) and \(\xi_{cHp}\), and corresponding values of \(F_{c,sl}\) from table 3.2, the transition times predicted by the above equations are \(\tilde{t}_{SVHt} = 5.5, 5.3, \) and \(5.2, \) and \(\tilde{t}_{SVHp} = 5.2, 5.4, \) and \(5.5, \) at \(Re = 895, 3450, 8950, \) respectively.

For the case of \(Re = 895, \) the estimated value of \(\tilde{t}_{SI}\) is larger than the predictions for \(\tilde{t}_{SV}\) by both theories, which indicates that this current transitions directly from the slumping phase to the viscous phase. From figure 3.8(b) it can be observed that the current departs the constant velocity phase for \(\tilde{t} \approx 5\) which is in good agreement with the predictions above for \(\tilde{t}_{SV}.\) For the cases of \(Re = 3450\) and \(Re = 8950,\) the predicted values of \(\tilde{t}_{SV}\) are larger than the predicted values for \(\tilde{t}_{SI},\) which suggest the presence of an inertial phase at these \(Re.\)

The transition time from inertial to viscous phase, \(\tilde{t}_{IV},\) can be predicted by matching the front velocity from equation (3.10) with equation (3.12) or (3.14) as

\[
\tilde{t}_{IVHt} = \left(\frac{\xi_{cHt}/4}{\pi^{1/4} \xi_{c}/2}\right)^4 \left(\frac{\tau_0^2 \bar{\tau}_o}{F_{c,sl}^{4/3}}\right)^{1/3} Re^{1/3}, \quad \text{and} \quad \quad (3.31)
\]

\[
\tilde{t}_{IVHp} = \left(\frac{\xi_{cHp}/8}{\pi^{1/4} \xi_{c}/2}\right)^{8/3} \left(\frac{\tau_0^2 \bar{\tau}_o}{F_{c,sl}^{8/7}}\right)^{1/3} Re^{1/3}. \quad (3.32)
\]

65
For the cylindrical currents both theories also predict the same \( Re \) dependence. The predicted transition times using the best fit values are \( \tilde{t}_{IV} = 6 \) and 8.3 for both theories at \( Re = 3450 \) and \( Re = 8950 \), respectively. From figure 3.8(b) it can be estimated that the currents leave the inertial-phase scaling at \( \tilde{t} \approx 17 \) for both \( Re \). This time is larger than the theoretical prediction by a factor of 2. It must be cautioned that the above estimates are based on an average constant velocity during the slumping phase. However, the actual front velocity slowly varies during the slumping phase.

Matching equation (3.27) with (3.29) or (3.30) the critical \( Re \) for the inertial phase to exist can be estimated as

\[
Re_{crHt} = \left( \frac{\pi^{1/4} \xi_c/2}{F_{c,sl}} \right)^{18} \left( \frac{F_{c,sl}}{\xi_{cHt}/4} \right)^{12} \bar{\tau}_0 \bar{h}_0^{1/2} \text{ for Hoult (1972), and} \tag{3.33}
\]

\[
Re_{crHp} = \left( \frac{\pi^{1/4} \xi_c/2}{F_{c,sl}} \right)^{14} \left( \frac{F_{c,sl}}{\xi_{cHp}/8} \right)^{8} \bar{\tau}_0 \bar{h}_0^{1/2} \text{ for Huppert (1982).} \tag{3.34}
\]

With the best estimates of the constants we obtain \( Re_{crHt} = 880 \bar{\tau}_0 \bar{h}_0^{1/2} \) and \( Re_{crHp} = 870 \bar{\tau}_0 \bar{h}_0^{1/2} \). For \( Re > Re_{cr} \) there will be an inertial phase, while for lower \( Re \) the current will transition directly from the slumping to the viscous phase without an inertial phase. This prediction of \( Re_{cr} \) is also in good agreement with our numerical results since we observe the \( Re = 895 \) case to transition directly to the viscous phase, while the simulations for \( Re = 3450 \) and \( Re = 8950 \) present an inertial phase.

It must be emphasized that the above estimates of transition times are sensitive to the exact value of the prefactors used with the different scaling laws. For instance, if we were to use the original prefactors presented in section 3.3.2 for \( \xi_p, \xi_c \), etc., the resulting estimate for critical Reynolds number for the existence of inertial phase is much larger and differs significantly from the experimental and computational observation.
3.4.4 Three-dimensionality of the flow

At the lowest $Re$ considered ($Re = 895$) the planar current remains 2D and the cylindrical current remains axisymmetric at all times. The 3D disturbances introduced in the initial condition do not grow over time at this $Re$. At the higher two $Re$, however, instabilities grow over time and the flow eventually becomes 3D forming a well defined pattern of lobes and clefts and a turbulent front and body. Differences between 2D and 3D simulations were reported by Cantero (2002) for saline currents and by Necker et al. (2002) in the context of planar particulate gravity currents. Here we identify and explain the mechanisms behind the spurious time variation of the front velocity during the viscous phase in 2D simulations.

During the acceleration phase, the 3D disturbances have not grown to sufficient strength to change the evolution of the current. Therefore, the 3D and 2D simulations yield nearly identical results during the acceleration phase in all the cases considered. As seen in figure 3.7, near the end of acceleration phase the interface is marked by the complete development of a 2D Kelvin-Helmholtz rolls (or toroidal rolls in case of cylindrical currents). During the slumping phase, these rolls rapidly undergo 3D instability and a fully developed 3D state quickly follows. The three-dimensionality of the current, however, does not have a strong influence on the speed of the current during the slumping phase. Some small variations can be seen in the inset in figure 3.5 between the 2D and 3D simulation results, but the overall evolution of the current remains unaffected.

The three-dimensionality of the current becomes important during the inertial and viscous phases. Substantial differences can be observed between the 2D and 3D results at later times in the insets in figures 3.5(b) and 3.5(c). Importantly, the 3D current moves faster than the 2D approximation and this difference is observed both in the planar and cylindrical currents. This implies that the speed of actual currents, which are invariably 3D, will be underestimated by 2D models. The other significant observation
is that, while the speed of the 3D currents smoothly decrease with time, the 2D currents present periods of strong acceleration and deceleration. This non-monotonic behavior of the 2D current is related to the presence of strong coherent vortices and their episodic interaction in the form of vortex pairing.

### Lobes, clefts and spanwise variation

The front of the current does not advance forward as one fixed entity. The propagation of the front presents some variation along the span (or along the circumferential direction) due to the formation of lobes and clefts (Simpson, 1972). Figure 3.10(a) shows, for example, the flow at the front of the 3D small-release planar current for \( \text{Re} = 8950 \) at \( \tilde{t} = 21.2 \). In this figure, the location of the front is visualized by a thick solid line contour of \( \tilde{\rho} = 0.01 \), the horizontal flow is visualized by vectors and the vertical flow is visualized by thin line contours (solid line contours correspond to positive vertical velocity and dash line contours to negative vertical velocity). The horizontal flow tend to diverge from the center of the lobes and to concentrate in the clefts. Also the near-bed flow moves upward in the clefts and downward in the lobes.

The 3D lobe and cleft structure of the advancing front can be seen in figure 3.11, which shows the flow structure of the 3D small-release planar current for \( \text{Re} = 8950 \) at \( \tilde{t} = 21.2 \). At this time the current is in the viscous phase of spreading. In figure 3.11(a) the flow is visualized by a surface of constant density (\( \tilde{\rho} = 0.05 \)), and figure 3.11(b) shows contours of span-averaged \( \tilde{\rho} \). The spanwise variation in front propagation continues after the initial formation of lobes and clefts and, as a result, the number and location of lobes and clefts constantly rearrange along the front. For example, figure 3.10(b) shows a composite picture of the front plotted on the \( \tilde{x} - \tilde{y} \) plane (top view) with several equispaced time intervals superposed for the 3D small-release planar current for \( \text{Re} = 8950 \). At the beginning (toward the left end of the plot) the front is nearly flat, but small random disturbances introduced in the initial condition quickly develop into
well formed lobe and cleft structures. This figure illustrates the footprint of clefs on the horizontal $\bar{x} - \bar{y}$ plane as they advance over time. A complex pattern is etched by the clefs as the front advances, with repeated merger of the clefs and splitting of the lobes.

The locations of transition from acceleration to slumping, from slumping to inertial and from inertial to viscous phases are marked in the figure by dotted lines. The lobe and cleft dynamics for the other cases simulated is similar and not shown here.

The lobe and cleft structure of the front has been observed and well documented in several laboratory experiments (see for example Simpson, 1972; Spicer & Havens, 1987; García & Parker, 1989; Simpson, 1997). With increasing time the instantaneous Reynolds of the flow, $Re_F = u_F h_H/\nu = Re \bar{\pi_F} \bar{h}_H$, decreases and this decrease in $Re_F$ has the dominant influence on the increase in the length scale of the lobe and cleft pattern. Figure 3.10(c) shows the normalized lobe size, $\tilde{\lambda}/\bar{h}_H$, as a function of $Re_F$ for the 3D small-release planar currents at $Re = 3450$ and $Re = 8950$. Also in the figure are the experimental data presented by Simpson (1972) and his empirical prediction $\tilde{\lambda}/\bar{h}_h = 7.4 Re_F^{-0.39}$. The numerical results present very good agreement with the experimental data. This $Re$ effect on the wavelength of the lobe and cleft pattern is in agreement with the results on the most unstable mode from the linear stability analysis of Härtel et al. (2000a).

The non-uniform structure of the front is better illustrated in figure 3.12, where the front location $\tilde{x}_F$ is plotted as a function of the spanwise coordinate ($\bar{y}$ for planar currents and $\theta$ for cylindrical currents). Only the 3D highest $Re = 8950$ simulations are considered. Figure 3.12(a) shows the front location of the large-release planar current at $\tilde{t} \approx 7.6$, after the front has traveled approximately 3 dimensionless units from the lock ($\bar{\pi_F} = 11.5$). For the small-release planar current figure 3.12(b) shows the front location at two different time instances, one at the earlier slumping phase ($\tilde{t} \approx 7$) when the front is located approximately at $\tilde{x} \approx 4$, and another one at a later time in the viscous phase ($\tilde{t} \approx 23$) after the front has traveled to $\tilde{x} \approx 10$. Figure 3.12(c) shows the front location.
for the cylindrical current during the slumping phase (\( \tilde{t} \approx 3 \)) and during the viscous phase (\( \tilde{t} \approx 18 \)) for only one quadrant of the complete computational domain.

The variation in the front location can also be measured in terms of the root mean square (rms) deviation in front location from the mean and is shown in table 3.5 for the two larger \( Re \) in the slumping and viscous phases for the different cases considered in figure 3.12. A comparison of the two different time instances shown in figure 3.12(b) shows that the rms fluctuation is larger in the viscous phase than in the slumping phase, both in the planar and cylindrical currents. Thus, not only does the size of lobes increase with time as indicated by figure 3.10(c), but also with increasing time or diminishing local \( Re_F \) these lobes become more prominent as indicated by these rms values. A comparison of the cylindrical and planar results shows that the cylindrical configuration presents larger variation of front location.

### Three-dimensionality of the current body

The body of the current is much more complex and 3D than that is shown in figure 3.11. The undulations seen in the span-averaged contours (figure 3.11(b)) provide clear indication of Kelvin-Helmholtz instability and the presence of a periodic train of rolled-up vortices. The imprint of these rolls can be seen in the 3D contour of density (figure 3.11(a)), where they appear to be bent, stretched, and eventually broken up into smaller scale structures. These small structures can be observed in the body of the current.

<table>
<thead>
<tr>
<th></th>
<th>( Re )</th>
<th>Planar large</th>
<th>Planar small</th>
<th>Cylindrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slumping phase</td>
<td>3450</td>
<td>4.1 × 10^{-2} (8.2)</td>
<td>4.6 × 10^{-2} (7.5)</td>
<td>0.9 × 10^{-2} (3.1)</td>
</tr>
<tr>
<td></td>
<td>8950</td>
<td>3.5 × 10^{-2} (7.6)</td>
<td>4.55 × 10^{-2} (7.1)</td>
<td>1.95 × 10^{-2} (3.0)</td>
</tr>
<tr>
<td>Viscous phase</td>
<td>3450</td>
<td>–</td>
<td>0.9 × 10^{-1} (24.7)</td>
<td>0.65 × 10^{-1} (19.7)</td>
</tr>
<tr>
<td></td>
<td>8950</td>
<td>–</td>
<td>4.8 × 10^{-2} (22.8)</td>
<td>1.4 × 10^{-1} (17.7)</td>
</tr>
</tbody>
</table>

Table 3.5: Front location rms for selected times in slumping and viscous phases. The results are reported with the time from release between brackets: rms (\( \tilde{t} \)), to correlate these results with insets in figures 3.5(a), 3.5(b) and 3.5(c).
behind the leading front giving the appearance of a turbulent wake that eventually dissipates toward the tail of the current. Similar observations were made in cylindrical currents (Cantero et al., 2006) and will not be presented here in detail. The turbulent 3D structure of the body of the current has been observed and well documented in numerous experiments (see for example Simpson & Britter, 1979; García & Parker, 1989; Alahyari & Longmire, 1996; García & Parsons, 1996; Simpson, 1997; Parsons & García, 1998; Thomas et al., 2003).

The complex 3D vortical structure of the wake is not entirely apparent in the density isosurface presented in figure 3.11(a). The corresponding isosurface of swirling strength, $\tilde{\lambda}_{ci}$, defined as the absolute value of the imaginary portion of the complex eigenvalues of the local velocity gradient tensor\(^2\), is shown in figure 3.13. As discussed in Zhou et al. (1999) and Chakraborty et al. (2005) the swirling strength provides a clean measure of the compact vortical structures of the flow as it picks out regions of intense vorticity, but discriminates against planar shear layers, where vorticity is balanced by strain-rate. Thus, as can be seen from figure 3.13, the 3D vortical structure of the high $Re$ planar current is well extracted by $\tilde{\lambda}_{ci}$. At the time instance shown the mean and rms values of dimensionless $\tilde{\lambda}_{ci}$ are 0.16 and 0.57, respectively. In figure 3.13(a) the isosurface of $\tilde{\lambda}_{ci} = 2.12$ is plotted and thus it captures intense vortical regions. The flow is dominated by inclined vortical structures and several hairpin vortices can be observed (indicated by arrows). These structures are similar to those observed in a turbulent wall layer, where the vortical structures are tilted from the wall in the flow direction. The gravity current shown in figure 3.13(a) flows to the right. In a frame of reference moving with the front, the flow within the current can be from right to left which explain the observed orientation of the vortical structures within the current. The net effect of the vortical structures on the concentration (density) field is seen in figure 3.11. The isosurface of

\[^2\]The local velocity gradient tensor has 3 eigenvalues. If all three eigenvalues are real then swirling strength is zero. If only one eigenvalue is real, then the other two are complex conjugates and there is local swirling motion.
swirling strength confirms what was observed earlier in the density isosurface, that is the body of the current is far more 3D than the head.

Also plotted in frames (b) and (c) of figure 3.13 are the contours of span-averaged swirling strength for the 3D planar simulation and the swirling strength for the 2D planar simulation at $Re = 8950$, respectively. The substantial difference between the 3D evolution and the 2D approximation is well highlighted in this figure. At the instance shown, the signature of a sequence of Kelvin-Helmholtz vortices can be seen in the 2D swirling strength at $\tilde{x} \approx 5.9, 6.6, 7.7, 8.2$ and 8.6. The span-averaged vortex signature in the 3D simulation is not as well defined as in the 2D simulation. The 2D result shows several coherent primary (counterclockwise rotating) vortices along the length of the current. Also present are secondary (clockwise rotating) vortices induced by the interaction of the primary vortices with the no-slip wall. In figure 3.13(c) the vortices are undergoing a complex interaction process of pairing and merging. Another striking difference between the 3D and 2D simulations is the strength of the rolled up vortices, which are indicated in figure 3.13 by the inset numbers. In the 2D approximation the vortices just formed upstream of the head are two to three times stronger than their 3D counterpart. Furthermore, as the vortices move upstream of the head, they lose their strength more rapidly in the 3D simulation, whereas in the 2D approximation, the coherence of the vortices is preserved and, therefore, their decay is not as strong. Also overlaid on figures 3.13(b) and 3.13(c) as a dashed line is the current interface plotted in terms of the span-averaged density contour of $\tilde{\rho} = 0.05$. The 2D approximation shows large variation in the current height, which is clearly associated with the presence of strong coherent vortices. The height variations are present even in the 3D simulation (see also figures 3.3 and 3.4), but the undulations are much weaker.
Three-dimensionality of the flow during the slumping phase

We now address three-dimensionality of the current in the constant-velocity slumping phase. Figure 3.14(a) shows the isosurface of $\tilde{\lambda}_{ci} = 2.12$ for the $Re = 8950$ large-release planar current at $\tilde{t} = 12$. At this instance the mean and rms values of $\tilde{\lambda}_{ci}$ are 0.28 and 0.74, respectively. Also shown in frames (b) and (c) are contours of $\tilde{\lambda}_{ci}$ from the span-averaged result of the 3D and 2D simulations, respectively. In the isosurface visualization only the advancing front of the heavy fluid ($\tilde{x} \geq 0$) is shown. The vortical structure of the current is qualitatively very similar to that seen in figure 3.13 for the small-release planar case in the viscous phase. An important difference is that the rolled up vortices are farther upstream of the front. At the instance shown the vortices are generally at least 1 dimensionless unit upstream of the front, while in the viscous phase shown in figure 3.13 the vortices are much closer to the front. For instance, the dominant vortex at $\tilde{x} \approx 1$ is about 4 dimensionless units upstream of the front, while in figure 3.13 the dominant vortex is only 1 dimensionless unit upstream of the front. Differences between 3D and 2D results are still present in terms of stronger Kelvin-Helmholtz vortices. However, these vortex cores are farther upstream of the front so they do not interact with it. Consequently, their influence on front velocity is not strong and the 2D and 3D fronts advance at about the same mean velocity.

As indicated in the insets in figure 3.5, only the average speeds of the 2D and 3D currents are the same in the slumping phase. Instantaneous location and velocity of the current fluctuates as the interface rolls up to form new vortices and as the older vortices interact. As can be seen at the instance shown in figure 3.14 the 2D current has advanced slightly ahead of the 3D current. A comparison of figure 3.14 in the slumping phase with figure 3.13 in the viscous phase shows, that in the slumping regime, the strong rolled up vortices are located farther upstream of the front, while in the inertial and viscous phases they are closer to the front of the current. This, we believe, is the
reason why three dimensionality has a stronger influence on the propagation speed of the current in the viscous phase.

**Time variation**

It can be observed in the insets in figure 3.5 that the 2D simulations present strong time variation in the mean front velocity during the viscous phase. This time variation is related to strong vortex interaction, which is absent in the 3D simulations. In the 2D approximation the spanwise vortices remain coherent and exhibit strong interaction in terms of pairing and leap-frogging. Lack of three-dimensionality prevents vortex stretching and break-up, and therefore the rolled up vortices maintain their strength and remain in the flow for longer time allowing them to interact with each other in a strong way. Figure 3.15 shows the mean front velocity in the 2D small-release planar current for $Re = 8950$. In the same figure a sequence of vortex pairing is shown as insets (a) to (e), each separated with a time interval of 0.71 time units. These times are also marked in the plot of front velocity to allow connection between the dynamics of vortex pairing in relation to the mean front velocity variation. In the insets the front of the current is visualized by density contours. Solid lines represent $\tilde{\rho} < 0.3$ and dashed lines represent $\tilde{\rho} \geq 0.3$. Five vortical structures marked 1 to 5 can be identified in the inset 3.15(a). Vortices 1, 2, 4 and 5 rotate counter-clockwise, and vortex 3 rotates clockwise. The vortex pairing begins with the interaction of vortices 3 and 4, which elongates vortex 4 (see inset (b)) giving space for the growth of vortex 3. Following the growth of vortex 3, the interaction of vortices 2 and 3 begins and, as a consequence, vortex 2 acts as a pump of heavy fluid at the expense of its own kinetic energy. The center of mass of vortex 3 is raised and this results in an increase of its potential energy, which can be visualized in frame (c). At this time the acceleration of the front begins. The acceleration of the front is driven by the enhanced potential energy of vortex 3, whose center of mass moves down as the front advances (see frames (d) and (e)).
The above sequence is just an example. Similar vortex interaction processes are responsible for other pronounced variations in the front velocity observed in figure 3.15. The details of vortex interaction subtly varies, but the interaction process qualitatively remains the same and is present in the cylindrical currents as well. Vortex interaction occurs in 3D simulations as well, however, due to their reduced strength and coherence, the interactions are not nearly as powerful. Detailed investigation of the flow shows strong vortex interactions in the 2D simulations even in the constant velocity slumping phase. These interactions are father upstream of the front than in the viscous phase, and as a result the velocity variation in the slumping phase is less pronounced.

3.5 Summary and conclusions

In this work we present highly resolved 2D and 3D simulations of gravity currents performed at three different $Re = 895, 3450, \text{ and } 8950$. The particular choice of $Re = 3450$ corresponds to a Grashof number $Gr = 1.5 \times 10^6$, the dimensionless parameter used by HärTEL et al. (2000b) in their planar current simulations. The other two $Re$ correspond to $Gr = 10^5$ and $Gr = 10^7$. We consider both planar and cylindrical configurations, and in the planar currents we vary the volume of the heavy fluid released into the lighter ambient fluid. The simulations have been conducted with a de-aliased spectral code for the planar 2D and 3D configurations, and for the cylindrical 3D configuration. A spectral multi-domain code has been used for the axisymmetric (2D cylindrical) configuration. These highly accurate numerical formulations allow the capture of all relevant length scales present in the flow.

The main objectives of the study are to examine the effect of planar vs. cylindrical nature of the current, the influence of the volume of release on the propagation of the front, the transition between phases, and the three-dimensionality of the flow during the various phases of spreading. The planform area of a planar current increases linearly,
while it does quadratically for a cylindrical current, resulting in fundamental differences in the spreading rate. In all the cases as the front accelerates from rest, the front velocity increases and reaches a maximum, after which the front velocity somewhat decreases before settling to a near constant value. The value of peak front velocity reached by the current increases with $Re$. Due to the quadratic spreading, the intensity of the cylindrical current weakens more rapidly and, as a result, the peak velocity is lower than the planar counterpart for the same initial $Re$. Interestingly, this maximum is reached after the front has advanced about 0.3 height units regardless of the $Re$ of the flow and the geometrical setting. This is, however, for the ideal case of infinite gate lift velocity, and some deviation may be expected in real experiments due to the finite time it takes to release the gate.

A close look at the interface between the heavy and light fluids shows that at about the time the front velocity peaks the interface begins to roll up and at about the time the roll up process saturates the deceleration of the front ends and the constant velocity phase begins. In the constant velocity slumping phase of the planar currents, we observe the dimensionless height of the current to be lower than the theoretical prediction of $1/2$ (Benjamin, 1968; Shin et al., 2004) for energy-conserving currents. The corresponding constant dimensionless front velocity is also observed to be lower than $1/2$. Based on the limited range of $Re$ investigated in the present study, a crude estimation for the constant velocity for the planar currents at asymptotically large $Re$ can be obtained to be about 0.44. This asymptotic value is lower than theoretical prediction by both Benjamin (1968) and Shin et al. (2004), but is in reasonable agreement with previously reported data presented in table 3.2 and with the best fit to the experimental data presented in figure 3.9(a). Härtel et al. (1999) have considered the start-up stages of gravity currents with slip surfaces, and their results show the presence of slight deceleration in the current speed past the peak value. Also, Härtel et al. (2000b) report for slip surfaces front velocities to be lower than the theoretical prediction of 0.5 even at large $Re$. These
cases show that the reduction in the front velocity occurs regardless of the existence of the bottom boundary layer. In the present simulations, in addition to energy loss to wall friction, part of the potential energy goes towards maintaining internal recirculation within the rolled-up vortices. Only the balance goes towards the kinetic energy of the advancing front. This partitioning of energy has not been accounted for in the existing theories. It can be conjectured that perhaps by accounting for internal fluid motion it may be possible to better predict the actual current height and velocity (see also Härtel et al., 2000b).

Over the entire computed time interval the large-release planar currents for the larger two $Re$ are not substantially affected by viscous effects and remain in the constant velocity slumping phase. The small-release planar and cylindrical currents remain in the slumping phase only for a finite time interval and transition to the inertial or viscous phase. In the case of the planar currents, the nearly constant velocity of spreading during the slumping phase is not affected by the size of release.

The $Re$ of the present planar small-release simulations are not adequate to permit an inertial phase to clearly develop. The transition is directly to the viscous phase for the lower two $Re$ simulations, while the larger $Re$ simulation enters the inertial phase for a brief period of time. In the case of the cylindrical currents the larger two $Re$ simulations present an inertial phase of spreading, while the lower $Re$ simulation transitions from the slumping phase directly to the viscous phase. The transition times between phases can be estimated by matching velocity at the same time using the different scaling laws. We have revisited the prefactors in the slumping, inertial and viscous phase scaling laws in the light of previously reported experimental data. With the revised scaling laws we predict transition from the slumping to the inertial phase to occur at $t/T_0 \approx 10$ for planar currents and at $t/T_0 \approx 4$ for cylindrical currents, independently of the volume of release. Here $T_0 = x_0/\sqrt{g' h_0}$ (or $T_0 = r_0/\sqrt{g' h_0}$). These predictions are in very good agreement with the experimental data and our experimental results.
For the present small-release planar configuration it can be estimated that the $Re$ needs to be greater than about $3400 \bar{\tau}_0 \bar{h}_0$ for the inertial phase to exist, and for the cylindrical configuration the $Re$ must be greater than about $= 880 \bar{\tau}_0 \bar{h}_0^{1/2}$.

The actual evolution of the front differs from simple theoretical prediction in several significant ways. The Kelvin-Helmholtz vortices formed at the interface strongly interact among themselves and with the bottom boundary in a complex chaotic manner. In response to vortex pairing, leap-frogging and other such interaction processes the propagation of the front undergoes episodic rapid acceleration and deceleration. The vortex interaction is stronger in case of 2D (or axisymmetric) approximation and as a result the undulations in front velocity is pronounced. The spanwise (or circumferential) coherence of the Kelvin-Helmholtz vortices is broken in 3D simulations. In these cases, vortex interaction is substantially weaker and result in much weaker undulations in front velocity.

At the lowest $Re$ of 895, in all the cases considered the 3D disturbance introduced in the initial condition decays and the current remains 2D (or axisymmetric) at all times. At the higher two $Re$ the current becomes fully 3D. The front of the current deforms into a lobe and cleft structure, which undergoes constant rearrangement and leads to uneven front propagation. Our numerical results agree very well with experimental observations. The body of the current is strongly 3D and qualitatively resembles a turbulent boundary layer populated with quasi-streamwise and inclined hairpin vortices. These 3D vortical structures have a strong influence on the density field and the interface between the heavy and light fluids is highly distorted.

Three-dimensionality of the current has a strong influence on the propagation speed in the inertial and viscous phases. Two-dimensional approximation substantially underpredicts the mean speed of the current and thus will over estimate the arrival time. In the inertial and viscous phases, the coherent Kelvin-Helmholtz vortices are drawn closer to the front and, thus, influence the speed of the front substantially. On the other
hand, in the slumping phase the coherent vortices are farther upstream of the head of
the current and do not affect the front velocity nearly as much. As a consequence, in
the slumping phase both 2D and 3D simulations predict the same propagation speed.
Figure 3.2: Time evolution of the large-release planar gravity current plotted in terms of the span-averaged equivalent current height as defined by equation (3.16). Contours are shown with time intervals of 3.54 time units. Frame (a): 3D simulation for $Re = 895$, frame (b): 3D simulation for $Re = 8950$, and frame (c): 2D simulation for $Re = 8950$. 
Figure 3.3: Time evolution of the small-release planar gravity current plotted in terms of the span-averaged equivalent current height as defined by equation (3.16). Contours are shown with time intervals of 7.08 time units. Frame (a): 3D simulation for $Re = 895$, frame (b): 3D simulation for $Re = 8950$, and frame (c): 2D simulation for $Re = 8950$. 
Figure 3.4: Time evolution of the cylindrical gravity current plotted in terms of the circumferentially-averaged equivalent current height as defined by equation (3.16). Contours are shown with time intervals of 3.54 time units. Frame (a): 3D simulation for $Re = 895$, frame (b): 3D simulation for $Re = 8950$, and frame (c): axisymmetric simulation for $Re = 8950$. 
Figure 3.5: See caption in figure 3.5(c).
Figure 3.5: Time evolution of front location for all the simulations in table 3.1. Frame (a): large-release planar current, frame (b): small-release planar current, and frame (c): cylindrical current. The dash lines represent constant velocity spreading slope, that is $x_F - x_0 \approx \tilde{t}$ or $r_F - r_0 \approx \tilde{t}$. The insets show the corresponding time evolution of front velocity. For the insets, the dashed lines represent the theoretical value from the hydraulic theories of Benjamin (1968) and Shin et al. (2004) developed for planar currents.
Figure 3.6: Front velocity during the acceleration phase as a function of traveled distance for all the simulation in table 3.1. The dashed line represents the theoretical value from the hydraulic theory by Benjamin (1968) and Shin et al. (2004). The maximum of the front velocity is reached at $x_F - x_0 \approx 0.33$ regardless of geometrical configuration and $Re$.

In the figure LP: large-release planar current, SP: small-release planar current, and C: cylindrical current. The inset figure shows the peak velocity, $u_{F,\text{peak}}$, in the acceleration phase as a function of $Re_{\text{peak}}$ as defined by equation (3.18). The plot is in log-log scale. The collapse of $u_{F,\text{peak}}$ with $Re_{\text{peak}}$ provides support for the difference in peak velocity for the different geometrical settings.
Figure 3.7: Flow evolution during the acceleration phase for the 3D small-release planar current for $Re = 8950$. The dashed line shows the interface between the heavy and light fluids visualized by the span-averaged density contour $\bar{\rho} = 0.5$. Solid lines represent contours of span-averaged spanwise vorticity. The front accelerates from rest and reaches the peak velocity at $\tilde{t} \approx 1$ (see also inset in figures 3.5 and 3.6). At this time, the roll-up of the interface begins and continues to develop till $\tilde{t} = 2.5$ ($\bar{\rho}_F - \bar{\rho}_o \approx 1$), when deceleration of the front ends and a constant front velocity is reached.
Figure 3.8: Evolution of front velocity. Frame (a): time evolution of front velocity for planar currents with small release from 3D simulations. The plot also includes experimental data from two of the lower Re experiments from Marino et al. (2005) with $\bar{x}_0 = 1$ and $\bar{h}_0 = 1$. Included are also the theoretical predictions for all the phases of spreading. The viscous phase predictions are for $Re = 8950$ $\bar{r}_0 = 1$ and $\bar{h}_0 = 1$. Frame (b): time evolution of front velocity for cylindrical currents from 3D simulations. The plot also includes experimental data from experiment S2 by Hallworth et al. (2001) with $\bar{r}_0 = 1.25$ and $\bar{h}_0 = 0.965$ Included are also the theoretical predictions for all the phases of spreading. For this case, the planar case slumping phase theory prediction is used. The viscous phase predictions are for $Re = 8950$ $\bar{r}_0 = 1$ and $\bar{h}_0 = 1$. 

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Figure 3.9: See caption in figure 3.9(c).
Figure 3.9: Front velocity during the slumping, inertial and viscous phases of spreading for planar and cylindrical currents. The plots include experimental data (open symbols) from tables 3.3 and 3.4, and the 3D simulation results (closed symbols). The front velocity has been scaled according to the different scaling laws in order to remove the influence of initial condition and $Re$ on $\bar{u}_F$. Frame (a): slumping phase for planar currents with small release. Frame (b): inertial phase for planar currents with small release. Frame (c): viscous phase for planar currents with small release for scaling law by Hoult (1972). Frame (d): viscous phase for planar currents with small release for scaling law by Huppert (1982). Frame (e): slumping phase for cylindrical currents. Frame (f): inertial phase for cylindrical currents. Frame (g): viscous phase for cylindrical currents for scaling law by Hoult (1972). Frame (h): viscous phase for cylindrical currents for scaling law by Huppert (1982). Each plot also presents the best fit to the experimental data.
Figure 3.10: Time evolution of lobe and cleft pattern. Frame (a): near-bed flow pattern at the front of the small release gravity current for \( Re = 8950 \) for \( \tilde{t} = 21.2 \) when the front is located at \( \tilde{x} \approx 9.5 \). Vectors show the horizontal flow at \( \tilde{z} = 0.03 \). Thin line contours show vertical flow velocity at the same height, solid line for positive vertical velocity and dash line for negative vertical velocity. The thick solid line indicate the front location visualized by a bottom density contour of \( \tilde{\rho} = 0.01 \). Frame (b): time evolution of lobe and cleft pattern in the 3D small-release planar current for \( Re = 8950 \) visualized by contours of constant density \( \tilde{\rho} = 0.01 \). The time separation between contours is \( \Delta \tilde{t} = 0.014 \). Dotted lines mark the transition locations between acceleration/slumping, slumping/inertial and inertial/viscous phases. Frame (c): lobe size as a function of local Reynolds number \( Re_F = Re \overline{u}_F \overline{H} \). The figure includes our results from the 3D small-release planar current for \( Re = 3450 \) (closed diamonds), \( Re = 8950 \) (closed circles), and from experimental data by Simpson (1972) (open squares). The line is the empirical prediction by Simpson (1972): \( \dot{\lambda}/\dot{h}_H = 7.4 Re_F^{-0.39} \).
Figure 3.11: Flow structure of the 3D small-release planar current for $Re = 8950$ at $t = 21.2$. Frame (a): isosurface of $\tilde{\rho} = 0.05$, and frame (b): contours of span-averaged $\tilde{\rho}$. At this time the current is in the viscous phase of spreading and shows a rather complex state of three-dimensionality.
Figure 3.12: Span-wise (circumferential) variation of the front location for $Re=8950$. Frame (a): front location for the 3D large-release planar configuration at $\tilde{t} \approx 7.6$ ($\tau_F \approx 11.5$). Frame (b): front location for the 3D small-release planar current at two different time instances, one at the earlier slumping phase, $\tilde{t} \approx 7$, (open symbol) when the front is located approximately at $\tilde{x} \approx 4$, and another one at a later time in the viscous phase, $\tilde{t} \approx 23$, (closed symbols) after the front has traveled to $\tilde{x} \approx 10$. Frame (c): front location for the 3D cylindrical current during the slumping phase, $\tilde{t} \approx 3$, (open symbol) and during the viscous phase $\tilde{t} \approx 18$, (closed symbol).
Figure 3.13: Contours of $\tilde{\lambda}_{ci}$ for the small-release planar current for $Re = 8950$ at $\tilde{t} = 21.2$. Frame (a): isosurface of $\tilde{\lambda}_{ci} = 2.12$ from the 3D simulation, frame (b): contours of span-averaged $\tilde{\lambda}_{ci}$ from the 3D simulation, and frame (c): contours of $\tilde{\lambda}_{ci}$ from the 2D simulation. In frame (a) several hairpin vortices are pointed with arrows to help their visualization. The dashed line in frames (b) and (c) represents the interface visualized by the contour of span-averaged density $\tilde{\rho} = 0.05$. In frames (b) and (c) the inset numbers indicate local values of span-averaged and 2D $\tilde{\lambda}_{ci}$, respectively.
Figure 3.14: Contours of $\tilde{\lambda}_{ci}$ for the large-release planar current for $Re = 8950$ at $\tilde{t} = 12$. Frame (a): isosurface of $\tilde{\lambda}_{ci} = 2.12$ (shown only for $\tilde{x} > 0$) from the 3D simulation, frame (b): contours of span-averaged $\tilde{\lambda}_{ci}$ from the 3D simulation, and frame (c): contours of $\tilde{\lambda}_{ci}$ from the 2D simulation. The dashed line in frames (b) and (c) represents the interface visualized by the contour of span-averaged density $\tilde{\rho} = 0.5$. In frames (b) and (c) the inset numbers indicate local values of span-averaged and 2D $\tilde{\lambda}_{ci}$, respectively.
Figure 3.15: Vortex interaction in the 2D small-release planar current for $Re = 8950$. The main figure shows the mean front velocity as a function of distance traveled by the front. The insets show the head of the current visualized by density contours. Five different time instances with time interval of 0.71 are shown, which are marked as (a), (b), (c), (d), and (e) in the main figure. Solids lines represent contours of $\tilde{\rho} < 0.3$ and dashed lines represent contours of $\tilde{\rho} \geq 0.3$. Five different individual vortex cores (billows) are shown in the different frames.
Chapter 4

High resolution simulations of cylindrical density currents

4.1 Introduction

Density or gravity currents are flows that are driven by horizontal pressure gradients generated due to the action of gravity over fluids with different density (Allen, 1985; Simpson, 1997). In many real applications and laboratory experiments the current is canalized and is confined to flow between parallel lateral walls. In such situations, if the viscous effects of the lateral walls can be ignored, the current moves as a statistically two-dimensional (2D) flow with a nominally planar front (planar current). There are a number of other applications, such as the release of heavy gas into an open space, the collapse of an axisymmetric volcanic plume, or a point discharge into a lake or ocean, in which the gravity current is not canalized and is allowed to spread out over the entire horizontal plane. In such situations, the current moves as a statistically axisymmetric flow with a nominally cylindrical front (cylindrical current). Another example of a cylindrical current is a directed release where the current does not spread all around, but forms a conical planform. Density currents in such sector-shaped geometric tanks are often studied in the laboratory, instead of a full cylindrical current, due to their simplicity. More examples of engineering, environmental and geological applications can be found in the books by Allen (1985) and Simpson (1997).

The dynamics of planar density currents is reasonably well understood (see for example Marino et al., 2005). Planar currents form a coherent front and a relatively long highly turbulent body behind the front (Cantero et al., 2007c). As the currents spread,
the planform increases linearly with front location and the they pass through different phases, namely slumping, inertial and viscous (Huppert & Simpson, 1980). On the other hand, as cylindrical currents spread the planform increases quadratically, which induces a faster decays on the intensity of the current as it evolves. The concentrated vorticity at the head of the cylindrical current initially intensifies as the current flows out due to intense vortex stretching (Patterson et al., 2006), and eventually the currents present a highly turbulent front and a relatively shallow calm body (Cantero et al., 2006). Despite these fundamental differences, cylindrical currents are also thought to pass through a similar sequence of different phases of spreading (Huppert & Simpson, 1980; Ungarish & Zemach, 2005; Patterson et al., 2006).

Several experiments have been performed to study the dynamics and structure of planar density currents. Allen (1971) and Simpson (1972) have devoted great effort to study the lobe and cleft pattern observed at the front of planar density currents. Simpson & Britter (1979) studied the dynamics of the head of a gravity current. Huppert & Simpson (1980), Rottman & Simpson (1983) and more recently Marino et al. (2005) have studied the different phases of spreading of a current produced by the release of a fixed volume of denser fluid in a lighter ambient. García & Parsons (1996) and Parsons & García (1998) have studied the similarity of planar density currents fronts, to mention but a few.

In comparison, experimental investigation of cylindrical currents has been relatively limited. The earliest work to study the spreading rate of cylindrical currents was reported by Martin & Moyce (1952b). Bonnecaze et al. (1995) analyzed the spreading rate and deposition patterns of cylindrical particle-driven gravity currents. Alahyari & Longmire (1996) performed Particle Image Velocimetry of a cylindrical gravity current and described the vortex dynamics in the early stages of the flow. Hallworth et al. (1996) studied experimentally the ambient fluid entrainment in 2D and axisymmetric currents. Hallworth et al. (2001) studied experimentally the effect of rotation on the propagation
of cylindrical currents. Recent experiments in a sector-shaped tank for varying fractional depth of release has been reported by Patterson et al. (2006).

Lately, three-dimensional (3D) highly-resolved simulations have been performed for planar gravity currents at modest Reynolds numbers (Lee & Wilhelmson, 1997a,b; Härtel et al., 2000b; Necker et al., 2002; Özgökmen et al., 2004; Cantero et al., 2007c). Such detailed information for cylindrical currents is missing and it is highly desired. For example, linear stability analysis of a planar front of a gravity current has shown that the spanwise wavelength of the most unstable mode, and as a result the spanwise length scale of the lobe and cleft pattern, will increase with decreasing Reynolds number (Härtel et al., 2000a). Consistent with this theoretical prediction, as the planar current advances forward, its instantaneous $Re$ decreases and a coarsening of the lobe and cleft structure is observed during all the phases of spreading (Cantero et al., 2007c). In case of a cylindrical current, the instantaneous $Re$ of the current decreases more rapidly due to the quadratic increase in planform. However, as the current flows out the circumferential length of the current increases. An interesting question that arises is whether the number of lobe and cleft structures increase or decrease over time.

In this work we center our attention on the release of a fixed cylindrical volume of homogeneous fluid in a slightly less dense environment and the time evolution of the resulting cylindrical current. We perform highly resolved simulations for 3D cylindrical currents at three different Reynolds numbers, document the results and compare them to experimental observations.

### 4.2 Numerical formulation

We consider flows in which the density difference is small enough that the Boussinesq approximation can be adopted. By this approximation density variations are only in-
The two dimensionless numbers in equations (4.1)–(4.3) are, respectively, the Reynolds and Schmidt number, defined as

\[ Re = \frac{U H}{\nu}, \quad Sc = \frac{\nu}{\kappa}. \]  

(4.5)

Figure 4.1: Sketch of a cylindrical density current showing the main features of the flow and nomenclature for this work.
where $\nu$ is the kinematic viscosity and $\kappa$ is the diffusivity of temperature or chemical species producing the density change. A cylindrical volume of height $H$ (full-depth) and radius $r_0$ containing the heavier fluid is released into the surrounding lighter fluid. Figure 4.1 shows the nomenclature used in this work and the initial setting of the flow. In this work we will concentrate on the condition $r_0 = H$.

The computational domain is a rectangular box of size $\tilde{L}_x \times \tilde{L}_y \times \tilde{L}_z$. Since the current spreads radially outward along the entire $\tilde{x} - \tilde{y}$ plane we choose $\tilde{L}_x = \tilde{L}_y$. Periodic boundary conditions are employed along the $(\tilde{x})$ and $(\tilde{y})$ directions for all variables. At the top and bottom walls no-slip and zero-gradient conditions are enforced for velocity and density, respectively. The planform of the periodic box is taken to be $15 \times 15$ in order to allow unhindered development of the current for sufficiently long time. Härtel et al. (2000b) indicated that for planar currents the interaction of the advancing front with the boundary becomes important when the front reaches within one depth scale from the boundary. This effect is expected to be less significant for cylindrical currents (see Cantero et al., 2006). The flow was started from rest with a minute random disturbance prescribed in the density field. The details of the initial conditions can be found in Cantero et al. (2006). The use of a rectangular grid to solve a cylindrical problem may seem odd. However, a rectangular grid provides better resolution away from the center of the domain than a cylindrical grid capturing better the fine structures of the flow at the front (lobes and clefts). The governing equations are solved using a de-aliased pseudospectral code whose details can be found in Cantero et al. (2007c).

In this work, three different $Re$ are considered: $Re = 895$, 3450 and 8950. The corresponding Grashof number, defined as $Gr = g'(H/2)^3/\nu^2$ (Härtel et al., 2000b) are $10^5$, $1.5 \times 10^6$ and $10^7$, respectively. The intermediate Reynolds number corresponds to that considered by Härtel et al. (2000b) in the planar configuration. With increasing $Re$ the complexity of the flow increases, and this requires increased resolution. The grid resolutions employed for the three different simulations are $280 \times 280 \times 72$, $512 \times 512 \times 110$.
and $880 \times 880 \times 180$ and thus they involve, 5.6, 28.8 and 139.4 million grid points. The numerical resolution for each simulation was selected to have between 6 and 8 decades of decay in the energy spectrum for all the variables. The time step was selected to produce a Courant number smaller than 0.5. The simulation at $Re = 8950$ required about 1 month of continuous run on 64 processors of the new SGI Altix 3000 (put to production on April 2005 at NCSA, University of Illinois at Urbana-Champaign), about 70Gb of RAM memory to run, 600Gb of storage for raw data, and 18Tb (18000 Gb) of storage for visualization postprocessing made by NCSA scientists.

In the planar case, along the spanwise direction, the width of the periodic domain is typically chosen to be 1.5 dimensionless units, which is adequate to include several spanwise lobe and cleft structures (Härtel et al., 2000a). Thus the present simulations of the cylindrical current are an order of magnitude more computationally expensive than the planar simulations, for similar Reynolds number and grid resolution. By imposing symmetry along the $x = 0$ and $y = 0$ planes the computational domain and correspondingly the computational cost can be reduced by factor 4. However, in the context of the present spectral simulations, it is more complex to impose such symmetry boundary conditions. Despite the increased computational cost there are advantages with the larger domain. First, if there are low circumferential wavenumber ($k_\theta = 1$ or 2) instabilities in the propagation of the front, such instabilities can only be captured in the larger domain, without imposed symmetries. The initial perturbation included a wide range of circumferential modes. Thus, the present simulations will capture the low wavenumber instabilities if they are present in a real system. If the computed results do not show such low wavenumber undulation in the propagation of the front, then these modes are not important and the use of symmetry boundary conditions will be supported. Furthermore, we make sure that the initial perturbation in the four quadrants is dissimilar, and consequently, the evolution of the current in the four different quadrants, although statistically identical, is not exactly the same. Thus, at the higher $Re$ turbulent flows,
we have four times larger data base for more accurate extraction of the statistics.

4.3 Results and discussion

4.3.1 Roll up of the interface

\( Re = 895 \)

After the release of the denser fluid, an intrusive front forms. Figure 4.2 shows the structure of the current as visualized by a density isosurface of \( \tilde{\rho} = 0.15 \) for the lower \( Re \) of 895. The structure of the flow visualized by this isosurface does not change substantially with the choice of \( \tilde{\rho} \) as long as it is maintained in the range \((0.01, 0.35)\) (see for example figure 4.5(a) for the larger \( Re \) flow). The figure shows only one quarter of the computational domain to allow for better visualization of the flow structures. Two different dimensionless time instances are shown, \( \tilde{t} = 7 \) and 10. At both times a well defined head of the current can be seen. The head initially consists of a coherent anti-clockwise rotating vortex ring (marked A1 on the \( \tilde{x} - \tilde{z} \) plane in figure 4.2) that forms first due to the roll up of the interface shear layer between the heavy and light fluids. As the front of the current progresses radially out, the vortex ring A1 slightly lifts up above the ground and a new anti-clockwise vortex ring A5 (numbering will become clear below) forms ahead of it at about \( \tilde{t} \approx 5 \) when the current is located at \( \tilde{r} \approx 2.7 \). At the earlier time of \( \tilde{t} = 7 \) shown in figure 4.2, the presence of two different vortex rings, one behind the other, can be clearly observed and together these two vortex rings form the distinct raised head of the current seen in the density isosurface. At this \( Re \) the currents remains axisymmetric at all times and the initial non-axisymmetric disturbance introduced into the flow decays away quickly.
Figure 4.3 shows the structure of the current at the intermediate $Re$ of 3450. At the earlier time of $\tilde{t} = 7$ a sequence of four vortex rings, one right behind the other, can be seen. The anti-clockwise vortex marked A1 is the first one to form, as a result of the roll up of the interface that develops soon after the release of the heavy fluid. With increase in time, the front of the heavy fluid propagates radially out, while the front of the light fluid propagates back towards the axis. As new interface develops, subsequent roll up of the interface results in the formation of vortex ring A2. In the present case the back propagating disturbance reaches the axis by about $\tilde{t} \approx 3$, after which the mean interface between light and heavy fluids increases only due to the forward propagation of the heavy front. At this same time the interface begins to roll up again to form the third vortex ring A3. These later vortices can be observed to be progressively weaker than the first one. At the earlier time of $\tilde{t} = 7$ the three vortices A1, A2 and A3 are nearly equispaced. The formation of vortex ring A5 ahead of A1 occurs at $\tilde{t} \approx 4.5$ when the current is located at $\tilde{r} \approx 2.6$ and will be discussed below in the context of the the higher $Re$ case (see section 4.3.1). By about $\tilde{t} \approx 10.0$ the vortex rings A1 and A5 merge and form a single head of the current, which can be observed at the later time of $\tilde{t} = 14$ shown in figure 4.3. Also at this time, due to their faster propagation, we observe the combined vortex ring A1+A5 is farther ahead of the weaker vortex rings A2 and A3.

The dynamics of vortex propagation is explored further for the larger $Re$ case in section 4.3.1.

At this $Re$ the initial disturbance introduced into the flow grows over time and quickly the flow forms a lobe-and-cleft structure. The initial random disturbance introduced at the interface includes a wide range of circumferential modes. It is, however, clear from the figure that there is a preferred most amplified circumferential mode of instability. In the frame for $\tilde{t} = 7$ about 20 lobe and cleft structures can be observed within the
quadrant that is visualized, while in the frame for \( \tilde{t} = 14 \) slightly lower number of lobe and cleft structures are observed. Also, at early times (not shown here) the instability can be observed only at the head of the current, but at later times it propagates into the subsequent vortex rings as well as can be clearly observed in the inset frame for \( \tilde{t} = 7 \). The lobes and clefts are observed to be reasonably well organized at the earlier time shown in figure 4.3, while at the later time their structure is less regular, suggesting the strong role of nonlinearity. The height of the current is elevated only at the location of the vortex rings. In between the vortex rings the current appears to be relatively thin and calm, devoid of strong instabilities. The remnant heavy fluid observed along the axis \((\tilde{x} = \tilde{y} = 0)\) and near the top boundary are artifacts of the no-slip boundary condition applied at the top boundary, but changing this to a free slip top boundary does not alter the physics of the current to be discussed in this work.

\[ Re = 8950 \]

Figure 4.4 shows the time development of the flow structure for the higher \( Re \) of 8950. Initially, the flow evolves as a nearly axisymmetric flow in which Kelvin-Helmholtz rolls develop and form along the front and body of the current. At this time of \( \tilde{t} = 2.7 \) the front nose of the current is slightly lifted away from the bottom wall and, as a consequence of no-slip condition, a layer of light fluid penetrates below the raised nose resulting in unstable stratification. It can be observed that at this early time some incipient lobes and clefts are starting to form. However, the three-dimensionality is primarily limited to the nose of the current. The vertical location of the nose does not depends strongly on the particular value of \( \tilde{\rho} \) used to visualize the front as can be observed in figure 4.5(a). This figure shows circumferentially-averaged density contours (as defined in equation (4.6)) for \( \tilde{t} = 5.3 \) and \( Re = 8950 \). The thick solid line indicates the contour of \( \tilde{\rho} = 0.15 \), dash lines the contours of \( \tilde{\rho} \) between 0.01 and 0.35, and dot lines the contours of \( \tilde{\rho} = 0.5 \) and 0.65. Figure 4.5(b) shows the streamline pattern at
the front of the current for the same conditions of figure 4.5(a). The flow direction in a frame of reference moving with the front is indicated by arrows. The front of the current show strong recirculation and it is difficult to identify in the field of view of the figure the stagnation streamline. It is clear from the figure, however, that the location of the stagnation streamline does not coincides with the location of the nose.

The roll up process of the interface can be better observed from circumferentially-averaged variable plots. For any flow variable \( f \), the circumferential average is indicated by an overbar and computed as

\[
\bar{f}(\tilde{r}, \tilde{z}) = \frac{1}{2\pi} \int_{0}^{2\pi} f(\tilde{r}, \theta, \tilde{z}) \, d\theta.
\]  

(4.6)

In figure 4.6 circumferentially-averaged velocity vector plots are presented at several time instances. To allow better visualization of the rolled-up vortices, the velocity field is plotted in a frame of reference moving with the front of the current together with contours of circumferentially-averaged density. The interface between the heavy and light fluids is taken to be marked by \( \tilde{\rho} = 0.15 \). At the earliest time shown (\( \tilde{t} = 1.77 \)) only the incipient roll up of the anti-clockwise vortex ring A1 can be seen. By \( \tilde{t} = 2.7 \) the back propagating front has almost reached the axis and in addition to the initial vortex ring A1, two additional anti-clockwise vortex rings A2 and A3 can be observed upstream (see also figure 4.4). Also, as a consequence of the bottom no-slip boundary condition, the strong anti-clockwise vortex ring A1 results in the formation of a clockwise rotating vortex ring C1 closer to the bottom wall. At \( \tilde{t} = 3.54 \) the pair of vortex rings A1 and C1 form the head of the current. Vortex rings A2 and A3 have merged and have resulted in the formation of the clockwise vortex ring C2. A new rolled up anti-clockwise vortex A4 and the associated clockwise vortex ring C3 can be seen as well. The resulting vortex ring structure and the interface can be verified in figure 4.4. The presence of counter-rotating vortices has been observed in cylindrical gravity currents.
in the experiments of Alahyari & Longmire (1996). Figure 4.7 shows the near-bed
($\tilde{z} = 0.017$) circumferentially-averaged radial pressure gradient ($\frac{dp}{d\tilde{r}}$) for the case
of $Re = 8950$ at $\tilde{t} = 3.54$. At this time, three regions of adverse pressure gradient
($\frac{dp}{d\tilde{r}} > 0$) can be observed with peaks at 2.14, 1.27 and 0.63, which correspond very
well with the location of the clockwise rotating vortices C1, C2 and C3, respectively
(compare to frame for $\tilde{t} = 3.54$ in figure 4.6). This local adverse pressure gradients
induce local boundary layer separation and the formation of the clockwise rotating
vortices as postulated by Alahyari & Longmire (1996).

By $\tilde{t} = 4.67$ vortex ring C1 has lifted the dominant vortex ring A1 further away
from the bottom wall lowering the outward propagation velocity of this structure. The
front thus advances faster than the vortex ring A1 and results in the formation of a
new anti-clockwise vortex ring A5, downstream of A1. For this $Re$ the formation of A5
occurs at $\tilde{t} \approx 4.25$ when the current is located at $\tilde{r} \approx 2.6$. This mechanism is the cause
of vortex ring marked A5 in the two lower $Re$ cases as well. The coherence of the Kelvin-
Helmholtz vortices is maintained until $\tilde{t} \approx 4.5$ when the current is located at $\tilde{r} \approx 2.7$ as
a consequence of the stabilizing effect of azimuthal stretching. At this time instabilities
grow and destabilize the Kelvin-Helmholtz rings A1, A2+A3 and A4 which undulate in
the azimuthal direction and eventually break up to smaller scale turbulent structures
(see section 4.3.2). As it is shown later, this time corresponds to the departure of the
current from the slumping phase. The location of the front at the time of the transition
is in good agreement with the findings of Patterson *et al.* (2006).

By $\tilde{t} = 7$ in figure 4.4 the vortex rings have lost the azimuthal coherence and present
a wide range of scales. Also a fully developed pattern of lobes and clefts can be observed
at the front. At $\tilde{t} = 21$ only A5 that forms the front of the current and the remnant of
the original vortex ring A1 can be observed. Despite the vortex rings decay, the lobe and
cleft pattern is still present. The level of turbulence has decayed substantially and only
the front of the current presents some vortical structures. The body of the current has
become a relatively calm region, where most of the flow features manifest as interface waves.

4.3.2 Interface roll-up and vortex rings dynamics

The roll up of the flows into coherent vortex rings is clearly associated with vorticity at the interface. This process can be quantified by the integral of the azimuthal component of vorticity in an area surrounding the interface (shaded region $S_1$ in the left advancing front in figure 4.1 for example), which is directly related to the circulation at the interface, $\Gamma$, by means of Stokes’ theorem, i.e.

$$\Gamma = \oint_{\gamma} \mathbf{u} \cdot d\mathbf{s} = \int_{S} \mathbf{\omega} \cdot dS.$$  \hfill(4.7)

Here $\mathbf{\omega} = \nabla \times \mathbf{u}$ is the vorticity and the line integral is done over the path $\gamma$ surrounding the surface $S$ (marked by dash line and arrows in the left advancing front in figure 4.1 for example).

Inset frame (a) in figure 4.8 shows the evolution of the circumferentially-averaged interface circulation, $\bar{\Gamma}$, for the case of $Re = 8950$. The analysis in this section is performed for the case of $Re = 8950$ which present less effect of the viscous term in order to highlight the effect of vortex stretching and baroclinic production. The integral is taken through a path line surrounding the interface with vertical bounds $\tilde{z} = 0.02$ and $\tilde{z} = 0.98$ to avoid the top and bottom boundary layer, and with horizontal bounds $\tilde{r} = 0.19$ and $\tilde{r} = 7.5$ to avoid the high shear layer in the vicinity of $\tilde{r} = 0$. This path is indicated in frame (b) of figure 4.8 by a thick long dash line. The values for the vertical and horizontal bounds were selected by inspection of the solution for several times of the simulation and the behavior of $\bar{\Gamma}$ is insensitive to these particular values as long as the path encompasses the interface and leave the boundary layers at the top and bottom walls and the high shear layer at the center of the domain outside of the analysis.
presents an increase with time until \( \tilde{t} \approx 2.3 \), consistent with the formation of vortex rings A1, A2 and A3 at the interface (see section 4.3.1 and figure 4.6). Then, \( \Gamma \) reaches a nearly constant value until \( \tilde{t} \approx 3.6 \) followed by a monotonically decay until \( \tilde{t} \approx 16 \).

A rough estimation of circulation at the interface can be made based on the front velocity. The jump of the circumferentially-averaged velocity across the nearly horizontal interface between light and heavy fluids is proportional to the velocity of the propagating front, \( \overline{u}_F \). The constant of proportionality is 1.0 for a deeply submerged current and is equal to 2.0 for a current of height half the layer depth and for all other intermediate current depths the constant of proportionality is between 1.0 and 2.0. The net circulation at the interface can be estimated then as

\[
\Gamma \sim \overline{u}_F \tilde{t}_F, \quad (4.8)
\]

where \( \tilde{t}_F \) is the length of the current. If a \( \overline{u}_F \sim \tilde{t}^\beta \) behavior is assumed for the front velocity, then the interface circulation follows the a power-law of the form

\[
\Gamma \sim \tilde{t}^{2\beta+1}. \quad (4.9)
\]

It can be readily seen from the equation above that only for \( \beta > -1/2 \), net circulation at the interfaces increases with time. If the front velocity decays more rapidly (i.e., for \( \beta < -1/2 \)), then net circulation at the interface decreases.

A cylindrical gravity current spreads in several phases (Fay, 1969; Fannelop & Waldman, 1971; Hoult, 1972; Huppert & Simpson, 1980). After a brief acceleration phase, the current moves at nearly constant speed (\( \beta \approx 0 \)) in what is called the slumping phase. Then, the currents enters a self-similar phase (provided that the \( Re \) is large enough to overcome viscous effects) in which the effects of flow initiation are forgotten. This phase is called inertial phase and is characterized by a front velocity \( \overline{u}_F \sim \tilde{t}^{-1/2} \) (\( \beta \approx -1/2 \)).
Finally, when viscous effects become important the current spreads in the viscous phase with $\bar{u}_F \sim \tilde{t}^{-7/8}$ ($\beta \approx -7/8$) (Huppert, 1982). Figure 4.9 shows the log-log plot of velocity of the front as a function of time for the three different $Re$. Also show in the figure are the $\tilde{t}^{-1/2}$ and $\tilde{t}^{-7/8}$ power-laws with corrected prefactors presented in Cantero et al. (2007c), which are straight lines in the log-log plot. Although a perfect constant velocity is not observed, a brief period of slow variation in velocity is observed, which can be taken to be the slumping phase of a cylindrical current (Cantero et al., 2007c). In the inertial and viscous phases, the velocity falls off more rapidly than in the slumping phase, and the computed decay in the front velocity is in good agreement with what is predicted by theories. The transition times for slumping and inertial/viscous phases are demarcated qualitatively in the figure. According to the model for interface circulation described above, $\Gamma \sim \tilde{t}$ during the initial slumping phase, $\Gamma \sim \tilde{t}^0$ during the self-similar inertial phase, and $\Gamma \sim \tilde{t}^{-3/4}$ during the final viscous phase.

Cantero et al. (2007c) have shown that a full-depth cylindrical current of unit initial radius will pass through all the phases of spreading if $Re > 900$, and showed that the dimensionless transition time from the slumping to the self-similar inertial phase, $\tilde{t}_{SI}$, and from the self-similar inertial to the viscous phase, $\tilde{t}_{IV}$, can be estimated as

$$\tilde{t}_{SI} = \frac{0.67}{F_{c,sl}^2} \quad \text{and} \quad \tilde{t}_{IV} = 0.4 \, Re^{1/3},$$

(4.10)

respectively. Here $F_{c,sl}$ is the approximate constant dimensionless velocity of the front in the slumping phase, which takes an approximate value of 0.41 at $Re = 8950$ (Cantero et al., 2007c). According to this equation $\tilde{t}_{SI} = 4$ and $\tilde{t}_{IV} = 8.3$ for the case of $Re = 8950$, which are in reasonable agreement with the results presented in figure 4.9.

Based on the simple scaling proposed above, the interface roll-up should intensify and $\Gamma$ should increase linearly with time until $\tilde{t} \approx 4$, then $\Gamma$ should remain approximately constant until $\tilde{t} \approx 10$, and decay as $\Gamma \sim \tilde{t}^{-3/4}$ from there on. As observed in figure 4.8,
Γ follows these scaling but with a shift on time, Γ increases only until \( \tilde{t} \approx 2.3 \) and then remain constant until \( \tilde{t} \approx 3.6 \). It is clear, then, that the evolution of interface roll-up is more complicated than what the simple model described above can predict and a deeper analysis is needed.

The evolution of the vortex rings and interface roll-up can be better understood by examining the vorticity dynamics dictated by (see for example Wu et al., 2006)

\[
\frac{D\tilde{\omega}}{Dt} = \tilde{\omega} \cdot \nabla \tilde{u} + \nabla \tilde{\rho} \times e + \frac{1}{Re} \nabla^2 \tilde{\omega}. \tag{4.11}
\]

The first term on the right hand side is responsible for vortex stretching and tilting, the second one for baroclinic production of vorticity and the third one for vorticity diffusion.

The vorticity evolution in a fixed oriented surface is

\[
\frac{\partial}{\partial t} \int_S \tilde{\omega} \cdot dS = \int_S (-\tilde{u} \cdot \nabla \tilde{\omega}) \cdot dS + \int_S (\tilde{\omega} \cdot \nabla \tilde{u}) \cdot dS + \int_S (\nabla \tilde{\rho} \times e) \cdot dS + \int_S \left( \frac{1}{Re} \nabla^2 \tilde{\omega} \right) \cdot dS. \tag{4.12}
\]

In the case of \( dS = dSe_\theta \) this equation dictates the evolution of the azimuthal component of vorticity in a radial slice of the flow (surface \( S_1 \) in figure 4.1 for example). The circumferential average can then be computed for each term in equation (4.12) to get

\[
\frac{d\Gamma}{dt} = \mathcal{A} + \mathcal{S} + \mathcal{B} + \mathcal{D} \tag{4.13}
\]

where \( \mathcal{A}, \mathcal{S}, \mathcal{B} \) and \( \mathcal{D} \) are, respectively, advection, stretching, baroclinic production and diffusion of \( \Gamma \). The surface \( S \) is taken with the same horizontal and vertical bounds as described above.

Figure 4.8(c) shows the time evolution of every term in equation (4.13). The baro-
clinic production of vorticity, $\overline{B}$, indicated by a dash-dot line, is always positive and nearly constant until $\tilde{t} \approx 2$. After this time $\overline{B}$ decays abruptly to a negligible value by $\tilde{t} \approx 3.6$. On the other hand, the stretching term, $\overline{S}$, indicated by a dash line, has a negligible (negative) value until $\tilde{t} \approx 2$ when it increases to peak at $\tilde{t} \approx 3.2$ following a rapid decay afterward. These two terms collaborate to increase $\Gamma$ defined positive for anti-clockwise rotating vortices. The interface roll-up or $\Gamma$ increase is counter-balanced by advection, $\overline{A}$, indicated by a dot line. Figure 4.8(c) shows that $\overline{A}$ is the main factor balancing the increase of $\Gamma$ and the subsequent strong decay. A detailed inspection of the results and computation the lateral and vertical azimuthal vorticity fluxes at the horizontal and vertical bounds of $S$ show that the main contribution to $\overline{A}$ is the inflow of negative azimuthal vorticity (clockwise rotating vortices) at the bottom boundary rather than an outflow of positive azimuthal vorticity. The inflow of negative azimuthal vorticity is produced by the formation and uplift of clockwise rotating vortices C1, C2 and C3 (see also figures 4.4 and 4.6). This is clearly seen in figure 4.8(b) which shows vorticity contours (dash line for positive vorticity and solid line for negative vorticity) for $Re = 8950$ at the time $\overline{A}$ peaks, $\tilde{t} = 4.03$. The long term dissipation of the vortex rings is produced by diffusion, $\overline{D}$, indicated by a dash-dot-dot line. Observe in figure 4.8(c) that while all the other terms on the right hand side of equation (4.13) tend asymptotically to zero, $\overline{D}$ tends to a negative constant value. The negative increase of $\overline{D}$ for $\tilde{t} < 4$ is associated with larger gradients of the azimuthal component of vorticity associated to the tilting and bending of the interface vortex rings that we explore next.

The tilting and bending of the vortex rings manifest as the generation of radial and vertical vorticity. We will focus on the evolution of the radial component, but a similar analysis can be performed for the vertical component as well. Consider equation (4.12) with $dS = dS e_r$. In this case equation (4.12) dictates the evolution of the radial component of vorticity in a circumferential slice of the flow (surface $S_2$ in figure 4.1 for example). In this case the surface $S$ in the integrals of equation (4.12) is taken with
vertical bounds $\tilde{z} = 0.02$ and $\tilde{z} = 0.98$ to let the boundary layer outside of the analysis, and horizontal bounds $\theta = 0$ and $\theta = 2\pi$ due to symmetry. The most important terms are the stretching and baroclinic production of radial vorticity. However, due to the symmetry of the flow, both stretching and baroclinic production of radial vorticity are zero since negative and positive radial vorticity are produced at the same rate. We will, then, analyze the evolution of the root mean square values

$$\tilde{S}_r = \sqrt{\int_S [(\tilde{\omega} \cdot \nabla \tilde{u}) \cdot \mathbf{e}_r]^2 dS} \quad \text{and} \quad \tilde{B}_r = \sqrt{\int_S [(
abla \tilde{\rho} \times \mathbf{e}) \cdot \mathbf{e}_r]^2 dS} \quad (4.14)$$

which are equivalent to $\tilde{S}$ and $\tilde{B}$, respectively, for the radial vorticity evolution analysis. The radial average of this quantities can be computed as

$$\overline{S}_r = \frac{1}{\tilde{L}_r - 0.19} \int_{0.19}^{\tilde{L}_r} \tilde{S}_r \quad \text{and} \quad \overline{B}_r = \frac{1}{\tilde{L}_r - 0.19} \int_{0.19}^{\tilde{L}_r} \tilde{B}_r \quad (4.15)$$

which are equivalent to $\overline{S}$ and $\overline{B}$, respectively.

Figure 4.10(a) shows the time evolution of $\overline{S}_r$ (solid line) and $\overline{B}_r$ (dash line) for the case of $Re = 8950$. It can be observed that $\overline{S}_r$ is 6 times larger than $\overline{B}_r$ and is the main responsible of vortex tilting. $\overline{S}_r$ peaks at $\tilde{t} \approx 4.6$ which is the time at which the vortex rings destabilize and the rapid break-up starts (see figure 4.4). Figure 4.10(b) shows the radial distribution of $\tilde{S}_r$ (solid line) and $\tilde{B}_r$ (dash line) for the time $\overline{S}_r$ peaks, $\tilde{t} = 4.67$. $\tilde{S}_r$ shows three clear peaks located at $\tilde{r} \approx 2.3, 1.5$ and 0.85. It can be verified in figure 4.6 that these peaks correspond to the locations of $A1+C1+A5$, $A2+A3$ and $A4$, respectively. Then, the radial component of vortex stretching destabilize the vortex rings and produce the rapid break up of the vortices.

The understanding of the dynamics of vortex tilting and bending by vortex stretching can be complemented by the following analysis. Consider, for example, a vortex tube
located at the center of the vortex ring A1. Let \((s, n, b)\) be the tangent, principal normal (pointing to the center of curvature of the vortex tube), and bi-normal orthonormal basis vectors, respectively. Let \(\nu\) and \(\chi\) be the curvature and the torsion of the vortex tube \(^1\), respectively, and \(\tilde{u} = (\tilde{u}_s, \tilde{u}_n, \tilde{u}_b)\) the velocity based on this basis. Then, the stretching term is

\[
\tilde{\omega} \cdot \nabla \tilde{u} = \tilde{\omega} \left[ \left( \frac{\partial \tilde{u}_s}{\partial s} - \tilde{u}_n \nu \right) s + \left( \frac{\partial \tilde{u}_n}{\partial s} + \tilde{u}_s \nu - \tilde{u}_b \chi \right) n + \left( \frac{\partial \tilde{u}_b}{\partial s} + \tilde{u}_n \chi \right) b \right]. \tag{4.16}
\]

Here \(\tilde{\omega} = |\tilde{\omega}|\). Consider the simplest case in which, \(\tilde{u}_s = 0\), \(\partial / \partial s = 0\) (no azimuthal variation), and that the vortex tube is slightly perturbed (see for example frames for \(\tilde{t} = 2.7\) and \(\tilde{t} = 3.6\) in figure 4.4) so that the vortex tube has torsion \(\chi \neq 0\). Consistent with this last assumption is to assume a small \(u_b \neq 0\). In this case

\[
\tilde{\omega} \cdot \nabla \tilde{u} = \tilde{\omega} (-\tilde{u}_n \nu s - \tilde{u}_b \chi + \tilde{u}_n \chi b) \tag{4.17}
\]

The component along \(s\) accounts for the intensification of azimuthal vorticity, and the components along \(n\) and \(b\) account for vortex tilting. Once the vortex rings form, they undergo initially azimuthal stretching which stabilize and intensify them. After the vortex rings are slightly perturbed, vortex stretching in the radial and vertical direction produce tilting and bending starting a self reinforced process that leads to rapid breakup of the structures. As it is clear from equation (4.17) this process is intensified by the azimuthal component of vortex stretching and intensification of the azimuthal component of vorticity.

In Cantero et al. (2007\(^c\)) it was observed that the formation of the first vortex ring (A1) marked the start of the slumping phase after a short acceleration period of the flow (acceleration phase). Here we note that the formation of all new anti-clockwise vortex rings happens over \(\tilde{t} \lesssim 5.0\), i.e. during the slumping phase. At \(Re = 895\) only two vortex rings

\(^1\)The torsion of the vortex tube accounts for the curvature in the axial-azimuthal plane.
rings (A1 and A5) form in the slumping phase. At $Re = 3450$ and 8950 four and five vortex rings form, respectively, before the inertial phase. After $t \approx 5.0$, i.e. during the inertial and viscous phases, no new vortices are formed for any of the cases simulated. The formation of vortex A5 coincides with the departure time from the slumping phase. The number of vortex rings that form in a cylindrical current is primarily a function of both $Re$ and the aspect ratio of the volume of release $r_0/h_0$, where $h_0$ is the initial height of the release. With increasing aspect ratio of the volume of release, the time spent in the slumping phase increases (Cantero et al., 2007c) and during this period circulation at the interface continues to increase. With increasing $Re$ the frequency at which the interface rolls up to form coherent vortices increases. After the formation of the last vortex ring (A5) the Kelvin-Helmholtz rolls start to destabilize in the azimuthal directions and eventually decay to concentrated regions of smaller scale turbulence.

### 4.3.3 Qualitative experiments

For comparison, we have also performed a laboratory experiment in a rectangular tank of $2m \times 2m \times 0.5m$. At one corner of the box a quarter cylinder of radius $0.165m$ is filled with salt water of slightly higher density. The rest of the tank is filled with fresh water and the two regions are initially separated with a cylindrical lock. Both the fresh and salt water are maintained to the same height of $0.165m$ and therefore here we consider a full-depth release with $r/H = 1$. The salt water is also dyed with potassium permanganate in order to visualize the front as it propagates out. The present experiment can thus be considered as a cylindrical current in a $90^\circ$ sector. The density difference and the height of the layer are chosen to yield a Reynolds number of 8950, and thus the results can be directly compared against those presented in figure 4.4. The bottom of the tank presents a smooth no-slip surface for the flow and the free surface at the top presents a slip condition. In the experiment, the heavy fluid is released in a finite amount of time (the time to lift the gate) and thus, the initial condition is not exactly the same as in
the simulations. Despite the differences in the experimental set up and the numerical simulation, the results resemble very closely. Figure 4.9 shows the front velocity during the inertial and viscous phases from simulations and for the present experiment in very good agreement. Also, the lobe and cleft pattern is qualitatively well captured by our simulation. Figure 4.11 shows the lobe-and-cleft pattern from the experiment for four time instances when the front is located at approximately \( \tilde{r} = 3.6, 4.1, 4.5 \) and 5.0. Each frame shows the complete front (that is the \( \pi/2 \) circular sector) of the current in the experiment. Through postprocessing of the images, the lobes and clefts could be clearly demarcated. Lobes are indicated with arrows to allow for a quantitative comparison with simulation results. Note that the Schmidt number for salt in water is 700, while the computations employ \( Sc = 1 \). The good agreement between numerical results and experimental observations (to be shown below) offers some justification to findings that as long as \( Sc > O(1) \) the flow and the lobe and cleft pattern are not very sensitive to the precise value of \( Sc \) (Härtel et al., 2000b; Cantero et al., 2006).

4.3.4 Lobes and clefts

The 3D lobe and cleft structure of the advancing front can be seen in figures 4.3 and 4.4 for the larger two \( Re \) considered in this paper. The circumferential variation in front propagation continues after the initial formation of lobes and clefts and, as a result, the number and location of lobes and clefts constantly rearrange along the front. The front of the current identified by contour of \( \tilde{\rho} = 0.015 \) at the bottom boundary is plotted on the \( x - \tilde{y} \) plane (top view) in figure 4.12. The front location at several equispaced time intervals of \( \Delta \tilde{t} = 0.354 \) are superposed for \( Re = 3450 \) (frame (a)) and \( Re = 8950 \) (frame (b)). The composite picture provides a clear view of the formation of lobes and lefts and the footprint the clefts leave on the horizontal \( x - \tilde{y} \) plane as the front advances over time. At the beginning (toward the center of the figure) the front is nearly axisymmetric, but small random disturbances introduced in the initial condition quickly develop into well
formed lobe-and-cleft structures. Different instability mechanisms for the formation of lobe and cleft structure of the front have been proposed in the context of planar currents (Allen, 1971; Simpson, 1972; Härtel et al., 2000a). These mechanisms are likely to be active and responsible for the lobe and cleft structure of the cylindrical currents as well. Even after they are fully formed, the speed of the current continues to vary along the circumference of the front, thus resulting in repeated splitting and merging of existing lobes and clefts. A complex pattern is etched by the clefts as the front advances, with repeated formation of new ones and merger between older ones, which is well captured in figure 4.12.

Figure 4.13 shows a snapshot of the near-bed flow at the front of the current for $Re = 3450$ at $\tilde{t} = 21$. The inset in the figure indicates the location of the horizontal section at $\tilde{z} = 0.04$ relative to the front where the flow is shown. The front is visualized by the density contour of $\tilde{\rho} = 0.015$ at the bottom wall (denoted by a thick solid line). The vector field shows the horizontal (velocity components $\tilde{u}_x$ and $\tilde{u}_y$) flow in a fixed frame of reference, and the contour lines show the vertical flow (solid line: positive $\tilde{u}_z$, and dashed line: negative $\tilde{u}_z$). The vector field shows clearly that the horizontal flow in the clefts is slower than in the lobes, and that there is a circumferential component that directs from the center of the lobes into the clefts. This flow pattern had been postulated by Allen (1985). The contours of vertical velocity show upward flow at the clefts and downward flow at the lobes. The composition of these results shows that each lobe has a pair of counter rotating vortices as originally postulated by Allen (1985). Some evidence of vortical activity at the front of the current can be seen in figure 4.16, which will be discussed in detail in section 4.3.5.

Figure 4.14 shows in details the lobe and cleft pattern observed in the present experiments performed at $Re = 8950$. The figure shows the front of the current when it is located at $\tilde{r} \simeq 6.5$. At this late time the height of the front and the concentration of potassium permanganate at the front have decreased enough to permit clear visualiza-
tion of the lobes and clefts in the flow. In the photographs, long streaks of fresh clear water trapped between the bottom of the tank and the current can be clearly identified. These streaks mark the path traversed by the clefts and have been demarcated in the figure with dashed lines to help their visualization. The photograph not only provides information on the instantaneous structure of the front, but also captures the path etched by the clefts in the recent past. Thus the experimental photograph can be compared with the simulation results presented in figure 4.12 and the qualitative features are in agreement. For example, what appears to be initiation of new clefts and merges between existing clefts (marked with arrows) can be observed in the figure.

Quantitative information on the lobe and cleft structure for both $Re = 3450$ and 8950 are presented in Table 4.1. At several selected time instances the mean radial location of the front ($\bar{r}_F$), the front velocity ($\bar{u}_F$), the current head height ($\overline{h}_H$), the local instantaneous Reynolds number of the front ($Re_F = Re\overline{h}_H\bar{u}_F$), the mean number of lobes observed within a 90° sector ($N_{90}$), and the mean wavelength of the lobe ($\tilde{\lambda}_l = \pi\bar{r}_F/2N_{90}$) are presented in the table. Also presented in the table are wavelength of the lobe extracted from the present experiment at $Re = 8950$ (see figure 4.11). The counting of lobes from figures 4.11 and 4.12 is subject to some interpretation, especially at instances of incipient mergers and splitting. In table 4.1 the number of lobes is therefore presented with an error bar. In addition to counting we have also computed both Fourier spectra and two-point correlation of the front location as a function of $\theta$. Due to the complex nature of the front, both the spectral and correlation present a set of broad peaks. Nevertheless, their results are consistent with those obtained from simple counting.

The amplitude of disturbance introduced in the initial condition is the same at all $Re$. So from figure 4.12 it can be seen that at the higher Reynolds number the disturbance grows more rapidly and result in earlier formation of distinct lobe and cleft structure. At both $Re$ the number of lobes initially goes through a brief period of adjustment where
<table>
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<th>Source</th>
<th>( Re )</th>
<th>( t )</th>
<th>( \bar{r}_F )</th>
<th>( \bar{u}_F )</th>
<th>( \bar{h}_H )</th>
<th>( Re_F )</th>
<th>( N_{90} )</th>
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<td>0.12</td>
<td>SP</td>
</tr>
<tr>
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<td>7.1</td>
<td>3.6</td>
<td>0.29</td>
<td>0.29</td>
<td>774</td>
<td>25.25( \pm )3</td>
<td>0.22</td>
<td>IP</td>
</tr>
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<td>4.5</td>
<td>0.25</td>
<td>0.32</td>
<td>711</td>
<td>27( \pm )3</td>
<td>0.26</td>
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<td>5.3</td>
<td>0.22</td>
<td>0.28</td>
<td>542</td>
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<td>0.31</td>
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<td>6</td>
<td>0.19</td>
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<td>446</td>
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<td>6.6</td>
<td>0.14</td>
<td>0.25</td>
<td>301</td>
<td>28.25( \pm )3</td>
<td>0.37</td>
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<td>3.6</td>
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<td>25( \pm )3</td>
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<td>EXP - C</td>
<td>8950</td>
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<td>28( \pm )3</td>
<td>0.23</td>
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<tr>
<td>EXP - C</td>
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<td>EXP - C</td>
<td>8950</td>
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<td></td>
<td></td>
<td>31( \pm )3</td>
<td>0.25</td>
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Table 4.1: Quantitative information on the lobe and cleft structure. In the table: \( \bar{r}_F \) is the mean radial location of the front, \( \bar{u}_F \) is the front velocity, \( \bar{h}_H \) is the current head height, \( Re_F = Re \bar{h}_H \bar{u}_F \) is the local instantaneous Reynolds number of the front, \( N_{90} \) is the mean number of lobes observed within a 90° sector, and \( \bar{\lambda}_l = \pi \bar{r}_F / 2 N_{90} \) is the mean wavelength of the lobe. C refers to cylindrical currents and P to planar currents. SP: slumping phase, IP: inertial phase, and VP: viscous phase.
$N_{90}$ first increases followed by some reduction. After this initial adjustment the number of lobes continue to increase with time. The increase is more prominent for $Re = 3450$, while at the higher $Re = 8950$ the increase is only modest. At $Re = 8950$ an increased competition for lobes to grow can be observed. For example, one can observe in figure 4.12 instances where a new lobe gets erased through merger before it reaches the mean size. Furthermore, careful observation of the details in figure 4.12(b) shows instances where out of a single lobe multiple smaller lobes are formed, but only one grows to the mean size. In contrast, at the lower $Re$ of 3450 almost every lobe grows to reach the mean size of a lobe for that radial location as can be seen from figure 4.12(a). Typically, a lobe splits into two with both growing over time. There is, however, lobes merging as can be seen in the details of figure 4.12(a), but this process is less intense than in the case of $Re = 8950$.

Figure 4.15(a) shows the lobe size normalized by the current head height, $\overline{h}_H$, from the numerical simulations as a function of $Re_F$. Also in the figure are the experimental data for planar currents by Simpson (1972) and the best fit to his data. The data for $Re = 3450$ is presented with solid symbols and the data for $Re = 8950$ with open symbols. The data is also separated by the corresponding phase of spreading. It is clearly seen in this picture that there is not as good as a collapse of the data for the cylindrical currents as there is for the planar current data by Simpson (1972). It is also clear the behavior depends on the phase of spreading, which is not the case for planar currents (Cantero et al., 2007c). Better collapse of the data can be obtained if the lobe size is normalized by the equivalent current height from mass conservation (box model height), $\overline{h}_r = (\overline{r}_0/\overline{r})^2$. Frame (b) of figure 4.15 shows the same set of data normalized by $\overline{h}_r$ as a function of $Re_r = Re \overline{u}_F \overline{h}_r$. The ranges on frames (a) and (b) are the same to allow comparison of the collapse of data in each case. Also in this figure is the data from figure 4.11 for the experiment with $Re = 8950$ in good agreement with the corresponding
numerical simulation. The best fit of all the data presented in this figure is

$$\tilde{\lambda} = 207 Re^{-0.87}.$$  

(4.18)

4.3.5 Turbulent structures

The complex 3D vortical structure of the current is not entirely apparent in the density isosurface presented in figure 4.4. The corresponding isosurface of swirling strength at the later three times is shown in figure 4.16. Here the swirling strength, $\tilde{\lambda}_{ci}$, is defined as the absolute value of the imaginary portion of the complex eigenvalues of the local velocity gradient tensor. As discussed in Zhou et al. (1999) and Chakraborty et al. (2005) the swirling strength provides a clean measure of the compact vortical structures of the flow. Swirling strength picks out regions of intense vorticity, but discriminates against planar shear layers, where vorticity is balanced by strain-rate. As can be seen from figure 4.16, the 3D vortical structure of the high $Re$ current is well extracted by $\tilde{\lambda}_{ci}$.

The root mean square swirling strength within the vortical region at the three different times are 1.05, 0.93 and 0.32, respectively. The isosurface of $\tilde{\lambda}_{ci} = 2.12$ is plotted in figure 4.16 for the first two time instances of $\tilde{t} = 7$ and 14. For $\tilde{t} = 21$ the isosurface of $\tilde{\lambda}_{ci} = 0.35$ is plotted. Thus, the figure captures only the intense vortical regions of the flow for all the times displayed. At the first instance shown in figure 4.16, the strong vortical structures extracted by $\tilde{\lambda}_{ci}$ near the head of the current are associated with the Kelvin-Helmholtz vortex rings A1 and A5. The effect of lobes and clefts on the vortical structure can be clearly observed. Two other rings of intense turbulence can be seen. The one centered around $\tilde{r} \approx 1.75$ is associated with the merged vortex ring A2+A3. The final weaker ring of turbulent structure is associated with vortex ring A4. By $\tilde{t} = 14$, all except the vortex ring associated with the front of the current have already decayed and any turbulence associated with them are no longer captured by
\( \tilde{\lambda}_{ci} = 2.12 \). However, this is an artifact of the relatively high threshold of 2.12 set for visualization. In figure 4.16 for \( \tilde{t} = 21 \) the isosurface of \( \tilde{\lambda}_{ci} = 0.35 \) shows that active turbulence is still present. A low level of turbulence can be observed behind the vortex rings. In addition we observe turbulence associated with a second vortex ring (A2+A3) located at \( \tilde{r} \approx 3.75 \) and close to the origin.

Apart from the vortex rings and the lobe and cleft structure the turbulent region of the flow behind the head is dominated by inclined vortical structures and several hairpin vortices can be observed (see for example inset for \( \tilde{t} = 14 \)). These structures are similar to those observer in a turbulent wall layer, where the vortical structures are tilted from the wall in the flow direction. In case of cylindrical gravity current the flow is directed radially out. However, in a frame of reference moving with the front, the flow within the current is radially inward, which explain the observed orientation of the vortical structures within the current. Similar trains of inclined vortical structures were observed for the case of planar gravity currents as well (Cantero et al., 2007c). The difference, however, is that at the present \( Re \) in the planar current the region of turbulence extended over a large portion of the body of the current, while in the cylindrical case the turbulence is limited to only vortex rings and the head of the current. The net effect of the vortical structures on the concentration (density) field is seen in figure 4.4.

### 4.3.6 Subcritical vs. supercritical flow

As seen in figure 4.6 the presence of vortex rings gives rise to significant local variation on the height and velocity of the current inducing regions of subcritical and supercritical flow. A local Froude number of the flow can be defined as

\[
Fr = \frac{u(r)}{\sqrt{g' \, h(r)}}
\]  

(4.19)
where the local height and velocity of the current are defined as

\[
h(r) = \int_0^H \left( \frac{1}{2\pi} \int_0^{2\pi} \int_0^r \frac{\rho(r, \theta, z) - \rho_0}{\rho_1 - \rho_0} \, d\theta \right) \, dz,
\]

and

\[
u(r) = \frac{\int_0^H \left( \frac{1}{2\pi} \int_0^{2\pi} \int_0^r u(r, \theta, z) \frac{\rho(r, \theta, z) - \rho_0}{\rho_1 - \rho_0} \, d\theta \right) \, dz}{\int_0^H \left( \frac{1}{2\pi} \int_0^{2\pi} \int_0^r \frac{\rho(r, \theta, z) - \rho_0}{\rho_1 - \rho_0} \, d\theta \right) \, dz}.
\]

It must be stressed that \( Fr \) is not the dimensionless front velocity, but provides a dimensionless measure of the local fluid velocity compared to that of local speed of wave propagation. The above definitions of \( \overline{u} \), \( \overline{h} \) and \( Fr \) are not unique, and some caution is required in interpreting regions of \( Fr > 1 \) and \( Fr < 1 \) as supercritical or subcritical flow regions. Nevertheless, we observe that with the definition used here the \( Fr \) at the head of the current consistently remains very close to critical which is indicative of a right choice of scales.

Figure 4.17 shows \( \overline{u} \) (solid line), \( \overline{h} \) (dash line) and \( Fr \) (dash-dot line) as a function of \( \tilde{r} \) at several times from \( \tilde{t} = 1.77 \) to 21. The current presents a well-defined front, where \( \overline{h} \) reaches its maximum, and a shallower body and tail. This distinct feature of a raised head can be clearly observed even at the later stages of the current. At the front of the current \( \overline{u} \) and \( \overline{h} \) approach zero, however, the decrease of \( \overline{u} \) is slower than that of the \( \overline{h} \) and, as a result, the local Froude number rapidly increases. The supercritical nature of the flow at the front is consistent with the front-like character (“shock”) of the current. The current at all times has an extended shallow body behind the raised head. At early times during the slumping phase of spreading (\( \tilde{t} \lesssim 5.5 \)) several undulations are present along the body of the current. It can be observed that \( \overline{h} \) increases at the location of the vortex rings, while \( \overline{u} \) shows local peaks in regions in between vortex rings, where the current is shallower. The combined effect creates larger amplitude fluctuations in the local Froude number. At later times, as the vortex rings dissipate during the inertial
and viscous phases, the undulations in $\overline{u}$, $\overline{h}$ and $Fr$ diminish.

During the slumping phase of spreading ($\tilde{t} \lesssim 5.5$), $Fr > 1$ over the entire length of the current. At intermediate times, during the inertial phase of spreading ($5.5 \lesssim \tilde{t} \lesssim 17$), $Fr > 1$ only in the region between the vortex rings A1 and A5. Eventually, as the current decays during the viscous phase, the Froude number becomes subcritical ($Fr < 1$) over the entire length of the current, except at the head where the critical value is maintained for all time. When the front is located at about $\tilde{r} = 3.5$, the flow presents an elevated front, which produces a region of higher pressure over $2.5 < \tilde{r} < 3.5$. The resulting lateral pressure gradient acts as the driving force to continue moving the front radially outward, but also presents a force to the shallow body trailing the head of the current. This interaction induces the separation of the head from the body of the current, which is indicated by the dip on the current height at $\tilde{r} \simeq 2.9$ at $\tilde{t} = 10.6$. Although subtle, this dip on the height persist in the flow for all time, and can be visualized in 3D figures as a circular wave at $\tilde{r} \simeq 4$ (compare to times $\tilde{t} = 21.0$ from figures 4.4 and 4.17). It is interesting to note that there are instances where the pressure gradient points radially inward so that the velocity of the current and correspondingly the Froude number becomes negative.

### 4.4 Summary and conclusions

In this work we present highly resolved 3D simulations of cylindrical density currents performed at three different $Re = 895, 3450, \text{ and } 8950$ with the objective of addressing the structure and dynamics of cylindrical density currents. The simulations have been conducted with a de-aliased spectral code, which captures all relevant length scales present in the flow.

As the front spreads, a shear layer forms between the heavy and the light fluids inducing the formation of Kelvin-Helmholtz vortices at the interface. The dynamic of the
interface and the number of rolls that form are highly dependent on the Reynolds number of the flow. For the lower Reynolds analyzed in this work only two vortices are formed, which remain axisymmetric during all the duration of the simulation. The interface stays smooth and nearly horizontal behind the front. For the larger two Reynolds considered 4 and 5 vortices are formed, which are destabilized in the azimuthal direction and eventually break up into smaller scale turbulence. In these cases the interface is very irregular with large undulations and evolves more dynamically than in the lowest Reynolds number case. The interface roll-up is not sustained during all the duration of the flow and occurs only during the initial slumping phase of spreading. Along with the formation of Kelvin-Helmholtz vortices at the interface, a series of counter-rotating vortices form at the bottom boundary and their eventual interaction results on the dissipation of the counter-rotating vortex, and the lift up and retardation of the original Kelvin-Helmholtz vortex A1. The retardation of A1 results in the formation of last vortex ring A5. As the flow evolves, these two vortices may or may not pair, but nevertheless, they both form the head of the current.

As mentioned above, regions of high turbulence in the flow are associated to the vortex rings break-up, which is induced primarily by radial and vertical vortex stretching. These regions are populated with trains of hairpin vortices tilted toward the axis. Similar findings have been reported by Cantero et al. (2007c) for planar currents, however, in that case the hairpin vortices are distributed evenly all over the head and body of the current.

For the larger two Reynolds number flows, a clear pattern of lobes and clefts develops. The minute perturbations in the initial condition grow very fast, originally at the lower part of the leading front, but very rapidly extend to the upper and rear part of the front. In the case of the lower Reynolds flow, the perturbations are not amplified and the flow progresses axisymmetrically. After lobes and clefts are formed, they evolve very dynamically presenting merging of clefts and splitting of lobes into new ones. The
process is similar for both larger Reynolds numbers, however, the larger Reynolds flow is characterized by smaller scales. The wavelength of the lobes grows with time as the front spreads and the local Reynolds number of the flow decreases. This is consistent with previous studies on planar currents that show that the most unstable wavelength decreases with the Reynolds of the flow (Härtel et al., 2000a). However, for cylindrical currents the number of lobes is maintained over time and the increase of wavelength is associated to the increase of circumferential length of the current.

A laboratory experiment for the largest $Re$ was performed in order to compare with the numerical simulations. Although the setting of the experiment is not exactly as the simulations, the front velocity and lobes wavelength from numerical results and experimental observations agree very well. The experiment was performed with salt water spreading in quiescent fresh water. The Schmidt number for this experiment is 700, while for the numerical simulations is 1. The good agreement observed offers some justification to previous findings that as long as $Sc > O(1)$ the flow and the lobe and cleft pattern are not very sensitive to the precise value of $Sc$ (Härtel et al., 2000b; Cantero et al., 2006).

As a result of the vortex formation the current high varies strongly, and together with it so does velocity. A local Froude number can be defined to discriminate between region of sub and supercritical flow. For early times, during the slumping phase, the flow is supercritical everywhere, while for later times during the inertial and viscous phases the flow is subcritical everywhere but at the front of the current. Interestingly, the local Froude number remains close to critical for all time at the head of the current and tend to supercritical at the leading front. This is consistent with the front-like propagation of the heavy fluid into an otherwise undisturbed quiescent ambient lighter environment.
Figure 4.2: Flow visualized by isosurface of $\tilde{\rho} = 0.15$ for $Re = 895$. The figure shows only one quadrant of the computational domain. The head consists of two coherent anti-clockwise rotating vortex rings marked A1 and A5 on the $\tilde{x} - \tilde{z}$ plane. For this $Re$ the flow progresses axisymmetrically. The insets show a front view of the flow.
Figure 4.3: Flow visualized by isosurface of \( \tilde{\rho} = 0.15 \) for \( Re = 3450 \). The figure shows only one quadrant of the computational domain. The interface rolls up and forms three vortex rings marked A1, A2 and A3 on the \( \tilde{x} - \tilde{z} \) plane. Then A5 is formed, which eventually merges with A1 to form the head of the current at later times. For this \( Re \), the front instabilities of the initial condition grow with time and form a clear pattern of lobes and clefts. The insets show a front view of the flow.
Figure 4.4: See caption in figure 4.4, frames for $\tilde{t} = 14$ and 21.
Figure 4.4: Flow visualized by isosurface of $\tilde{\rho} = 0.15$ for $Re = 8950$. The figure shows only one quadrant of the computational domain. Initially, the flow evolves axisymmetrically. Kelvin-Helmholtz vortex rings develop forming the front and body of the current marked A1, A2, A3 and A4 on the $\tilde{x} - \tilde{z}$ plane. Later, three-dimensionality develops and destabilize the vortex rings in the azimuthal direction which break up to smaller scale turbulence. Eventually, the turbulence decays and the body of the current becomes a calm region where most of the flow manifest as interface waves. For this $Re$, the front instabilities of the initial condition grow very rapidly with time and form a pattern of lobes and clefts. The insets show a front view of the flow.
Figure 4.5: Density contours and streamlines at the front of the current. Frame (a): circumferentially-averaged density contours at the front of the current for $\tilde{t} = 5.3$ and $Re = 8950$. The thick solid line indicates the contour of $\tilde{\rho} = 0.15$. The contours of $\tilde{\rho}$ between 0.01 and 0.35 are indicated by dash lines and the contours of $\tilde{\rho} = 0.5$ and 0.65 by dot lines. Observe that the shape of the front as indicated by $\tilde{\rho} = 0.15$ does not depend significantly on the particular value of $\tilde{\rho}$ as long as it is in the range between 0.01 and 0.35. Frame (b): streamlines at the front of the current for $\tilde{t} = 5.3$ and $Re = 8950$. Due to the complexity of the flow at the head it is difficult to identify the stagnation streamline. Nevertheless, it is readily seen that the location of the stagnation streamline does not coincide with the location of the nose of the current.
Figure 4.6: Flow field visualized by circumferentially-averaged velocity vectors. To allow for better vortex visualization, the nose velocity has been subtracted in each frame. The current interface is visualized by the contour of $\tilde{\rho} = 0.15$. Density contours are also shown to help visualize the current structure.
Figure 4.7: Near-bed ($\tilde{z} = 0.017$) mean radial pressure gradient for $Re = 8950$ at $\tilde{t} = 3.54$. The location of the peaks are 2.14, 1.27 and 0.63, which correspond very well with the location of the clockwise rotating vortices C1, C2 and C3, respectively.
Figure 4.8: Azimuthal stretching evolution. Frame (a): the solid line shows the time evolution of $\Gamma$ for $Re = 8950$. The figure also shows with dash lines the three scaling laws $\Gamma \sim \widetilde{t}^{\beta+1}$ for $\beta = 0, 1/2$ and $-7/8$. Frame (b): Vorticity contour for $Re = 8950$ at $\widetilde{t} = 4.03$. Solid lines shows negative vorticity contour corresponding to clockwise rotating vortices (C1, C2 and C3). Dash lines indicate positive vorticity corresponding to anti-clockwise rotating vortices (A1, A2+A3 and A4). The long dash thick line indicates the path over which circulation is computed. This path avoids the top and bottom boundary layers as well as the high shear layer in the vicinity of $\tilde{r} = 0$. Frame (c): time evolution of $\Gamma$ (solid line), $\mathcal{A}$ (dot line), $\mathcal{F}$ (dash line), $\mathcal{B}$ (dash-dot line) and $\mathcal{D}$ (dash-dot-dot line) for $Re = 8950$. The solid line with closed circle symbols is $\mathcal{A} + \mathcal{F} + \mathcal{B} + \mathcal{D}$ (see equation (4.13)).
Figure 4.9: Front velocity as function of time. The plot is in log-log scale. The transition times between phases of spreading are qualitatively indicated in the figure. AP: acceleration phase, SP: slumping phase, IP: inertial phase, and VP: viscous phase. The inset shows front location as a function of time. Solid line: $Re = 8950$, dashed line: $Re = 3450$ and dashed-dot line: $Re = 895$. Circle: data from experiment for $Re = 8950$. Square: data from Hallworth et al. (2001) for $Re = 152000$, $r_0/H = 1.25$ and $h_0/H = 0.965$. 
Figure 4.10: Radial stretching evolution. Frame (a): time evolution of $\bar{S}_r$ (solid line) and $\bar{B}_r$ (dash line) for Re = 8950. Frame (b): radial distributions of $\bar{S}_r$ (solid line) and $\bar{B}_r$ (dash line) for Re = 8950 at the time of peak for $\bar{S}_r$, $\bar{t} \approx 4.6$. Three peaks are observed for $\bar{S}_r$ that corresponds to the locations of A1+C1+A5, A2+A3 and A4. This can be verified in figure 4.6. The distribution of $\bar{B}_r$ is more homogeneous but also shows peaks corresponding to the vortex ring locations.
Figure 4.11: Lobes and clefts from an experiment for $Re = 8950$ and $Sc = 700$. Each frame shows the entire front in the experiment which extend a circular section of $\pi/2$. From top to bottom, the frames correspond to time instances when the front is located at $\tilde{r} = 3.6, 4.1, 4.5$ and $5.0$, respectively. Through postprocessing the lobe-and-cleft pattern has been demarcated in the pictures. Each lobe is indicated with an arrow. Locations where a lobe is not clearly seen and it was not counted (but could still had been counted) are demarcated with an (+) sign. On the other hand, locations where a lobe has been counted but was not clearly seen are demarcated with a (-) sign. In this way, an error can be estimated in the number of lobes counted.
Figure 4.12: Composite picture of the front location over time. Front visualized by bottom contours of $\tilde{\rho} = 0.015$. The time separation between contour is $\Delta \tilde{t} = 0.354$. The details show several lobe splitting and merger. Frame (a): $Re = 3450$, and frame (b): $Re = 8950$. 
Figure 4.13: Near-bed flow at the front of the current for $Re = 3450$ at $\tilde{t} = 21$. The inset indicates the location of the section at $\tilde{z} = 0.04$ relative to the front. The front is visualized by the corresponding bottom density contour of $\tilde{\rho} = 0.015$ (thick solid line). The vector field shows the horizontal flow (velocity components $\tilde{u}_x$ and $\tilde{u}_y$) and the contour lines show the vertical flow (solid line: positive $\tilde{u}_z$, and dashed line: negative $\tilde{u}_z$). By the vector field it can be seen that the horizontal flow in the clefts is slower than that in the lobes. It is clear also, that there is a circumferential horizontal flow from the center of the lobes into the clefts. The vertical flow is upward (solid line contours) at the cleft locations and downward at the lobes location (dashed line contours).
Figure 4.14: Detail of lobes and clefts from an experiment for $Re = 8950$ with $Sc = 700$. The front is located at $\tilde{r} \simeq 6.5$. At this time the concentration of potassium permanganate at the front has decreased enough to allow the visualization of cleft by streaks of fresh clear water trapped between the bottom boundary and the current in the near-front region. These streaks has been demarcated in the figure with dashed lines to help their visualization. Two merger of clefts (marked with arrows) can be observed in the figure. Compare to figure 4.12.
Figure 4.15: Lobe size for cylindrical currents as a function of local $Re$. Frame (a): lobe size normalized by current head height, $\overline{h}_H$, as a function of $Re_F$. Also in this frame are the experimental data for planar currents by Simpson (1972) and the best fit to his data. Frame (b): lobe size normalized by current head height, $\overline{h}_r = (\overline{\tau}_\theta/\overline{\tau})^2$, as a function of $Re_r = Re u_F \overline{h}_r$. Also in this frame is the data from figure 4.11 for the experiment with $Re = 8950$. The data from numerical simulations is discriminated by the corresponding phase of spreading. SP: slumping phase, IP: inertial phase, and VP: viscous phase.
Figure 4.16: Flow structures visualized by an isosurface of $\tilde{\lambda}_{ci}$. For $\tilde{t} = 7$ and 14 the isosurface of $\tilde{\lambda}_{ci} = 2.12$ is shown, while for $\tilde{t} = 21$ the isosurface of $\tilde{\lambda}_{ci} = 0.35$ is shown. The insets show detailed views. The inset in frame for $\tilde{t} = 14$ shows in detail a train of asymmetric hairpin vortices that have formed behind the front.
Figure 4.17: Evolution of depth-averaged variables for $Re = 8950$. Solid line: $\bar{u}$, dashed line: $\bar{h}$, and dash-dot line: $Fr$. The dot line indicates $Fr = 1$. Observe that $Fr \sim 1$ at the front location for all time. On the body of the current $Fr$ transitions from larger than 1 to smaller than 1 at $\tilde{t} \sim 6$. 
Chapter 5

An Eulerian-Eulerian model for gravity currents driven by inertial particles

5.1 Introduction

Gravity or density currents are flows driven by lateral pressure gradients produced by the action of gravity on fluids with different density. The density difference can be due to scalar fields such as temperature or salinity for which the excess density is conserved over the bulk, or by particles in suspension that may settle or be re-entrained into the flow. For this reason particle-driven gravity currents are also known as non-conservative gravity currents (García, 1992).

Particulate gravity currents, commonly known as turbidity currents, can be observed in many engineering, environmental and geological applications and they are the focus of the present work. In most of the cases the currents are dilute and the density difference is only a few percents, however, this is enough to trigger swift flows with a large transport capacity of mass and energy. Bagnold (1962) was among the first to discuss the conditions for auto-suspending turbidity currents. The sediment deposits generated by turbidity currents have also been of great interest to petroleum geologists (Kuenen & Migliorini, 1950). One of the most interesting property of particulate gravity currents is that they can modify their driving force via deposition and resuspension of particles. If particle entrainment prevails over deposition, the current will eventually accelerate and attain very high velocities (Parker et al., 1986). In the ocean, for example, sediment slumps can trigger gravity currents capable of traveling very long distances (Heezen & Ewing, 1952; Mohrig et al., 1998). These strong flows can carve out submarine canyons
(Fukushima et al., 1985) and mold the seabed producing different bedforms patterns such as ripples, dunes, antidunes and gullies (Allen, 1985). The dynamics of particulate gravity currents is also relevant for dusty thunderstorm fronts (Droegemeier & Wilhelmson, 1987), pyroclastic flows produced in volcano eruptions (Sparks et al., 1991), aerosol releases in the environment, flows originated by the discharge of a sediment-laden flow into a lake (Normark & Dickson, 1976) and snow avalanches (Hopfinger, 1983). Many more examples can be found in the books by Simpson (1997) and Allen (1985).

Particulate gravity currents have received a vast amount of attention in the past. García & Parker (1989) investigated the formation of internal hydraulic jumps produced when a current finds a change in slope. The formation of the jump is associated with a change in the flow regime and influences sediment transport capacity. García & Parker (1993) studied the erosion capacity of gravity currents and developed an empirical entrainment function that links the bottom shear stress with the rate of sediment entrainment into suspension. Depositional particulate gravity currents have been studied by Gladstone et al. (1998) for bidisperse particles and by Altinakar et al. (1990) and García (1994) in the context of poorly sorted sediment. Bonnecaze et al. (1993), Bonnecaze et al. (1995), Bonnecaze & Lister (1999) and de Rooij & Dalziel (2001) have studied depositional gravity currents in planar and cylindrical geometrical settings. Best et al. (2001) have focused on mean flow and turbulence structure of particulate gravity currents. Shallow water equation models have been used to study the dynamics of particulate currents and resulting deposition patterns (Bonnecaze et al., 1993; Choi & García, 1995). Recently, fully resolved simulations (Necker et al., 2002, 2005; Blanchette et al., 2005) have been performed for particle-driven gravity currents. These simulations have allowed clear interpretation of the flow dynamics and their relation to depositional patterns.

Particulate currents differ from thermal or saline currents in their fundamental feature that the particles, which are the source of density variation, do not exactly follow
the fluid. In contrast, in the case of scalar currents the thermal or saline concentration fields are advected at the local fluid velocity. In the limit of small particles the primary source of relative velocity between the particles and the surrounding fluid is due to gravity induced settling of particles. Most prior theoretical and numerical investigations of particulate currents have been limited to this regime. The velocity of particles in these studies is chosen to differ from the local fluid velocity by a constant settling velocity in the direction of gravity and the settling velocity is typically assumed to be the same as that of an isolated particle freely settling in still fluid.

Several geological phenomena involve the transport of coarse particles by gravity currents. Such finite sized particles do not follow the surrounding fluid exactly and settling velocity is only one of the mechanisms contributing to the relative velocity. The finite inertia of the particles becomes important with additional contribution to relative velocity arising from the inertial response of particles in regions of strong fluid acceleration. In regions of rectilinear fluid acceleration (or deceleration), inertial particles may lag (or lead) the fluid substantially. Also, in regions of curved streamlines the particle pathlines may not curve as rapidly as the fluid surrounding them. In the context of high Reynolds number turbulent flows it is well known that inertial particles tend to exhibit preferential concentration, with local accumulation in regions of high strain-rate and avoiding regions of high vorticity (Squires & Eaton, 1991; Wang & Maxey, 1993). In the context of gravity currents, this has implications for the distribution of particles in the highly vortical regions of the front of the current and along the interface between the heavy and the light fluids. Since the density difference due to the suspended particles is the main cause of the flow, the redistribution of particles will in turn alter the dynamics of the current. Furthermore, processes such as deposition, erosion and resuspension can also be influenced by the inertial behavior of the particles.

In this work we focus attention on the effect of particle inertia on the dynamics of the particulate gravity current. We will also address the influence on flow features
and deposition patterns. We use the equilibrium Eulerian approach (Ferry & Balachandar, 2001, 2002; Ferry et al., 2003) to account for the inertial effect of particles. The advantage of this approach is that the relative velocity between the particles and the surrounding fluid flow is given by an explicit expression, without having to solve additional equations for the particulate velocity field. The equilibrium Eulerian velocity and the mathematical model to be employed is presented in section 5.2, which is followed by a brief description of the formulation of the problem in section 5.3. In section 5.4 the numerical methodology is described. Then, we present two-dimensional (2D) direct numerical simulations and assess the effect of finite inertia on the current structure, front velocity, bed shear stress and deposition pattern in section 5.5. Finally, summary and conclusions are drawn in section 5.6.

5.2 Mathematical model

We are interested in simulating buoyant flows driven by the presence of solid particles of finite size. In this situation particles not only modify the bulk density as in the dusty gas formulation (Marble, 1970), but move at a velocity that differs from the local surrounding fluid velocity. We limit attention to dilute suspensions where the volume concentration of particles ($\phi_d$) is taken to be small. Particle concentration will be considered to be low not only in the mean, but also locally even in regions of preferential accumulation, and thus complex issues surrounding local inter-particle interactions will be ignored. The effective density variation within the flow will be sufficiently low that we will employ Boussinesq approximation. The inertial effect of the particles is the focus of this study and thus the relative velocity between the particles and the surrounding flow is due to both gravitational settling and particle inertia. However, dimensionless particle inertia, defined in terms of Stokes number ($\tilde{\tau}$ - ratio of particle time scale to fluid time scale), will be considered sufficiently small that equilibrium approximation can be used (Ferry
& Balachandar, 2001, 2002; Ferry et al., 2003). By limiting \( \tilde{\tau} \) to sufficiently small values we can express the particle velocity in terms of surrounding fluid velocity. For the case of large \( \tilde{\tau} \), a Lagrangian treatment of particles is required since the effect of initial conditions does not decay sufficiently rapidly.

Here we develop a new formulation that includes the role of particle inertia in the interest to simulate gravity currents at environmental and geological scales. In this section we present an Eulerian-Eulerian model based on an asymptotic expansion of the two-phase flows equations in parameters describing the (dimensionless) particle inertia (\( \tilde{\tau} \)) and the (dimensionless) particle volumetric concentration (\( \tilde{\phi}_d \)). The model is formally exact to \( O(\tilde{\tau} \tilde{\phi}_d + \tilde{\tau}^2 + \tilde{\phi}_d^2) \) and consists of conservation equations for the continuous phase (carrier fluid), an algebraic equation for the disperse phase (particles) velocity, and a transport equation for the particle volume fraction.

Let indices \( c \) and \( d \) denote the continuous and disperse phases, respectively. We denote the densities, volume fractions, and velocities of each phase by \( \rho_c, \phi_c, u_c \), and \( \rho_d, \phi_d, u_d \), respectively. In the case of constant density phases and no mass transfer between phases the mass conservation equations can be expressed as (Zhang & Prosperetti, 1997; Drew & Passman, 1998)

\[
\frac{\partial \phi_c}{\partial t} + \nabla \cdot (\phi_c u_c) = 0 \quad \text{and} \quad \frac{\partial \phi_d}{\partial t} + \nabla \cdot (\phi_d u_d) = 0,
\]

where \( \phi_c + \phi_d = 1 \).

The process of obtaining the ensemble-averaged momentum equations and their closure have been discussed in detail in the literature (Joseph & Lundgren, 1990; Zhang & Prosperetti, 1997; Drew & Passman, 1998; Machioro et al., 1999; Prosperetti, 2001). The resulting momentum equations for the continuous and disperse phases can be expressed
as
\[
\phi_c \rho_c \frac{D_c \mathbf{u}_c}{Dt} = -\phi_c \nabla p + \mu_c \nabla^2 \mathbf{u}_c - \mathbf{F} \quad (5.3)
\]
\[
\phi_d \rho_d \frac{D_d \mathbf{u}_d}{Dt} = \phi_d (\rho_d - \rho_c) \mathbf{g} - \phi_d \nabla p + \mathbf{F} \quad (5.4)
\]

Here \( D_c / Dt \) and \( D_d / Dt \) indicate material derivatives following the continuous phase velocity and the disperse phase velocity, respectively, \( p \) is the dynamic pressure in the continuous phase (i.e. the potential \( \rho_c \mathbf{x} \cdot \mathbf{g} \) has been subtracted), \( \mu_c \) is the dynamic viscosity of the continuous phase, \( \mathbf{g} \) is the gravity vector, and \( \mathbf{F} \) is the net hydrodynamic interaction between phases. Observe that the viscous term is a function of the volume averaged velocity \( \mathbf{u}_v = \phi_c \mathbf{u}_c + \phi_d \mathbf{u}_d \) (see Machioro et al., 1999).

For small particles, whose time scale is sufficiently smaller than the flow time scale defined in terms of the maximal compressional strain-rate, an Eulerian field representation for particle velocity can be used and the equation of motion for the particles can be solved explicitly to obtain an explicit expansion for the particle velocity field as (Ferry & Balachandar, 2001)

\[
\mathbf{u}_d \approx \mathbf{u}_c + \mathbf{V}_s - \tau (1 - \beta) \frac{D_c \mathbf{u}_c}{Dt} \quad (5.5)
\]

where \( \tau \) is the particle response time, \( \beta \) depends on the particle to fluid density ratio \( (\rho = \rho_d / \rho_c) \), and \( \mathbf{V}_s \) is the still fluid settling velocity of the particle, which are given by

\[
\tau = \frac{d^2 (\rho + 1/2)}{18 \nu_c f}, \quad \beta = \frac{3}{2\rho + 1}, \quad \text{and} \quad \mathbf{V}_s = \tau (1 - \beta) \mathbf{g}. \quad (5.6)
\]

Here \( d \) is the particle diameter, \( f = 1 + 0.15 Re_P^{0.687} \) is the correction for non-Stokesian drag (Crowe et al., 1998) that depends on particle Reynolds number \( Re_P = d |\mathbf{u}_c - \mathbf{u}_d| / \nu_c \), and \( \nu_c \) is the kinematic viscosity of the continuous phase. The particles have been assumed to be spherical and as a result the added mass coefficient is taken to be 1/2.
In equation (5.5) it is assumed that the ratio of settling velocity to the ambient flow velocity is small and of the order of Stokes number. It can be shown that implicit in the equilibrium approximation for particle velocity given in equation (5.5) is the assumption

\[
\frac{D_d\mathbf{u}_d}{Dt} \approx \frac{D_c\mathbf{u}_c}{Dt}. \tag{5.7}
\]

Equation (5.7) can be used to eliminate \(D_d\mathbf{u}_d/Dt\) from equation (5.4), which can be combined with equation (5.3) to obtain

\[
\left[\rho_c + \phi_d(\rho_d - \rho_c)\right]\frac{D_c\mathbf{u}_c}{Dt} = \phi_d(\rho_d - \rho_c)\mathbf{g} - \nabla p + \mu_c \nabla^2 \mathbf{u}_v. \tag{5.8}
\]

We consider the setting depicted in figure 5.1, where a channel is filled at one end with the particulate mixture and is separated by a gate from the rest of the channel, which is filled with clear fluid. When the simulation begins the gate is lifted and the flow develops forming an underflow intrusion of the mixture into the clear fluid (denoted by a solid line in figure 5.1). Let the height of the channel \(H\) be the length scale, \(U = \sqrt{g \Phi (\rho - 1) H}\) be the velocity scale and the initial volume fraction \(\Phi\) be the particle volumetric concentration scale, where \(g\) is the magnitude of the gravitational acceleration. The time and pressure scales are correspondingly defined as \(H/U\) and \(\rho_c U^2\), respectively. We consider density variations to be small and use Boussinesq approximation. The resulting governing equations in the dimensionless form are

\[
\frac{D_c\tilde{\mathbf{u}}_c}{Dt} = \tilde{\phi}_d \mathbf{e}^g - \nabla \tilde{p} + \frac{1}{Re} \nabla^2 \tilde{\mathbf{u}}_v, \tag{5.9}
\]

\[
\nabla \cdot \tilde{\mathbf{u}}_v = 0, \tag{5.10}
\]

\[
\tilde{\mathbf{u}}_d = \tilde{\mathbf{u}}_c + \tilde{\mathbf{V}}_s - \tau \frac{D_c\tilde{\mathbf{u}}_c}{Dt}, \quad \text{and} \quad \frac{\partial \tilde{\phi}_d}{\partial t} + \nabla \cdot \left( \tilde{\phi}_d \tilde{\mathbf{u}}_d\right) = \frac{1}{Sc Re} \nabla^2 \tilde{\phi}_d. \tag{5.12}
\]
Here all dimensionless terms are denoted by a tilde on top, and \( \mathbf{e}^g \) is a unit vector pointing in the direction of gravity.

The Reynolds number, defined as \( \text{Re} = U H / \nu_c \) characterizes the strength of the current. \( \text{Sc} = \nu_c / \kappa \) is the Schmidt number, where \( \kappa \) is the diffusivity of particles. The other two controlling parameters define the suspended particles in terms of particle Stokes number, \( \tilde{\tau} \), and dimensionless settling velocity, \( \tilde{V}_s \), defined as

\[
\tilde{\tau} = \frac{\tau (1 - \beta) U}{H} \quad \text{and} \quad \tilde{V}_s = \frac{V_s}{U},
\]

respectively. These parameters characterize the inertial and settling effects of the particle, respectively.

Note that for numerical stability it is common practice to add a diffusion term to equation (5.12). In the limit of \( \tilde{\tau} \to 0 \) and \( |\tilde{V}_s| \to 0 \) the above governing equations reduce to those corresponding to a scalar gravity current for which this term accounts for the diffusion of the scalar field. In the present case of a particulate gravity current this term can be taken to account for the departure of particle motion from equilibrium prediction. Such departures arise from close interaction of particles, and in general, diffusivity is a function of both local particle concentration and local shear (Acrivos, 1995; Foss & Brady, 2000). Nevertheless, solution of equation (5.12) with little or no diffusion is numerically unstable, especially in the context of spectral simulations.

According to (5.11) for \( \tilde{\tau} = 0 \) the particle velocity is simply the sum of local fluid velocity and the still fluid settling velocity. This is the limit often considered in the case of particulate currents. The last term on equation (5.11) arises from the inertial behavior of particles and it accounts for the velocity difference due to the inability of finite inertia particles to move with the fluid in regions of fluid acceleration. It must be pointed out that this term is only the first order correction of \( O(\tilde{\tau}) \) and, as shown in Ferry & Balachandar (2001), higher order terms of the expansion can be formally derived.
starting from the equation of motion for the particles. Numerical tests in a variety of turbulent flows have shown that the $O(\tilde{\tau})$ correction included in equation (5.11) is adequate to capture important inertial behaviors such as preferential accumulation and turbophoretic migration of particles of $\tilde{\tau} \leq 0.3$ (Ferry & Balachandar, 2001, 2002; Ferry et al., 2003; Shotorban & Balachandar, 2006).

Note that from equation (5.11) we can express the volume averaged mixture velocity as $\mathbf{u}_v = \mathbf{u}_c + \phi_d \mathbf{V}_s + O(\tilde{\tau} \tilde{\phi}_d)$. From which it follows that to $O(\tilde{\tau} \tilde{\phi}_d)$ we can approximate

$$\nabla^2 \tilde{\mathbf{u}}_v \approx \nabla^2 \tilde{\mathbf{u}}_c \quad \text{and} \quad \nabla \cdot \tilde{\mathbf{u}}_c \approx 0.$$  \hspace{1cm} (5.14)

The set of governing equations (5.9), to (5.12), with the approximations in equation (5.14) form a complete Eulerian-Eulerian system of equations for two-phase flows that include particle settling and inertia effects. The equations are formally accurate to $O(\tilde{\tau} \tilde{\phi}_d + \tilde{\tau}^2 + \tilde{\phi}_d^2)$. The main advantage of this system compared to the original set of equations, i.e. equations (5.1), (5.2), (5.3) and (5.4), is that the momentum equation for the dispersed phase need not be solved as the particle velocity field is expressed algebraically in terms of local fluid velocity by equation (5.11). Another advantage is that the mathematical structure of the simplified governing equations is similar to the standard single fluid incompressible Navier-Stokes equations, and this allows the use of standard techniques developed for incompressible flows for the present problem.

### 5.3 Formulation of the problem

Here we will examine the importance of particle inertia and settling under typical scenarios encountered in industrial, geological and environmental applications. From the definition of the length, velocity and time scales introduced above, the particle Stokes
number and dimensionless settling velocity can be written as

\[
\tilde{\tau} = \left[ \frac{(\rho - 1)^{5/3} g^{2/3}}{18 f \nu_c^{1/3}} \right] d^2 \Phi^{2/3} Re^{-1/3}, \quad \text{and} \quad (5.15)
\]

\[
\tilde{V}_s = \frac{\tilde{\tau}}{(\rho - 1) \Phi}. \quad (5.16)
\]

Consider the case of a turbidity current with sand particles suspended in water. If we consider sand to water density ratio to be about \( \rho \approx 2.65 \), and if we assume the relative motion of particles with respect to the surrounding fluid to be in the Stokes regime \( (f = 1) \), then the prefactor within the square parenthesis in the above equation can be estimated to be \( 5.9 \times 10^7 \text{m}^{-2} \). Now if we consider a suspension of 250\( \mu \text{m} \) sand particles at a volume concentration of \( \Phi = 1\% \) in a modest gravity current of \( Re = 10000 \), the resulting Stokes number based on mean flow time scale is 0.0079. The Stokes number will increase for larger particles and at higher concentration, but will decrease slowly with increasing intensity of the current (i.e. increasing \( Re \)).

If we consider the example of dust storms, where sand particles are suspended in air \( (\rho \approx 2000) \), the prefactor in (5.15) becomes \( 2.4 \times 10^{11} \text{m}^{-2} \). Now consider a suspension of 50\( \mu \text{m} \) particles at a volume concentration of \( \Phi = 0.1\% \) in a current of \( Re = 10000 \). The corresponding Stokes number becomes \( \tilde{\tau} = 0.28 \).
From (5.16) it can be readily seen that in a dilute suspension ($\Phi \sim O(10^{-2})$) of light particles ($\rho \sim O(1)$), as in the case of turbidity currents, the relative magnitude of the settling velocity can be much larger than the Stokes number. On the other hand, for the case of heavy particles ($\rho \sim O(10^3)$) Stokes number can be much larger than dimensionless settling velocity at sufficiently large concentration.

It is reasonable to nondimensionalize the settling velocity of particles with the velocity scale of the current, to gauge the relative importance of particle settling. In contrast, the Stokes number as defined above in equation (5.13) accurately captures only the inertial response of particles to mean scale motion. It is of interest to explore how particle inertia and settling velocity scale with the smaller scales of the flow. Crude estimates of the Kolmogorov velocity and time scales ($u_k$ and $\tau_k$) can be expressed in terms of the Reynolds number of the flow as (see for example Pope, 2000)

$$\frac{T}{\tau_k} \sim Re^{1/2} \quad \text{and} \quad \frac{U}{u_k} \sim Re^{1/4}.$$  \hspace{1cm} (5.17)

From which it follows that

$$\tau^+ = \tau \frac{T}{\tau_k} \sim Re^{1/6} \quad \text{and} \quad V_s^+ = \frac{\bar{V}_s U}{u_k} \sim Re^{-1/12}.$$  \hspace{1cm} (5.18)

The inertial response of particles to turbulent eddies is at its maximum when the time scale of the eddies matches that of the particles. Eddies which are larger are of longer time scale and they simply advect the particles, while eddies much smaller are of shorter time scale and do not last long enough to affect the particle motion. As illustrated in (5.17), with increasing $Re$ a wide range of time scales can be expected within the flow. Thus, we see that even though $\bar{\tau}$, which is based on mean flow scaling, may be much weaker for inertial response of particles, at high enough Reynolds number, some of the smaller scales of motion will be of appropriate time scale for inertial response of the
particles. As we will see below in the simulations to be presented, even modest values of $\tilde{\tau} \sim 0.025$ result in significant inertial response from the particles.

In this work, we present 2D direct numerical simulations for $Re = 3450$. This particular choice corresponds to the same value of Grashof number of $Gr = g\Phi(\rho - 1)H^3/\nu_c^2 = 1.5 \times 10^6$ used by Necker et al. (2002). We address the effect of particle inertia on the flow structure, dynamics, bed shear stress and deposition patterns by varying the parameter $\tilde{\tau}$. In order to isolate the physics of particle inertia, the present investigation neglects any interaction with the bottom. We consider a pure depositional flow without any resuspension and thus avoid the use of empirical particle resuspension relations García & Parker (1993).

5.4 Numerical approach

The dimensionless governing equations are solved using a de-aliased pseudospectral code (Canuto et al., 1988). Fourier expansions are employed for the flow variables in the horizontal direction ($x$). In the inhomogeneous vertical direction ($z$) a Chebyshev expansion is used with Gauss-Lobatto quadrature points. An operator splitting method is used to solve the momentum equation along with the incompressibility condition. With this method the flow field is advanced from time $\tilde{t}^{(n)}$ to $\tilde{t}^{(n+1)}$ in two steps. First, an advection-diffusion equation is used to advance from time level $\tilde{t}^{(n)}$ to an intermediate time level. After the intermediate level velocity field is determined, a Poisson equation is solved to compute the pressure field. Finally, a pressure correction step is used to advance the flow velocities to the level $\tilde{t}^{(n+1)}$ (see for example Brown et al., 2001). A low-storage mixed third order Runge-Kutta and Crank-Nicolson scheme is used for the temporal discretization of the advection-diffusion terms. This scheme is carried out in three stages. The time step from level $\tilde{t}^{(n)}$ to level $\tilde{t}^{(n+1)}$, $\Delta \tilde{t}$, is split into three smaller steps, with pressure correction at the end of each step. More details on the implemen-
The computational domain is a box of size $\tilde{L}_x = 25 \times \tilde{L}_z = 1$, which extends from $\tilde{x} = -12.5$ to $\tilde{x} = 12.5$ and from $\tilde{z} = 0$ to $\tilde{z} = 1$. The flow is initialized from rest with $\tilde{\phi}_d = 1$ in $\tilde{x} \in (-1, 1)$ for all $\tilde{z}$, and $\tilde{\phi}_d = 0$ otherwise with a smooth transition. The details of the initial condition can be found in Cantero et al. (2006). This setting of the problem generates two currents moving from the center outward. The solution was advanced in time until the front reached location of $\tilde{x} = \pm 11.5$ to avoid the influence of finite domain size (Härtel et al., 2000b; Cantero et al., 2006, 2007c). The simulations were performed using a resolution of $N_x = 1536 \times N_z = 150$. It must be mentioned that more resolution is needed for the particulate flow simulations compared to the corresponding scalar case (i.e. same $Re$ with $\tilde{\tau} = 0$ and $\tilde{V}_s = 0$).

Periodic boundary conditions are enforced for all the variable in the horizontal direction. At the top and bottom walls no-slip and no-penetration conditions are enforced for the continuous phase velocity. For the normalized concentration of particles we apply

$$\bar{V}_{az} \tilde{\phi}_d - \frac{1}{ScRe} \frac{\partial \tilde{\phi}_d}{\partial \tilde{z}} = 0, \quad \text{and} \quad \frac{\partial \tilde{\phi}_d}{\partial \tilde{z}} = 0, \quad (5.19)$$

respectively for the top and bottom walls, where $\bar{V}_{az}$ is the wall normal component of the normalized particle settling velocity. Volume integration of the normalized concentration equation (5.12) on the computational domain $V$ shows that

$$\frac{d}{dt} \int_V \tilde{\phi}_d \, dV = \int_{\partial V} \left( \tilde{\phi}_d \tilde{u}_d - \frac{1}{ScRe} \nabla \tilde{\phi}_d \right) \cdot (-\mathbf{n}) \, dA \quad (5.20)$$

where $\mathbf{n}$ is the surface outward normal and $\partial V$ is the computational domain boundary. Here the first terms in the brackets on the right hand side corresponds to the convective flux of particles while the second term is the diffusive flux and together they account for the total flux of particles through the boundaries of the domain. At the top and
bottom walls due to no-slip and no-penetration conditions \( \mathbf{u}_d = \mathbf{V}_s \) and thus the boundary condition (5.19) at the top wall corresponds to zero net flux of particles. At the bottom wall since the concentration gradient is set to zero, the net flux of particles is due to settling of particles. Thus here we consider a depositional flow, where the net concentration of particles within the computational domain continually decreases due to the depositional flux of particles through the bottom boundary. In many physical situations there can be shear and turbulence induced resuspension of particles from the bottom boundary. Empirical models of resuspension yield a non-zero diffusive flux of particles at the boundary expressed as a function of wall shear and particle Reynolds number (García & Parker, 1993). In this work, as the first step towards understanding the role of particle inertia, we will avoid such empiricism and ignore resuspension of particles.

The solution of the concentration equation, even in the limit of a scalar field, can lead to sharp concentration gradients when diffusive effects are not adequately accounted for. In order to avoid resulting numerical difficulties, the Schmidt number of the scalar field is typically limited to \( O(1) \). In the context of particulate concentration, the velocity of particle advection, \( \mathbf{u}_d \), is different from the fluid velocity. More importantly, even though \( \nabla \cdot \mathbf{u}_c = 0 \), the corresponding divergence of particle velocity field will not be zero in case of inertial particles (i.e., if \( \tilde{\tau} \neq 0 \)). This non-zero divergence of particle velocity field results in preferential accumulation of particles in regions of high strain-rate and avoidance of regions of high vorticity. Strong accumulation of particles is observed even at moderate values of \( \tilde{\tau} (\sim 0.025) \) resulting in even sharper gradients. Thus the importance of the diffusion terms is enhanced in the case of particulate concentrations.

Tadmor (1989) and Karamanos & Karniadakis (2000) have shown that spurious numerical behavior of the solution can be controlled by the use of a spectral viscosity without sacrificing spectral accuracy. In this approach, diffusion is increased for high wavenumbers to avoid Gibb’s oscillations, but the effect on the large scales (small
wavenumbers) is minimized. However, since the flow has a predominant flow direction, numerical instabilities have been observed to more likely occur in the direction of spreading than in the vertical direction, suggesting that an anisotropic implementation is needed. Based on these observations, following Karamanos & Karniadakis (2000) and Rani & Balachandar (2003) the conservation of mass for the disperse phase is modified to

$$\frac{\partial \tilde{\phi}_d}{\partial t} + \nabla \cdot (\tilde{\phi}_d \tilde{u}_d) = \frac{1}{Re Sc} \left[ \frac{\partial}{\partial \tilde{x}} \left( Q_{k_x} \otimes \frac{\partial \tilde{\phi}_d}{\partial \tilde{x}} \right) + \frac{\partial^2 \tilde{\phi}_d}{\partial \tilde{z}^2} \right]. \quad (5.21)$$

Here $Q_{k_x}$ is a wavenumber dependent diffusivity kernel and $\otimes$ denotes the convolution operation in physical space, i.e.

$$\frac{\partial}{\partial \tilde{x}} \left( Q_{k_x} \otimes \frac{\partial \tilde{\phi}_d}{\partial \tilde{x}} \right) = - \sum_{k_x} k_x^2 \hat{Q}_{k_x} \hat{\phi}_d \exp(i k_x \tilde{x}) \quad (5.22)$$

where $\hat{\cdot}$ represents the Fourier coefficient and $k_x = -N_x/2, \ldots, N_x/2 - 1$ is the wavenumber along the horizontal direction. The diffusivity kernel is computed as:

$$\hat{Q}_{k_x} = \begin{cases} 
1 & \text{for } |k_x| \leq M \\
1 + (Sc/Sc_{sv}) \exp \left[ (k_x^2 - N_x^2/4) / (k_x^2 - M^2) \right] & \text{for } |k_x| > M.
\end{cases} \quad (5.23)$$

where $Sc_{sv}$ and $M < N_x/2$ are free parameters to be selected. Based on numerical considerations we have chosen $Sc = 0.7$, and, in agreement with the findings of Härtel et al. (2000b) (see also Cantero et al., 2007c,b), we also observe that the results to be presented are not sensitive to this choice as long as $Sc$ is kept order 1. Any attempt to control numerical oscillations by setting $Sc$ smaller than order 1 results in over diffusive solutions where vortex shedding and Kelvin-Helmholtz instabilities are strongly damped. Several numerical test were performed to select adequate values for the diffusivity kernel.
The optimal choice that yields the highest quality result is \( Sc_{sv} \sim 3 \times 10^{-4} N_x, M \sim N_x/16. \)

The numerical scheme requires the computation of the material derivative of the continuous phase velocity, \( D_c u_c / Dt \), at each time step to be used in the equilibrium approximation for the disperse phase velocity field (see (5)). The most stable, efficient and accurate way of computing this material derivative was employing a third order explicit approximation based on the continuous phase velocity over the four previous stages.

5.5 Results and discussion

First we explore the effect of inertia in isolation, without any gravitational settling of particles, by setting \( \tilde{V}_s = 0 \). Although this is an idealized case, it can be considered as the limiting case of small heavy particles for which \( \tilde{V}_s / \bar{\tau} \rightarrow 0 \). In the second part of the results section we include settling effects and explore the influence of particle inertia on deposition patterns.

5.5.1 Front velocity

The height of the heavy current, \( \tilde{h} \), as a function of the streamwise location can be defined as

\[
\tilde{h}(\tilde{x}, \tilde{t}) = \int_0^1 \tilde{\phi}_d(\tilde{x}, \tilde{z}, \tilde{t}) \, d\tilde{z}.
\]

Thus at locations entirely occupied by the heavy particle laden fluid the current height will be 1.0 and in regions of pure fluid devoid of suspended particles the current height will be zero. The streamwise location of the current front, \( \tilde{x}_F \), can be unambiguously defined as the point where the current height, \( \tilde{h} \), reaches zero. In practice, although \( \tilde{h} \) remains quite small, it does not become identically zero ahead of the current front due
to diffusion. As a result a small threshold is used to identify the front location and the results are insensitive to the precise choice of the threshold (see details of definition in Cantero et al., 2007c). The front velocity can then be computed as

\[ \tilde{u}_F = \frac{d\tilde{x}_F}{d\tilde{t}}. \]  

(5.25)

Figure 5.2 shows the time evolution of the front velocity for three different values of \( \tilde{\tau} = 0, 0.025 \) and 0.05 at \( Re = 3450 \). Three regimes can be clearly distinguished in this figure, an initial acceleration phase, followed by a constant velocity phase, and a final phase of decay. Cantero et al. (2007c) presented a detailed analysis of these velocity phases in the context of scalar currents (i.e. \( \tilde{\tau} = 0 \) and \( \tilde{V}_s = 0 \)). The current rapidly accelerates from its initial rest state and reaches a peak velocity at a dimensionless time of about \( \tilde{t} \sim 1 \).

It is interesting to note that during this initial acceleration phase the inertia of the suspended particles does not seem to make a large difference on the front velocity of the current. A close-up of the front velocity during the acceleration phase is shown in figure 5.2 as an inset. It can be seen that at the very early stages the inertia of the suspended particles tends to slow the current, which can be explained by noticing that as the current accelerates from the gate \( Dc\tilde{u}_c/D\tilde{t} \) is positive and, from (5.11), the streamwise velocity of the particles can be estimated to lag behind the fluid.

This is more thoroughly explored in figure 5.3, where frame (a) shows contours of carrier fluid horizontal velocity (solid line) together with the horizontal component of the particles velocity (dash line) for \( \tilde{t} = 0.21 \) for \( \tilde{\tau} = 0.05 \) and \( \tilde{V}_s = 0 \). The thick long-dash line is the contour of \( \tilde{\phi}_d = 0.5 \) and gives an indication of the front location. Figure 5.3(b) shows the horizontal component of the inertial correction \( -\tilde{\tau}D\tilde{u}/D\tilde{t} \) (solid lines), and inertial correction vector field. In this figure as well the thick long-dash line also represents the contour of \( \tilde{\phi}_d = 0.5 \).
As soon as the lock is released, the lateral pressure difference sets up a counterclockwise circulation which extends over the entire depth of the channel, but is narrowly confined in the horizontal direction around the interface. The vector field of $-\tilde{\tau}D\tilde{u}/D\tilde{t}$ shows that the effect of particle inertia is to oppose the counterclockwise circulation and thereby to slowdown the initial advancement of the front. Since the time period of acceleration is about 1.0, $D\tilde{u}/D\tilde{t} \sim \tilde{u}$, and therefore the difference between particle and fluid velocity is dictated by $\tilde{\tau}$ and quite small, which for the present case are 2.5% or 5.0%.

From the inset of figure 5.2 it is observed that at the later stages of acceleration the inertia of the particles tend to speed up the front and as a result the peak front velocity attained by the particulate current appears to be insensitive to $\tilde{\tau}$. Frames (a) and (b) of figure 5.4 show the same information that figure 5.3 but for $\tilde{t} = 0.56$. Observe that at this time the inertial correction to the horizontal component of the particles velocity is smaller than 2% everywhere. The velocity field is sufficiently complex that the inertial correction, $-\tilde{\tau}D\tilde{u}/D\tilde{t}$, is positive to the right of the current and negative to the left. At the very nose of the current, the particle velocity slightly exceeds the fluid velocity and contributes to the catching up of the front with the scalar current. Also it can be observed that, although minor, the non-solenoidal nature of the inertial correction field is readily apparent.

Figure 5.5 shows the same information that the previous two figures but for $\tilde{t} = 1.77$. At this time there is increased difference between the horizontal component of the fluid velocity and particles velocity as shown in frame (a). In frame (b) the vector field shows clearly the non-solenoidal nature of the inertial correction with the vector field converging at the top and bottom fronts. Observe that this convergence of the velocity field implies injection of particles into the heavy front at the bottom, which breaks the symmetry of the problem shown by the $\tilde{\phi}_d = 0.5$ line. The divergence of the inertial correction vector field is related to the newly formed Kelvin-Helmholtz vortices at the
heavy and light fronts. As will be explained later, particles migrate from vortical regions and accumulate along regions of high shear.

Following the peak velocity the propagation of the current somewhat slows down before reaching a near constant front velocity. This deceleration of the current has been observed to coincide with roll up of the interfacial shear layer between the heavy and light fluids into coherent vortices. In the context of a scalar current it was observed (Cantero et al., 2007c) that the incipient roll up of the Kelvin-Helmholtz vortices started at around \( \tilde{t} \sim 1 \) and was nearly complete by \( \tilde{t} \sim 2.5 \). Figure 5.6 shows contours of swirling strength at four time instances \( \tilde{t} = 1.06, 1.77, 2.47 \) and 4.24, which are also indicated in figure 5.2. Here the swirling strength, \( \tilde{\lambda}_{ci} \), is defined as the absolute value of the imaginary portion of the complex eigenvalues of the local velocity gradient tensor\(^1\). Frame (a) of this figure shows the results for \( \tilde{\tau} = 0.0 \), frame (b) the results for \( \tilde{\tau} = 0.025 \) and frame (c) the results for \( \tilde{\tau} = 0.05 \). The values of peak swirling strength for the dominant vortices are indicated. During the deceleration subphase (at \( \tilde{t} = 1.06, 1.77 \) and 2.47), the interface rolls up produces strong Kelvin-Helmholtz vortices, which seem to regulate the value of the constant velocity of spreading in the slumping phase. With the presence of inertial particles the strength of the rolled up vortices weakens as indicated by the values of swirling strength during the deceleration subphase. This can perhaps be explained by the fact that inertial particles lag the fluid and cannot spin at the same rate of fluid elements. The consequence is a reduction in the deceleration rate.

Following the acceleration-deceleration phase the current settles to a near constant velocity in the slumping phase. The front velocities during this phase are 0.407, 0.415 and 0.432 for \( \tilde{\tau} = 0, \tilde{\tau} = 0.25 \) and \( \tilde{\tau} = 0.05 \), respectively. Thus, for the largest inertial particles considered here the constant slumping phase velocity has increased by

\(^1\)The local velocity gradient tensor has three eigenvalues. If all three eigenvalues are real then locally the flow is not swirling and \( \tilde{\lambda}_{ci} \) is set to zero. If the local velocity gradient tensor has one real and a complex conjugate pair of eigenvalues, the imaginary part of the complex eigenvalue provides a clean measure of the local swirling strength (Zhou et al., 1999; Chakraborty et al., 2005).
about 6.1%. Cantero et al. (2007c) observed the front velocity of a scalar current in the slumping phase to be well captured by two-dimensional simulations, since the dominant rolled up Kelvin-Helmholtz vortices remain sufficiently behind of the front. This behavior can be expected to remain unaffected for the case of the particulate currents as well. In the present simulations the constant velocity slumping phase extends over only a short period due to the limited amount to heavy fluid released. In the case of a large-volume release the constant velocity phase will persist for a long duration and the increased front velocity in a particulate current can significantly alter the arrival time of the current. The increase in front velocity with increasing inertial effect of the particles is due to particle accumulation near the head of the current and will be discussed below.

At high enough Reynolds number the constant velocity slumping phase will transition to an inertial phase, where the dominant balance is between gravity and inertia. The asymptotic behavior of a scalar current in the inertial phase shows a slow decay in the front velocity as \( \ddot{u}_F \sim \ddot{t}^{-1/3} \) (Fay, 1969; Hoult, 1972; Huppert & Simpson, 1980). The time of transition from constant velocity to a slow inertial decay can be estimated to be (Cantero et al., 2007c)

\[
\tilde{t}_{SI} = \frac{0.94\bar{x}_0\bar{h}_0}{F_{p,sl}^3},
\]

where \( \bar{h}_0 \) and \( \bar{x}_0 \) are the dimensionless height and half-length of the initial release and \( F_{p,sl} \) is the approximate constant velocity of the front in the slumping phase. Thus, with increasing constant front velocity during the slumping phase, the transition to inertial phase occurs earlier. For the present case of \( \bar{h}_0 = \bar{x}_0 = 1 \) the slumping-to-inertial transition times can be estimated as \( \tilde{t}_{SI} = 13.9, 13.1 \) and \( 11.6 \) for the cases of \( \bar{\tau} = 0, 0.025 \) and \( 0.05 \), respectively.

However, at lower current strengths (i.e. at lower \( Re \)) a direct transition from slumping to viscous phase will occur without going through an inertial phase (Cantero et al.,
The transition time from slumping to viscous phase can be estimated as

$$\tilde{t}_{SV} = \frac{0.57(\tilde{h}_0\tilde{x}_0)^{3/4}}{F_{p,sl}^{5/4}Re^{1/4}}. \quad (5.27)$$

Here again the transition will occur earlier with increasing constant front velocity during the slumping phase. For the three \(\tilde{\tau} = 0, 0.025\) and 0.05 cases the transition times can be estimated as \(\tilde{t}_{SV} = 13.5, 13.1\) and 12.5. Thus, for the present modest \(Re\), the estimated slumping to viscous transition times are very close to the slumping to inertia transition times. In fact, simple theoretical arguments show that for a full-depth planar current of unit initial release \((\tilde{h}_0 = \tilde{x}_0 = 1)\) to enter the inertial phase the Reynolds number of release must be greater than \(\sim 3.4 \times 10^3\) (Cantero et al., 2007c). The \(Re = 3450\) of the present simulation is clearly in the critical range and as a result if an inertial phase were to exist its extent will be quite limited and the current can be expected to transition quickly to the viscous phase. The transition times observed in figure 5.2 are in reasonable agreement with the theoretical estimates for the cases of \(\tilde{\tau} = 0\) and 0.025. For the larger inertial particles, the computed transition from the slumping phase is observed to occur somewhat earlier. Clearly the theoretical predictions are for a scalar current and they do not account for the inertial effect of particles. In the viscous phase the front velocity of both the scalar and the particulate currents are observed to decay at about the same rate. Although, the velocity for the \(\tilde{\tau} = 0.05\) case is consistently a little lower than for the other two cases.

### 5.5.2 Preferential accumulation

The concentration equation (5.12) can be rewritten in the following form

$$\frac{D_d\tilde{\phi}_d}{Dt} = \frac{1}{ScRe} \nabla^2 \tilde{\phi}_d - \tilde{\phi}_d \nabla \cdot \tilde{u}_d. \quad (5.28)$$
From which it can be seen that in a scalar current, where $\nabla \cdot \tilde{u}_d = \nabla \cdot \tilde{u}_c = 0$, at all later times the local concentration of scalar is guaranteed to be lower than the initial uniform concentration before release in the heavier fluid. This is however not the case for particulate currents, where the divergence of particle velocity can be non-zero. The equilibrium approximation provides a convenient way to obtain the divergence of particle velocity. By taking the divergence of equation (5.11) we obtain

$$\nabla \cdot \tilde{u}_d = \tilde{\tau} \left( \|\Omega_c\|^2 - \|S_c\|^2 \right)$$

(5.29)

where $\Omega_c$ and $S_c$ are the skew-symmetric and symmetric parts of the local fluid velocity gradient tensor, respectively. Note that $\nabla \cdot \tilde{u}_d > 0$ when $\|\Omega_c\| > \|S_c\|$, which implies that particles migrate from regions of vorticity and accumulate in regions of high strain rate. This preferential migration of particles increases with increasing $\tilde{\tau}$.

Figure 5.7 shows the structure of the current in the scalar limit ($\tilde{V}_s = 0$ and $\tilde{\tau} = 0$) at four different time instances. The flow is visualized by contours of particle concentration. Soon after release an intrusion front forms with a lifted nose due to the no-slip boundary condition. As the current advances Kelvin-Helmholtz vortices form at the interface, which together with bottom drag, balances the initial acceleration of the heavy front. As a consequence, after the initial set-up of the Kelvin-Helmholtz vortices, the front moves at constant speed until dilution and viscous effects in the current become important. Then, the current slows down and eventually dissipates.

Figures 5.8 and 5.9 show the corresponding results for currents with inertial particles of negligible settling, that is for $\tilde{\tau} = 0.025$ and $\tilde{\tau} = 0.05$ with $\tilde{V}_s = 0$. The solid lines indicate contours of $\tilde{\phi}_d \leq 1$, and dash lines correspond to $\tilde{\phi}_d \geq 1$. Particulate currents differ from their scalar counterpart in several ways. First, regions of $\tilde{\phi}_d \geq 1$ are not present in the scalar current as can be expected on theoretical grounds. In contrast, significant regions of $\tilde{\phi}_d \geq 1$ can be observed in case of particulate currents. At early
times ($\bar{\tau} < 10$) these regions of increased concentration can be observed to extend right behind the head of the current. This provides support for the sustained increase in the constant velocity of the particulate current in the slumping phase. At later times, when the current enters the inertial and viscous phases, such enhanced concentrations are not observed and accordingly the propagation of the particulate currents is not faster.

Also shown in figures 5.7, 5.8 and 5.9 at early times are the particle concentration levels at the center of the rolled up Kelvin-Helmholtz vortices. It is clear that with particle inertia the concentration of particles at the cores of the vortices has reduced to zero. Particles, owing to their inertia, are expected to spin out of the coherent vortices resulting in vortex cores devoid of particles. In contrast to these vortex cores, the body of the current, below the vortex cores, correspond to regions of high strain-rate and thus constitutes regions where particles accumulate. Also, due to particle inertia, long tongues of heavy particle laden fluids can be seen to extend above the body of the current. Such flow features can be expected to have an impact on instantaneous wall shear stress and deposition patterns.

Figure 5.10 shows the vertical profile of streamwise-averaged particle concentration at $\bar{t} = 2.47$ defined as

$$\bar{\phi}_d^{(x)}(\bar{z}, \bar{t}) = \frac{1}{L_x/2} \int_0^{L_x/2} \bar{\phi}_d(\bar{x}, \bar{z}, \bar{t}) d\bar{x}. \quad (5.30)$$

The currents with inertial particles present a larger mean particle concentration for $\bar{z} < 0.3$, where the front of the current is located. The concentration of particles over the region $0.3 < \bar{z} < 0.6$ is lower for the inertial particles, since this is where the vortices are located and the particles are spun out of their cores. The relative difference between the maximum values of $\bar{\phi}_d$ for $\bar{\tau} = 0.0$ and $\bar{\tau} = 0.05$ is about 5%. However, it is observed from figures 5.8 and 5.9 that the increase in concentration is not distributed uniformly along the horizontal direction, but preferentially close to the head of the current. This localized increase in concentration is likely to be the main source of the 6% increase in
the front velocity observed in the slumping phase.

In the initial acceleration phase particles do not affect substantially the flow structure compared to the scalar case. Once Kelvin-Helmholtz vortices start forming (at the beginning of the deceleration subphase), inertial particles resist spinning as fast as the carrier fluid, and on average diminish the initial strength of interface roll-up. This scenario of weaker interfacial vortices for the inertial particles is accurate only in the deceleration subphase, and soon changes in the constant velocity slumping phase. The net circulation at the interface can be estimated as (Cantero et al., 2007b)

$$\Lambda(t) \sim \bar{u}_F \bar{x}_F. \quad (5.31)$$

Thus in the constant velocity slumping phase circulation increases linearly with time with the slope given by the front velocity. With the higher front velocity, the net circulation at the interface for the inertial particles is higher than that for the corresponding scalar case (non-inertial particles). Figure 5.11 shows the swirling strength for time $\tilde{t} = 10$. Frame (a) shows the results for $\tilde{\tau} = 0.0$ with $\tilde{V}_s = 0.0$, frame (b) shows the results for $\tilde{\tau} = 0.025$ with $\tilde{V}_s = 0.0$ and frame (c) shows the results for $\tilde{\tau} = 0.05$ with $\tilde{V}_s = 0.0$. The correspondence between the locations of intense vortices as seen in the swirling strength contours and the regions devoid of particles in the concentration contours confirm the role of intense vortices. Consistent with the estimate for interfacial circulation, the extent of vortical region observed for the $\tilde{\tau} = 0.05$ case is much larger than the $\tilde{\tau} = 0.0$ case. Also in the case of inertial particles, rolled up vortices can be observed to penetrate all the way up to the front of the current, while in the scalar case, the rolled up vortices are located away from the front. It has been argued that the dynamic low pressure associated with the coherent vortices that are located close to the front of the current lowers the driving horizontal pressure gradient and thereby reduce the speed of the current (Cantero et al., 2007b). Thus, despite the presence of coherent
vortices close to the front, the increased velocity of the current with inertial particles, indicates the important role of particle accumulation close to the front.

At a much later time of \( \tilde{t} = 20 \), the current with \( \tilde{\tau} = 0.05 \) presents a somewhat lower vortical activity in the near-front region, while the currents with \( \tau = 0.025 \) and \( \tilde{\tau} = 0.0 \) show relatively stronger vortical activity near the front of the current as seen in figure 5.12. This figure presents the same information as figure 5.11 at the later time of \( \tilde{t} = 20 \). In the viscous phase self similar theories predict the front velocity to be be either \( \tilde{u}_F \sim \tilde{t}^{5/8} \) or \( \tilde{u}_F \sim \tilde{t}^{4/5} \) depending on the relative importance of interfacial vs bottom wall friction (Hoult, 1972; Huppert, 1982). From (5.31), using either of the power laws for the front velocity, it can be estimated that in the viscous phase net circulation at the interface decreases with time and thus formation of new vortices is not expected. The increased level of coherent vortices in the \( \tilde{\tau} = 0.0 \) and \( \tilde{\tau} = 0.025 \) cases near the front is consistent with the higher current speed observed for these cases during the viscous decaying phase. Also, in these cases, the strong interaction between the vortices at the front of the current results in episodic increase and decrease in the current speed, which can be observed in figure 5.2 as undulations.

5.5.3 Bottom shear stress

The shear stress distribution at the bottom boundary plays an important role in the resuspension of particles and thus in the time evolution of bed morphology. Figure 5.13 shows the dimensionless bed shear stress, \( (1/Re)\partial \tilde{u}_c/\partial \tilde{z} \), at \( \tilde{t} = 5 \) when the front of the current is located at \( \tilde{x} \simeq 3 \) for the three simulations of different particle inertia. The inset in the figure shows the instantaneous structure of the current visualized by concentration contours for the case of \( \tilde{\tau} = 0 \). The overall structure of the current is similar for the other two cases as well (see figures 5.7, 5.8 and 5.9).

As is evident from the figure, considerable variation can be observed in the local shear stress distribution. The peak located at \( \tilde{x} \sim \tilde{x}_F \) in figure 5.13 is associated with
the front. The subsequent three peaks are associated with the vortices identified in the inset as B1, B2 and B3. In the case of the scalar current the peak associated with the vortex B1 is weak because the billow B1 has not grown strong enough. The stronger vortices for the cases of $\tilde{\tau} = 0.025$ and $\tilde{\tau} = 0.05$ with the inertial particles are responsible for the larger values of bed shear stress. With increasing inertial effect of the particles the vortices B1, B2 and B3 move farther away from the front, but contribute to increased variation in the wall shear stress. Such differences in the bottom shear stress distribution persists at later time instances as well.

### 5.5.4 Effect of particle settling

Figures 5.14, 5.15 and 5.16 show the structure of the current for the three different inertial effects ($\tilde{\tau} = 0$, 0.025 and 0.05), respectively, but for the case of weak particle settling given by $\tilde{V}_s = 0.005$. At early times ($\tilde{t} < 15$) the net loss of particles due to sedimentation is not significant to greatly alter the dynamics of the flow. The observed flow structures at these early times are quite similar to those observed without any settling effect. Thus the role of particle inertia persists with the presence of weak gravitational settling. At later times, however, the loss of particles through settling is sufficiently significant that the current looses its intensity and begins to die quite rapidly. As can be expected, even weak particle settling has a dramatic effect at long times. Simulations with larger settling effects are uninteresting as the current dies off too quickly. In reality, settling of particles must be balanced by resuspension of particles, and in this limit the effect of particle settling in the bulk of the current can be of interest.

The net instantaneous deposition of particles at the bottom boundary can be defined as

$$\hat{m}_s(\tilde{t}) = \int_0^{\tilde{t}_x} \tilde{V}_{sz} \tilde{\phi}(\tilde{x}, \tilde{z} = 0, \tilde{t}) \, d\tilde{x}$$

and the results computed for the three different values of $\tilde{\tau}$ is presented in figure 5.17.
Up to $\tilde{t} \simeq 15$ the deposition rate increases. The increase is mainly due to the increase in planform area covered by the current. At later times, although the planform of the current continues to increase at a slower rate, the reduction in concentration is sufficiently large that net deposition decreases with time. Interestingly, as observed in figures 5.14, 5.15 and 5.16, $\tilde{t} \simeq 15$ is about the time when the effect of particle settling begins to have a strong effect on the dynamics of the current. As can be seen from the figure, the net effect of inertia is to increase the deposition rate at early times. The deposition rate is increased due to the larger accumulation of particles in the body of the current ($\tilde{z} < 0.3$) as they are spun out of the vortices. At later times, due to increased reduction in the suspended mass of particles, the deposition rate for inertial particles decreases.

The cumulative deposition of particles can be computed as

$$\tilde{D}(\tilde{x}, \tilde{t}) = \int_0^\tilde{t} \tilde{V}_{sz} \tilde{\phi}_d(\tilde{x}, z \tilde{z} = 0, \tilde{t}) \, d\tilde{t}. \quad (5.33)$$

Figure 5.18 shows $\tilde{D}$ at three different time instances $\tilde{t} = 10, 20$ and 45 for the three different inertial particles. Frame (a) shows the results for $\tilde{\tau} = 0$, frame (b) for $\tilde{\tau} = 0.025$ and frame (c) for $\tilde{\tau} = 0.05$. As explained above, deposition is significantly enhanced by inertia. Not only the total deposition is increased but also the deposit pattern is substantially influenced. Preferential concentration of particles generate localized regions of increased deposition which explains the different peaks in frames (b) and (c). It is interesting to note for $\tilde{\tau} = 0.025$ a regular undulating pattern of enhanced and suppressed deposition is observed, which is somewhat less pronounced in the cases of no inertial effect and the largest inertial effect considered. It can also be observed that the spatial wavelength of the undulatory deposition pattern decreases with the increased inertial effect of the particles.
5.6 Summary and conclusions

We have presented simulations of particulate currents employing a two-phase flow model which includes both the settling and also the particle inertia effects. The model consists of conservation equations for the continuous phase, an algebraic equation for the particle velocity based on the equilibrium Eulerian approach (Ferry & Balachandar, 2001), and a transport equation for the particle volumetric concentration. By the incorporation of the equilibrium Eulerian approach we avoid solving additional differential equations for the conservation of particulate momentum, which constitutes a big saving in computational time.

The results presented in this work clearly show that particle inertia has an important influence on the structure and dynamics of the particulate currents. Particles migrate from the core of Kelvin-Helmholtz vortices and accumulate in the front and body of the current. As a result the concentration of particles near the front is observed to be even higher than the concentration in the original release. Such preferential concentration of particles at the front results in a measurable increase in the constant velocity of the current during the slumping phase. The level of increase in the constant slumping phase velocity increases with particle inertia. The change in the structure of the current modifies the vortex pattern and its intensity. As a consequence we observe the associated bottom shear stress to be more intense in the case of inertial particles. This can have a strong influence on erosion and resuspension of particles from the bed.

Particle inertia has a significant effect on the deposition rate. We observe local cumulative deposit to be more than 100% larger for the case of particles of weak inertia ($\tilde{\tau} = 0.05$) as compared to particles of negligible inertial effect ($\tilde{\tau} = 0.0$). This dramatic increase in the deposition rate is due to the preferential accumulation of particles closer to the wall ($\tilde{z} < 0.3$) as they are spun out of interfacial vortices. Not only the deposition rate is increased but also the deposition pattern is changed.
Figure 5.2: Effect of inertia on front velocity. Front velocity as a function of time for $\dot{V}_s = 0$. Solid line: $\tilde{\tau} = 0$, dash-dot line: $\tilde{\tau} = 0.025$ and dash line: 0.05. In the figure AP: acceleration phase, SP: slumping phase, and VP: viscous phase. Observe in the inset frame that during the initial acceleration phase the fronts corresponding with particulate front move slightly slower than the scaler case due to the inertial correction.
Figure 5.3: Inertial correction during flow initiation. Frame (a): horizontal component of fluid velocity (solid line) and particles velocity (dash line) for $\tilde{\tau} = 0.05$ and $\tilde{V}_s = 0.0$ at $\tilde{t} = 0.21$. The thick long-dash line is the contour of $\tilde{\phi}_d = 0.5$ and gives an indication of the front location. Frame (b): inertial correction $-\tilde{\tau} D\tilde{u}/D\tilde{t}$ for $\tilde{\tau} = 0.05$ and $\tilde{V}_s = 0.0$ at $\tilde{t} = 0.56$. Contour lines for horizontal component of the inertial correction. The vector field show the non-solenoidal nature of the inertial correction.
Figure 5.4: Inertial correction during flow initiation. Frame (a): horizontal component of fluid velocity (solid line) and particles velocity (dash line) for $\tilde{\tau} = 0.05$ and $\tilde{V}_x = 0.0$ at $\tilde{t} = 0.56$. The thick long-dash line is the contour of $\tilde{\phi}_d = 0.5$ and gives an indication of the front location. Frame (b): inertial correction $-\tilde{\tau} \tilde{D}\tilde{u}/\tilde{D}\tilde{t}$ for $\tilde{\tau} = 0.05$ and $\tilde{V}_x = 0.0$ at $\tilde{t} = 0.56$. Contour lines for horizontal component of the inertial correction. The vector field show the non-solenoidal nature of the inertial correction.
Figure 5.5: Inertial correction during flow initiation. Frame (a): horizontal component of fluid velocity (solid line) and particles velocity (dash line) for $\tilde{\tau} = 0.05$ and $\tilde{V}_k = 0.0$ at $\tilde{t} = 1.77$. The thick long-dash line is the contour of $\tilde{\phi}_d = 0.5$ and gives an indication of the front location. Frame (b): inertial correction $\tilde{\tau} D\tilde{u}/D\tilde{t}$ for $\tilde{\tau} = 0.05$ and $\tilde{V}_k = 0.0$ at $\tilde{t} = 1.77$. Contour lines for horizontal component of the inertial correction. The vector field show the non-solenoidal nature of the inertial correction.
Figure 5.6: Swirling strength during acceleration phase for $\bar{V}_s = 0.0$. Frame (a) $\bar{\tau} = 0.0$, frame (b) $\bar{\tau} = 0.025$ and frame (c) $\bar{\tau} = 0.05$. The inset numbers indicate the swirling strength value for the main vortical structure as the front advances. Observe that for early times ($\bar{t} \leq 2.47$) the strength of this vortex diminishes with the inertia of the particles.
Figure 5.7: Contours of particles concentration. Solid line: $0.1 < \tilde{\phi}_d < 1$, dash line: $1.0 < \tilde{\phi}_d$. Solution for $\tilde{\tau} = 0$ and $\tilde{V}_s = 0$. Inset numbers indicate local value of $\tilde{\phi}_d$. 
Figure 5.8: Contours of particles concentration. Solid line: $0.1 < \tilde{\phi}_d < 1$, dash line: $1.0 < \tilde{\phi}_d$. Solution for $\tilde{\tau} = 0.025$ and $\tilde{V}_s = 0$. Inset numbers indicate local value of $\tilde{\phi}_d$. 
Figure 5.9: Contours of particles concentration. Solid line: 0.1, dash line: 1.0. Solution for $\tilde{\tau} = 0.05$ and $\tilde{V}_s = 0.0$. Inset numbers indicate local value of $\tilde{\phi}_d$. 

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Figure 5.10: Mean vertical particle concentration ($\tilde{\phi}_d^{(x)}$) profile for $\tilde{V_s} = 0.0$ at $\tilde{t} = 2.47$. Solid line $\tilde{\tau} = 0.0$, dash-dot line $\tilde{\tau} = 0.025$, and dash line $\tilde{\tau} = 0.05$. Particles spun out of the interface Kelvin-Helmholtz vortices ($0.3 \leq \tilde{z} \leq 0.6$) accumulate in the head and body of the current ($\tilde{z} \leq 0.3$).
Figure 5.11: Contours of $\tilde{\lambda}_{ci}$ (solid line) for $\bar{t} = 10$. Dash line is the contour for particle concentration $\tilde{\phi}_d = 0.05$. Solution for $\bar{\tau} = 0.05$ and $\tilde{V}_s = 0.0$. 

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Figure 5.12: Contours of $\tilde{\lambda}_{ci}$ (solid line) for $\tilde{t} = 20$. Dash line is the contour for particle concentration $\tilde{\phi}_d = 0.05$. Solution for $\tilde{\tau} = 0.05$ and $\tilde{V}_s = 0.0$. 
Figure 5.13: Dimensionless bed shear stress for $\bar{V}_s = 0$ at $\bar{t} = 5$ after the currents have traveled about 3 dimensionless length units. Solid line: $\bar{\tau} = 0$, dash-dot line: $\bar{\tau} = 0.025$, and dash line: $\bar{\tau} = 0.05$. The inset shows the current structure for $\bar{\tau} = 0$ visualized by particle concentration contours. Three vortical structures are identified as B1, B2 and B3.
Figure 5.14: Contours of particles concentration. Solid line: $0.1 < \tilde{\phi}_d < 1$, dash line: $1.0 < \tilde{\phi}_d$. Solution for $\tilde{\tau} = 0$ and $\tilde{V}_s = 0.005$. 
Figure 5.15: Contours of particles concentration. Solid line: $0.1 < \tilde{\phi}_d < 1$, dash line:$1.0 < \tilde{\phi}_d$. Solution for $\tilde{\tau} = 0.025$ and $\hat{V}_s = 0.005$. 
Figure 5.16: Contours of particles concentration. Solid line: $0.1 < \tilde{\phi}_d < 1$, dash line: $1.0 < \tilde{\phi}_d$. Solution for $\tilde{\tau} = 0.05$ and $\tilde{V}_s = 0.005$. 
Figure 5.17: Sediment deposition rate as a function of time for $V_s = 0.005$. The net effect of inertia is to increase the deposition rate due to the larger accumulation of particles near the bottom.
Figure 5.18: See caption in figure 5.18(c).
Figure 5.18: A posteriori analysis of deposition (without resuspension included). The figure shows the influence of particle inertia on total deposition $\tilde{D}$. The deposit is visualized for three time instants: $\tilde{t} = 10, 20$ and $45$. Frame (a): $\tilde{V}_s = 0.005, \tilde{r} = 0$. Frame (b): $\tilde{V}_s = 0.005, \tilde{r} = 0.025$. Frame (c): $\tilde{V}_s = 0.005, \tilde{r} = 0.05$. 
Chapter 6

Discussion, conclusions and future work

6.1 Discussion and conclusions

This work presents a systematic study of scalar and particulate density currents. The objectives of this work are: 1) to develop a sound theoretical and computational model for scalar and particulate density currents, including both particle inertia and settling effects; 2) to assess the hydrodynamics of density currents, specifically oriented to the effect of three-dimensionality and turbulent structures on the macroscale features of the flow for planar and cylindrical settings; and 3) to assess the effect of particle inertia and settling on macroscale features of particulate density currents.

The work is divided in four main sections. The main objective in section 1 (chapter 2) is to validate the present computational methodology by comparing the numerical results with previously published experimental and numerical works and with laboratory experiments produced for this study. This section addresses, in part, objective 1 posed in the introduction.

The main objectives of the section 2 (chapter 3) are to examine the effect of planar vs. cylindrical nature of the current, the influence of the volume of release on the propagation of the front, and the influence of 3D effects. This section addresses objective 2 posed in the introduction. The difference in the planform increases between planar and cylindrical currents results in fundamental differences in the spreading rate. In all the cases the front accelerates from rest to reach a maximum for the front velocity. This occurs after the front has advanced about 0.3 height units regardless of the Reynolds
number of the flow and the geometrical setting. At the time the front velocity peaks the interface begins to roll up. This process saturates eventually inducing a constant velocity of spreading. In the constant velocity slumping phase, the planar current heights are lower than the theoretical prediction for energy-conserving currents. The same occurs for the front velocity, which is also observed to be lower than the theoretical prediction for energy-conserving currents. In the present simulations, in addition to energy loss to wall friction, part of the potential energy goes towards maintaining internal flow recirculation, and only the balance goes towards the kinetic energy of the advancing front. This partitioning of energy has not been accounted for in the existing theories and we conjecture that perhaps by accounting for internal fluid motion it may be possible to better predict the actual current height and velocity. In the case of the planar currents, the nearly constant velocity of spreading during the slumping phase is not affected by the size of release. The Reynolds number of the present planar small-release, however, are not adequate to permit a clear development of the inertial phase, and for the two lower Reynolds number studied the transition is directly to the viscous phase, while for the larger Reynolds number simulation the current enters the inertial phase for a brief period of time. For the present planar configuration it can be estimated that the Reynolds number needs to be greater than about $3400 \overline{\tau}_\theta \overline{h}_\theta$ for the inertial phase to exist. In the case of the cylindrical currents the larger two Reynolds number simulations present an inertial phase of spreading, while the lower Reynolds number simulation transitions directly to the viscous phase. For the present cylindrical configuration the Reynolds number must be greater than about $880 \overline{\tau}_\theta \overline{h}_\theta^{1/2}$ to allow the development of an inertial phase. Three-dimensionality of the current has a strong influence on the propagation speed in the inertial and viscous phases. The Kelvin-Helmholtz vortices formed at the interface strongly interact among themselves and with the bottom boundary in a complex chaotic manner. In response to vortex pairing, leap-frogging and other such interaction processes the propagation of the front undergoes episodic rapid acceleration.
and deceleration. Two-dimensional approximation substantially underpredicts the mean speed of the current and thus will over estimate the arrival time. In the inertial and viscous phases, the coherent Kelvin-Helmholtz vortices are drawn closer to the front and, thus, influence the speed of the front substantially. On the other hand, in the slumping phase the coherent vortices are father upstream of the head of the current and do not affect the front velocity nearly as much. As a consequence, in the slumping phase both 2D and 3D simulations predict the same propagation speed.

The objective of section 3 (chapter 4) is to examine the structure and dynamics of cylindrical density currents. This section addresses also objective 2. As the front spreads, a shear layer forms between the heavy forward advancing and the light backward retreating fronts. As the interface develops, it rolls up forming Kelvin-Helmholtz vortices, which present a very different dynamics from the planar case due to the azimuthal stretching of the vortical structures. The number of rolls that form is highly dependent on the Reynolds number of the flow. The interface roll-up is not sustained during all the duration of the flow. A simple scaling law for a cylindrical current shows that the interface circulation grows only during the near constant velocity slumping phase, which is consistent with the duration of Kelvin-Helmholtz vortex rings formation in the simulations. As the vortex rings propagate radially out, they are inherently stretched along the azimuthal direction, which provides additional stability. The Kelvin-Helmholtz rolls are then destabilized in the azimuthal direction, and eventually break up into smaller scale structures. In cylindrical currents, high turbulence regions in the flow are associated with the break-up of the vortices, while in planar currents the flow is turbulent in its entirety, i.e. head and body. In both cases, the turbulent regions are populated with trains of hairpin vortices tilted toward the axis. At the larger two Reynolds numbers cylindrical simulations, a clear pattern of lobes and clefts develops originally at the lower part of the leading front, but very rapidly extend to the upper and rear part of the front. Once formed, they evolve very dynamically presenting
merging of clefts and splitting of lobes into new ones. The process is similar at both Reynolds numbers, however, with increasing Reynolds number the flow is characterized by smaller scales. The wavelength of the lobes grows with time as the front spreads and the local Reynolds number of the flow decreases. This is consistent with previous studies on planar currents that show that the most unstable wavelength decreases with Reynolds number. However, for cylindrical currents the number of lobes is maintained over time and the increase in wavelength is associated with the increase in circumferential length of the current. A laboratory experiment was performed for the case of salt water spreading in quiescent fresh water in cylindrical configuration. The Schmidt number in the experiment is 700, while in the numerical simulations is 1. Although the setting of the experiment is not exactly as in the simulations, the front velocity, the wavelength of the lobe and cleft structures and their temporal dynamics computed in the numerical simulations are in good agreement with the experimental observations. The general agreement implies that the dynamics of the front and its propagation are primarily dictated by Reynolds number and the size of the release and are not greatly influenced by Schmidt number.

The main objective of section 4 (chapter 5) is to study the effects of inertia on the front velocity, flow structure, bed shear stress, and deposition patterns. This section addresses objectives 1 and 3 posed in the introduction. A novel Eulerian-Eulerian model is presented for particulate density currents, which includes particle inertia and settling effects. The model reduces to the well-accepted set of equation for the modeling of scalar density currents in the case that inertia and settling are negligible. As the flow develops, it show the same phases of spreading as in the case of scalar currents. In the initial acceleration phase, while the flow do not present high turbulence, particles do not affect substantially the flow. Once vortex shedding starts, particles resist to spin as fast as the carrier fluid, and on average diminish the initial strength of the vortices. This effect manifests as a lower interface resistance to the flow. After substantial turbulence
develops in the flow, inertial particles migrate from high vorticity regions to high strain regions, which produces larger concentrations of particles in the body and head of the current, and together with the lower interface resistance, lead to faster currents with increasing particle inertia. The faster currents are associated with higher local Reynolds, and for intermediate times while deposition is still not important, they present more vortical activity and different bed shear stress patterns. The migration of particles from the interface of the current and accumulation in the head and body increase bottom particles concentration, which increases deposition rates. However, as mentioned above, the bed shear stress distribution is also modified and a coupled analysis of erosion and deposition is needed to assess the net effect on the bed. The results presented in this work for particulate density currents are based on 2D simulations, and show that particle inertia has an important influence on the structure and dynamics of the flow. As indicated in chapter 3, three-dimensionality affects the flow structure during the final phases of spreading, and some of the waviness in the deposition profiles for large times may be an artifact of the 2D simulations and are likely to disappear in 3D simulations. With regards to deposition, it must also be stressed that the interaction of the flow with the bottom boundary is important, and the feedback from the morphology that the flow develops plays an important role. In this work we do not seek to address this problem and only focus on the effect of particle inertia on the sediment flux at the bottom, i.e. the deposition rate. The integral in time of the deposition rate is used as a surrogate for the deposit height to explore possible changes in deposition patterns.

Finally, we would like to stress that the study in this work is performed through highly resolved large scale direct numerical simulation of the developed set of equations. The accurate numerical technique employed allows to capture all the relevant scales of the flow without the need of any type of turbulence modeling for flows with Reynolds numbers up to about 10000. Although these are modest Reynolds numbers, they are representative of mature turbulent flows at laboratory scales, and are only one order
of magnitude lower than some urban-scale similar flows (García et al., 2005). The use of direct numerical simulations prevents us from simulating field scale flows with larger Reynolds numbers. Nevertheless, our goal in this work is to address the connection of turbulent structures with macroscopic features of the flow and to introduce a new model capable of capturing the effect of particle inertia rather than performing field scale simulations. In order to simulate such larger Reynolds number flows, different approaches should be used, such as LES (Large Eddy Simulation) or RANS (Reynolds Averaged Navier-Stoke), in which smaller scales of the flow are modeled instead of resolved. For these cases, results from direct numerical simulations at laboratory scale can be useful to calibrate and improve the turbulence models, and we foresee the use of this approach as a useful complimentary tool to laboratory experiments.

6.2 Future work

In the present work we have attempted to improve the understanding of the dynamics of density currents under simplified conditions, that is channelized currents and open space releases over fix smooth flat boundaries at moderate Reynolds numbers. These simplified settings allowed to clearly identify the processes inherent to the flow avoiding any back-coupling from processes still not well understood that need to be incorporated through empirical modeling such us turbulence closures for larger Reynolds flows and sediment resuspension. There are, however, many situations in which more complicated scenarios need to be modeled. The directions that we foresee to continue this study are:

- The study of larger Reynolds number flows. The inertial self-similar phase of spreading has not yet been studied in detail through numerical simulations, and will be reached only for larger Reynolds number flows. The next natural step is the implementation of a subgrid scale turbulence model to perform Large Eddy Simulations.
• The study of particle resuspension. The next natural step is the implementation of an entrainment function through a boundary condition to model particle entrainment into suspension.

• The study of more complex geometries. The next natural step is to study the flow dynamics when it finds a change of slope favorable or unfavorable.

• The study of bed morphology evolution and interaction with the flow.
References


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