CHARACTERIZATION OF THE FLOW TURBULENCE USING WATER VELOCITY SIGNALS RECORDED BY ACOUSTIC DOPPLER VELOCIMETERS

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# CONTENTS

1. Introduction 1

2. Requirements for making the water velocity signal representative of the turbulent process in the water 1
   2.1 Sampling strategy 2
      2.1.1 Sampling time 2
      2.1.2 Sampling Frequency 2
   2.2 Effects of the presence of noise in the recorded signal on the computation of turbulence parameters 6
   2.3 Despiking the water velocity signals 8
   2.4 Removing steadiness in the signal 9

3. Computation of turbulence parameters 11
   3.1 Turbulent kinetic energy 11
   3.2 Convective velocity 12
   3.3 Turbulent Flow scales 16
      3.3.1 Integral time scales 16
      3.3.2 Integral length scale 17
      3.3.3 Taylor microscale 17
      3.3.4 Kolmogorov microscale 18
   3.4 Dissipation rate of turbulent kinetic energy 18

4. TuDe (Turbulent description) code 20

5. Acknowledgments 22

Appendix A: Power spectrum computation via Finite Fourier Transforms 23

Appendix B: Autocorrelation function via Fast Fourier Transforms computations 27

Appendix C: Computation of Power Spectral in spatial domain 28

Appendix D: References 29

Appendix E: TuDe Matlab code 31

1. Introduction

Present water measurement technology allows researchers to capture water velocity signals with a relatively high sampling frequency. From these signals, the computation of parameters which describe the flow turbulence can be performed with a good accuracy. The results obtained from these measurements can be used to understand the physics of the flow to investigate complex flow phenomena and to validate theories or numerical models.

Information about the turbulence in a flow is required commonly for studies related to determination of the mass transfer coefficient between the fluid and suspended particles (sediment, bubbles); agglomeration rates of suspended solids; computation of mass transfer rates between the fluids and the walls; etc. The common turbulence parameters usually required in those studies are: turbulent kinetic energy ($TKE$), Reynolds stresses, convective velocities, rate of dissipation of the $TKE$, and turbulence scales in the flow such as the scales of the energy containing eddies, Taylor and Kolmogorov scales.

Not all the recorded water velocity signals have the necessary information to compute the turbulence parameters and some restrictions appear as a function of: the flow velocities, the length scales present in the flow, noise level and the sampling frequency. First, requirements for making the recorded water velocity signal representative of the turbulent process under investigation in the flow are discussed which include definitions of adequate both sampling time of the signal and sampling frequency. Also the evaluation of effects of the presence of noise in the recorded signal on the computation of the turbulence parameters must be performed. Then, a summary of the available techniques to estimate the turbulence parameters from water velocity signals is included as well as some recommendations for their applications. As a final product, a Matlab code called TuDe (Turbulence Description) was developed by the authors to perform the estimation of the flow turbulence parameters based in all the information included in this report. Even though most of the methodology included here can be used to analyze time water velocity digital signals recorded with different instruments, this paper focuses on the use of Acoustic Doppler Velocimeters.

2. Requirements for making the water velocity signal representative of the turbulent process in the water

Some issues must be discussed to make the recorded water velocity signal representative of the turbulent process present in the flow. The first issue is related to the sampling strategy used to resolve the flow turbulence. It includes the definition, depending on the flow conditions, of an adequate sampling time of the signal as well as a sufficiently high sampling frequency. The second issue consists on evaluation of the effects of the presence of noise in the recorded signal on the computation of the turbulence parameters. Minimization of noise energy during the measurement is required as well as the use of techniques to both detect the noise energy level on the recorded signal and correct its effects on turbulence parameters. Yet another issue is the presence of undesired spikes in the water velocity signals and their effect on the turbulence parameters estimated from
these signals. Despiking is recommended in those cases and issues related to this topic are also discussed here. Finally, the steadiness or presence of periodic components in the water velocity signals with length or time scales bigger than the turbulence scales must be removed before the computation of the turbulence parameters. Swinging motion of bubbles plumes, tidal processes, and turbine agitated vessels by impeller blades are presented as examples of this process.

2.1 Sampling strategy
2.1.1 Sampling time
Tennekes et al. (1972) considered the error obtaining the mean value of a signal due to the finite integration time. Being $U$ the true mean value and $U_T$ the mean computed using the finite integration time, Tennekes et al. (1972) showed that the mean-square error may be approximated by:

$$\left(\frac{U_T - U}{\sigma}^2\right) \approx 2\frac{T^2}{T_m}$$  \hspace{1cm} (2.1.1)

where, $u'$ is the velocity fluctuation, $T$ is the integral time scale of the turbulent process and $T_m$ is the integration or sampling time. This approximation is valid for integration times much longer than the integral time scale. Using equation (2.1.1) adequate sampling time can be defined after the accepted mean-square error is fixed. A sampling time:

$$T_m > 20 \cdot T$$  \hspace{1cm} (2.1.2)

must be chosen to get a relative mean-square error in the computation of the mean value lower than 10% the ensemble variance.

$$\left(\frac{U_T - U}{\sigma}^2\right) \leq 2\frac{T^2}{T_m} < 2\frac{1}{20} = 0.10$$  \hspace{1cm} (2.1.3)

To define the sampling time required to get reliable values of the turbulence parameters, a dimensionless power spectrum shown by García et al. (2004) is used (see figure 2.1.1). $E_{11}$ is the power spectrum in the wave number domain, $\sigma^2$ is the variance of the recorded water velocity signal, $F$ is a dimensionless number defined as $f L / U_c$ where $f$ is the frequency, $L$ is the energy containing eddy length scale of the flow and $U_c$ is the convective velocity.

A dimensionless number $F = f L / U_c = 0.20 \,(1/2F=0.1)$ is defined in this figure as the upper boundary of the plateau observed at low frequencies. This value is close to the universal Strouhal number $= 0.16$ proposed by Levi (1991) which establishes a relationship which applies to a variety of fluid-flow problems between the frequency of oscillation induced in a restrained fluid body by an external free flow, the velocity of the latter and a typical dimension of the body. Using longer sampling time $T_m$ does not give extra information to the computation of the total turbulent energy as the integral of the power spectrum. Thus, a sampling time $T_m = \frac{1}{f} = \frac{L}{0.2U_c} = \frac{5L}{U_c} = 5\,T$ is defined as the time needed for the analysis of one turbulent flow structure. Soulsby (1980) claimed that at least 30 biggest turbulent structures per sampling time must be recorded to give
reliable statistics. Thus, he finally suggested that the sampling time to get reliable values of the turbulent energy is:

\[ T_m > 150 \frac{L}{U_c} = 150 \cdot T \]  

(2.1.4)

This sampling time agrees with the one obtained from plots published by Lenschow et al. (1994) to estimate moments up to fourth order with a defined statistical significance (normalized error variance of the moments smaller than 5%).

Finally, Bendat et al. (1993) claimed that the random error estimating the power spectrum is equal to \( 1/n_d^{0.5} \) where \( n_d \) is the number of records used to compute the spectrum. Each record must be at least as long as one turbulent structure (5 \( L/U_c \)). To get an error smaller of 15% estimating the power spectrum, 45 records must be used which makes:

\[ T_m > 225 \frac{L}{U_c} = 225 \cdot T \]  

(2.1.5)

To get errors smaller than 10% a sampling time longer than 500 \( T \) is required.

Even though an initial estimate of the sampling time could be obtained from equations (2.1.2) and (2.1.4) and (2.1.5), an analysis of water velocity signal recorded in the present flow conditions must be performed. The analysis consists on the running average for each statistical parameter to be computed (variance, covariance, skewness, etc.). A running average is performed by increasing the number of samples used to define the parameter until the computed value does not change more than a stipulated value. Thus, the number of samples required to obtain a converged parameter value can be defined.
2.1.2 Sampling Frequency
The capability of Acoustic Doppler Velocimeters (ADV) to resolve flow turbulence for a specified flow and sampling conditions must be checked. Garcia et al. (2004) claim that, in general, the ADV produces a reduction of all the even moments in the water velocity signal due to the low pass filter (averaging) of the sampling strategy used by this technology. They analyzed also the impacts on the third order moment but a clear trend could not be detected. This is due to the fact that the third order moment (as well as all the odd moments) usually present a very small value which require a very long integration time to estimate them with a reasonable level of accuracy (Sreenivasan et al., 1978). If the skewness is exactly zero, the integration time required is indeterminate. Garcia et al. (2004) also affirm that the ADV’s sampling strategy affects the autocorrelation function (increasing $R_{xx}$ for small lag times), the time scales computed from it (which result to be high biased) and the power spectrum (less resolution of the inertial range). However, they claim that all these effects are rather negligible in cases where a value of the dimensionless frequency \( F = f_R L/U_c \) higher than 20 is used. Here \( f_R \) is the frequency with which the ADV records the data available to the user. Garcia et al. (2004) proposed that this ratio provides a new criterion to check the validity of the ADV measurements as a good representation of the turbulence in any flow with known convective velocity and length scales. In cases where this criterion cannot be satisfied (i.e., \( F \) results lower than 20) a set of curves were proposed by García et al. (2004) to estimate the values of the necessary corrections. These curves are presented in figures 2.1.2 and 2.1.3. The parameters representing flow statistics obtained for different user-set frequency \( f_R \) are made dimensionless using the corresponding value of the parameter computed from water velocity signals sampled at frequency \( f_s = 260.8 \text{ Hz} \) which is the frequency of the three-dimensional velocity measurement process of the ADV (no averaging effect of ADV is included; more details about this effect can be found in Garcia et al., 2004). The dimensionless parameters are plotted as a function of the dimensionless parameter \( F \) defined as:

\[
F = \frac{f_R}{U_c} = \frac{f_R}{f_T} = \frac{L}{d_R}
\]  

(2.1.5)

where \( f_T \) = characteristic frequency of large eddies present in the flow and \( d_R \) = diameter of the sampled volume set by the flow and sampling characteristics. So far, the sampling volume has been considered as a point. However, the spatial averaging performed by ADVs should be considered when the value of \( d_R \) is smaller than the diameter of the measurement volume, \( d \). For instance, in the case of the 10Mz Nortek Velocimeter \( (d = 6 \text{ mm}, \text{ and } f_R = 25 \text{ Hz}) \), \( d_R < d \) for convective velocities smaller than 15 cm/s. In those cases \( d \) must be used in equation (2.1.5) instead of \( d_R \) because spatial averaging becomes more important than the temporal averaging. This replacement is based on the assumption that the spatial average acts as a low pass filter with wavelength = \( 1/d \). The higher the ratio \( F \), the better the description of the turbulence that can be achieved with a specific instrument.

The evolution of variance (integral of the spectrum and second order moment of the signal) is shown in dimensionless form in figure 2.1.2, together with the corresponding
fourth order moment. The effects of the averaging on the latter is more important (higher reduction in the sampled moment) than on the variance.

Based in figures 2.1.1 and 2.1.2 no turbulence characterization could be performed from the recorded signal for $F < 1$.

The time scales computed from the autocorrelation function (as the integral of $R_{xx}$ up to the first zero crossing) are high biased due to the sampling averaging. Figure 2.1.3 quantifies this bias showing the variation of this time scale with $F$ in dimensionless terms.

Figure 2.1.2: Sampling effect (averaging) on second and fourth order moments.

Figure 2.1.3: Sampling effect (averaging) on integral time scale ($T_{11}$).
2.2 Effects of the presence of noise in the recorded signal on the computation of turbulence parameters.

The presence of noise in recorded water velocity signals can strongly affect the computed value of all the turbulence parameters. Special efforts must be used to reduce the noise level of the signal during measurement. After the water velocity signal is recorded, the noise energy level must be evaluated and its relative effect on the turbulent parameters computed. In cases of ADV measurements, Doppler noise constitutes the main source of error. García et al. (2004, see Appendix E) introduced a set of curves to evaluate the relative importance of the Doppler noise energy on the total measured energy using ADVs. In cases with high noise impact, the noise energy level needs to be defined and corrections to the turbulence parameters (turbulent kinetic energy, length and time scales and convective velocity) must be performed.

The Doppler noise has the characteristics of white noise (Nikora et al., 1998; Lemmin et al., 1999 and McLelland et al., 2000) with a flat power spectrum (Anderson et al., 1995) which definitely indicates the presence of uncorrelated noise (Lemmin et al., 1999). In low energy flow, the energy level of the white noise can be identified in a power spectrum as a flat plateau at high frequencies. Nikora et al. (1998) suggested that empirical spectra of the Doppler noise can be replaced by straight horizontal lines whose ordinates are equal to the average of noise spectral ordinates. This technique was called “spectral analysis” by Voulgaris et al. (1998). They calculate the noise energy using the noise energy level detected in the tail of the spectrum (frequency range is chosen so that there are 10 estimates for the calculation of the statistically significant average, i.e. 11.5-12.5Hz for sampling frequency = 25Hz).

García et al. (2004) concluded that in the cases where the Doppler noise energy is important in relation to the real turbulent energy of the signal; the noise energy level can be computed using the “spectral analysis” method because the white noise plateau is observed in the power spectrum. In cases of very high energy flow (or very small noise energy level), this method can not be used. However, it was proved (Garcia et al. 2004) that the noise energy in these cases is smaller than 10% of the real total energy. An upper estimated value for a noise energy level can be computed in these cases using the energy value from the spectrum at the Nyquist frequency \((f_R/2)\) assuming that the flat plateau would be present in the spectrum just after the Nyquist frequency.

Using the noise energy level detected with the “spectral analysis” technique, corrections to the turbulence parameters are performed; the corrected power spectrum for each velocity component can be simply obtained by subtracting the white noise energy level from the measured spectra. The corrected variances for each component are computed by integrating the corrected power spectra, and from them the corrected turbulent kinetic energy is computed. In addition, using the inverse fast Fourier transform of the corrected power spectrum, estimators of the autocorrelation function and corrected different time scales are obtained.
2.3 Despiking the water velocity signals

The undesired spikes present in the water velocity signals must be first detected and then replaced in two independent steps. The replacement of the detected spike is not required for computations of the bulk statistical moments, but it is essential for computations of all the turbulence parameters derived from power spectrum and autocorrelation functions.

Filtering of the signals can be performed by defining specific requirements for the signal quality parameters. In ADV measurements, the signal to noise ratio (SNR) and the correlation parameter are indicators of the quality of the signal. Generally, a correlation parameter higher than 70% and a SNR higher than 15db are required for a good turbulent representation of the flow. This analysis can be done using the average parameter for all three ADV beams or the minimum parameter from among the three beams. The latter is the option suggested for the authors for filtering based on signal quality parameters. These and other more complicated filtering techniques such as acceleration spike filter or phase-space threshold despiking technique (Goring et al. 2002, Wahl, 2003) are included in WinADV, a software developed and written by Tony L. Wahl, Hydraulic Engineer of the U.S. Bureau of Reclamation.

After the spikes are detected, different options can be used to replace the values. These options include interpolation from preceding values, use of the overall mean of the signal, or interpolation between ends of the spike. Goring et al. (2002) suggested using a third order polynomial through 12 points on either side of the spike as the best option. Another methodology consists on low pass filtering the recorded signal in the frequency domain. The selected cut-off frequency should be low enough to remove energy of the spikes but high enough to not significantly modify the resolve portion of the energy spectrum. The inverse fast Fourier transform of the filtered spectrum produces the interpolated signal without the spikes.

2.4 Removing steadiness in the signal

To obtain an accurate estimate of the turbulence parameters the steadiness present in the signal must be removed because it produces an overestimation of both the computed turbulent energy in the signal and the scales in the flow. Usually, the unsteady component in the water velocity signals presents time scales larger than the turbulence scales and is referred in the literature as pseudoturbulence. Rao et al. (1972) and Wu et al. (1989) claimed that it is necessary to remove the pseudoturbulence from the signal before the turbulence can be properly characterized. The reason is that the fluctuations due to pseudoturbulence are different from the purely random fluctuations. Bendat et al. (2000) summarized the different alternatives to remove steadiness when the signals were already recorded with unsteady component. It could be performed through: a) high pass filtering using Fast Fourier Transform, b) fitting a polynomial to the low frequency components, c) computing the moving average of the recorded signal and d) dividing the signal in sub samples.
A recorded water velocity signal is used here to represent the different methodologies. The selected signal corresponds to the radial water velocity component recorded at a bubble plume facility (Garcia et al., 2004) at a vertical location 3.05m from the axis of the tank and vertical location 5.3m above the diffuser. The air discharge at the diffuser at this time was 35scfm. The evolution of the total variance of the selected recorded signal with zero mean (which includes all the energy components as turbulence and pseudoturbulence) as the low-pass filter frequency increases is plotted in the figure 2.4.1. A random periodic motion of the bubble plume was detected during the measurement with a dominant (average value) time scale of 327sec, which produces the steadiness behavior in the recorded signal. The energy values are computed integrating the power spectrum up to the filter frequency $f_c$. It is clearly observed the two processes which are involved in the signal: turbulence and swinging motion (pseudoturbulence). The swinging motion is predominant for frequencies smaller than 0.003 Hz (period longer than 327sec). The turbulent process with a totally different trend is present for frequencies higher than 0.003 Hz.

![Figure 2.4.1: Accumulated energy of the signal as the cutoff frequency increases for the analyzed water velocity signal.](image)

Due to the random characteristics of the observed swinging motion, and to assure that all the fluctuations present in the signal due to this phenomenon are removed, a time scale of the filter equal to 120 seconds was used which is shorter than the mean observed period in this signal but longer enough to not significantly affect the turbulence components. Figure 2.4.2 shows the recorded signal and the low frequency signal from low-pass filtering the recorded one. The high pass filtering was performed making equal to zero all the Fourier component of the power spectrum of the recorded signal for frequencies lower than the filter frequency. Then the low pass filter is obtained by difference with the complete signal in the time domain.
Besides, polynomials of different orders were fitted to the recorded signal in order to detect the low frequency component of the signal. Figure 2.4.3 shows the evolution of the energy of the high frequency component of the signal (found by difference) as the order of the fitted polynomial increased. The figure shows also the separation of the process involved in the recorded signal (swinging motion and turbulence). Polynomials of order higher than 20 are required to represent the swinging motion of the plume but not important extra information is captured with polynomial of higher orders up to 50. Polynomial of order much higher than 50 are needed to represent the turbulence process.

Figure 2.4.2: Low frequency component of the analyzed water velocity signal.. Time scale of the low-pass filter = 120 seconds.

Figure 2.4.3: Energy of the high frequency component of the analyzed water velocity signal as the order of the polynomial used to fit the low frequency component increases.
Finally, a polynomial of order 20 is used to represent the swinging motion of the plume.

As another option, moving average of the selected signal was performed. The main issue here is selecting an averaging time for the detection of the low frequency component in the signal which provides an acceptable compromise between the bias error due to excessive long averaging time and the random error due to the use of short time averaging. An optimum averaging time $T_o$ is defined (Bendat et al., 1993) which minimize the mean square error (obtained as a sum of squares of both the bias and random errors) in the determination of the low frequency component of the signal. Bendat et al. (1993) derived the following expressions for $T_o$:

$$T_o = \left[ \frac{72}{B_m C_T U_x^2} \right]^{\frac{1}{2}} \tag{2.4.1}$$

where

$$B_m = \frac{1}{G_{xx}(0)} \tag{2.4.2}$$

and

$$C_T = \left[ \frac{d^2 \hat{u}}{dt^2} \right]_U^2 \tag{2.4.3}$$

$U_x$ is the mean value of the signal, $G_{xx}(0)$ is the power spectrum value in the frequency domain at frequency equal to zero and $\hat{u}$ is the low frequency component of the signal. To estimate the second derivative of $\hat{u}$ in equation (2.4.3) the polynomial of order 20 fitted before to the recorded signal is used. Using the information related to the signal analyzed here, the optimum averaging time $T_o$ was 111.55sec. Finally and averaging time of 120sec was used. The high frequency component of the signal is found by difference with the complete signal in the time domain. Finally, the recorded signal was also divided in sub samples of length smaller than the dominant swinging period in order to eliminate the steadiness. The main turbulence parameters were computed based in these sub samples and an average value of the parameter for the entire set of sub samples is used as representative of the high frequency energy (turbulence component) of the recorded signal. A contrast between the cited techniques is included in figure 2.4.4 and table 2.4. They show the low frequency component (swinging motion) and the turbulent energy respectively detected applying the different methodologies to the analyzed radial water velocity signal. No important differences are found with the only exemption of the technique which averages the observed values of sub samples. For this example, the high pass filtering of the signal using a time scale of the filter of 120 seconds was the methodology selected to analyze the three velocity components of water velocity signals.
### Table 2.4: Variance of both the complete signal and the high frequency component (turbulence) estimated using the different methodologies.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Variance [cm²/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete signal</td>
<td>18.03</td>
</tr>
<tr>
<td>High frequency component obtained by</td>
<td></td>
</tr>
<tr>
<td>a) High pass filtering</td>
<td>6.18</td>
</tr>
<tr>
<td>b) Fitting polynomial</td>
<td>6.41</td>
</tr>
<tr>
<td>c) Moving average</td>
<td>7.30</td>
</tr>
<tr>
<td>d) Averaging subsamples</td>
<td>7.97</td>
</tr>
</tbody>
</table>

### Figure 2.4.4: Contrast between low pass filtering (time scale=120sec), polynomial fit (order 20) and moving average (moving period of 120sec) for the analyzed water velocity signal.

#### 3. Computation of turbulence parameters

Following, the methodology available to compute the turbulent kinetic energy, convective velocity, turbulence scales and dissipation rates of turbulent kinetic energy using the stationary component of the recorded water velocity signal is detailed. Correction of each turbulence parameter due to the presence of Doppler noise is also included.

##### 3.1 Turbulent kinetic energy (TKE)

The turbulent Kinetic energy - TKE is computed by definition as:

\[
TKE = \frac{1}{2} \left( u^2 + v^2 + w^2 \right) \tag{3.1.1}
\]
where \( u^2 \), \( v^2 \) and \( w^2 \) indicate the variance of the signal for each Cartesian velocity component. However, since the measured energy using ADV is biased high due to Doppler noise (Lohrmann et al., 1994), the above variances have to be corrected. The noise effect must be removed firstly from the power spectrum of each recorded water velocity component signal using energy noise level defined before. It consists simply of subtracting this level from the measured spectra. The area below the corrected power spectrum in the frequency domain is the variance of the analyzed signal. Using the corrected variances for each water velocity component, the corrected value of turbulent kinetic energy is obtained. Let \( NL_x \), \( NL_y \), \( NL_z \), the white noise levels of each of the measured spectra. Then:

\[
TKE_{corrected} = TKE - \frac{fR}{2} (NL_x + NL_y + NL_z) \tag{3.1.2}
\]

### 3.2 Convective velocity \( U_c \)

Sometimes, spatial correlation information is required to characterize the flow turbulence which can be obtained from the time series using the Taylor frozen turbulence hypothesis. This hypothesis expresses the assumption that a spatial non uniformity in the velocity structure passes by the observation point so rapidly that it has no time to change during the transit time (Heskestad, 1965). For \( u' \) equal to the longitudinal component of the fluctuating velocity in a turbulent field being carried along the \( x_1 \) direction with the mean velocity \( U_1 \) (\( U_2 \) and \( U_3 \) are the mean velocities in the other Cartesian components are assumed to be equal to zero):

\[
- \frac{\partial u'_i}{\partial t} = U_1 \frac{\partial u'_i}{\partial x_i} \tag{3.2.1}
\]

Figure 3.2.1 shows how a triangular spatial turbulent structure in \( u'_1 \) that is carried to the left along \( x_1 \) with a convective velocity \( U_1 = -2 \text{cm/s} \).
By assuming that the spatial structure does not change its shape during the transit time, the time signal measurement at $x_1 = 20\text{cm}$ looks like that in figure 3.2.2.

![Figure 3.2.2: Temporal turbulent structure sampled at location $x_1 = 20\text{cm}$.

Based on this information:

$$\frac{\Delta u'_{i}}{\Delta t} = \frac{4\text{cm/s}}{3s} = 2\text{cm/s} = -U_1$$

$$\frac{\Delta u'_{i}}{\Delta x_i} = \frac{4\text{cm/s}}{6\text{cm}}$$

It is generally recognized that this estimate breaks down for large turbulence intensities. Wernersson et al. (2000) claimed that this equation is only valid in homogeneous flow at low turbulent intensities. Heskestad (1965) derived a relation between time and spatial derivatives taking into account this problem. This relation is valid for incompressible shear flows steady in the mean where the scale of non homogeneity is large compared to the Taylor microscale. Such flows include high Reynolds number, turbulent, free shear flow and wall-restricted shear flows away from the solid boundaries.

It is assumed in this paper that

$$\frac{Du'_{i}}{Dt} = \frac{\partial u'_{i}}{\partial t} + U_j \frac{\partial u'_{i}}{\partial x_j} + u' \frac{\partial u'_{i}}{\partial x_j} = 0 \quad (3.2.2)$$

This can be obtained from the momentum equation for $u'_{i}$ component assuming: the viscous term is considered negligible (the momentum associated with the turbulent motion is primarily governed by the large eddy structure which are not directly affected by the viscous forces); $\lambda \ll L$ (increasing Reynolds number) where $\lambda =$ Taylor microscale and $L$ is the characteristic length of the mean shear region; negligible pressure gradient which can be considered fair approach for $R_{\lambda} > 10^3$, and a good approximation for $R_{\lambda} > 10^4$, where $R_{\lambda} = u' \lambda / \nu$. Then,

$$-\frac{\partial u'_{i}}{\partial t} = U_j \frac{\partial u'_{i}}{\partial x_j} + u' \frac{\partial u'_{i}}{\partial x_j} \quad (3.2.3)$$
Analyzing the fluctuating velocity component in each direction, squaring and averaging this equation, and assuming that velocity products and product of velocity derivatives are uncorrelated (independence of Fourier components for distant wave numbers), the following relations between the time and spatial derivatives can be found:

\[
\left( \frac{du'_i}{dt} \right)^2 = U_i^2 \left[ 1 + 2 \frac{U^2_i}{U^2_1} + 2 \frac{U^2_2}{U^2_2} + \frac{\overline{u'_2}^2}{U^2_1} + 2 \frac{\overline{u'_3}^2}{U^2_2} + 2 \frac{\overline{u'_2}^2}{U^2_2} \right] \left( \frac{du'_i}{dx_1} \right)^2 \]  

(3.2.4)

\[
\left( \frac{du'_2}{dt} \right)^2 = U_2^2 \left[ 2 \frac{U^2_2}{U^2_1} + 1 + 2 \frac{U^2_1}{U^2_1} + 2 \frac{u'_1^2}{U^2_1} + \frac{\overline{u'_1}^2}{U^2_2} + 2 \frac{\overline{u'_1}^2}{U^2_2} \right] \left( \frac{du'_2}{dx_2} \right)^2 \]  

(3.2.5)

\[
\left( \frac{du'_3}{dt} \right)^2 = U_3^2 \left[ 2 \frac{U^2_2}{U^2_3} + 2 \frac{U^2_3}{U^2_3} + 1 + 2 \frac{u'_3^2}{U^2_3} + \frac{\overline{u'_3}^2}{U^2_2} + 2 \frac{\overline{u'_3}^2}{U^2_2} \right] \left( \frac{du'_3}{dx_3} \right)^2 \]  

(3.2.6)

Thus, the convective velocity \( U_{ci} \) in the \( i \) direction defined as

\[
\left( \frac{du'_i}{dt} \right)^2 = U_{ci}^2 \left( \frac{\overline{u'_i}}{\overline{U_i}} \right)^2 
\]

(3.2.7)

can be computed as (Wu et al., 1989)

\[
U_{ci}^2 = U_i^2 \left[ 1 + 2 \frac{U^2_i}{U^2_1} + 2 \frac{U^2_2}{U^2_2} + \frac{\overline{u'_2}^2}{U^2_1} + 2 \frac{\overline{u'_3}^2}{U^2_2} + 2 \frac{\overline{u'_2}^2}{U^2_2} \right] 
\]

(3.2.8)

where \( i, j, k \) denote the Cartesian components. This equation is commonly used for the computation of convective velocity in turbine agitated tanks (Wu et al., 1989, Kresta et al., 1993, Wernersson et al., 2000). Corrected values of the variances must be used to obtain corrected values of the convective velocity.

If the turbulence is isotropic and the mean flow direction is in the \( x_1 \) direction, equation (3.2.8) is reduced to (Wu et al., 1989 and Kresta et al., 1993):

\[
U_{ci}^2 = U_i^2 \left[ 1 + 5 \frac{\overline{u'_1}}{U_i^2} \right] 
\]

(3.2.9)

As an alternative approach, the convective part of the fluctuations can be also estimated from the power spectrum in the convective flow (Wernersson et al., 2000). Integration of the three dimensional power spectra up to the frequency \( f_{\text{conv}} \) gives the contribution of the fluctuation to the locally convective flow \( \left( u'_{f_{\text{conv}}} \right)^2 \). The frequency \( f_{\text{conv}} \) can be determined
by the intersection between the slope of the high energy eddies and the slope of the inertial subrange eddies (-5/3 slope) (Wernersson et al., 2000). Thus the convective velocity in the $x_1$ direction (assuming that $U_2 = U_3 = 0$) can be computed as:

$$U_{c1}^2 = U_1^2 \left[ 1 + 5 \frac{u_{conv}^2}{U_1^2} \right]$$  \hspace{1cm} (3.2.10)

The effects of using the complete equation (3.2.8) or the two different approximations (3.2.9) and (3.2.10) are analyzed for two different flow environments. First in test 1, a water velocity signal which represents open channel flow conditions was recorded in a 91cm wide tilting flume along the center line of the flume and at longitudinal location 7m from entrance and 10cm above the bed. The water discharge was 0.050m$^3$/sec and the water depth was 15cm. The bed was covered by 3/4 inch crushed stone chips which provided a rough fixed bed. A Sontek Micro ADV was used to sample the velocity signal for 2 minutes at 50 Hz sampling frequency. In addition in test 2, a water velocity signal was analyzed which was recorded by a Nortek Doppler velocimeter (NDV) 10 MHz in a flow generated by a bubble plume in a large experimental tank (digester) at a wastewater treatment plant. The tank is approximately 15 m in diameter, 7 m deep at the walls, and 8.2 m deep at the center. The air discharge analyzed was 23scfm. The signal was recorded at a radial distance of 107cm from the center of the tank and the measurement point was located 120cm above the diffuser. Both, the mean and the fluctuating velocities at the sampling point for each experimental condition as well as the convective velocities computed from the different options are included in the table 3.2.1. Corrections to the standard deviation values due to the presence of the Doppler noise were performed.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Test 1: Open channel</th>
<th>Test 2: Bubble plume</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_1$ [cm/s]</td>
<td>58.64</td>
<td>6.72</td>
</tr>
<tr>
<td>$U_2$ [cm/s]</td>
<td>0.83</td>
<td>5.26</td>
</tr>
<tr>
<td>$U_3$ [cm/s]</td>
<td>-1.78</td>
<td>-0.03</td>
</tr>
<tr>
<td>$u'_1$ [cm/s]</td>
<td>4.28</td>
<td>2.52</td>
</tr>
<tr>
<td>$u'_2$ [cm/s]</td>
<td>3.07</td>
<td>4.96</td>
</tr>
<tr>
<td>$u'_3$ [cm/s]</td>
<td>2.61</td>
<td>3.23</td>
</tr>
<tr>
<td>$U_{c1} \text{ (eq. 3.2.8)}$ [cm/s]</td>
<td>59.14</td>
<td>13.30</td>
</tr>
<tr>
<td>$U_{c1} \text{ (eq. 3.2.9)}$ [cm/s]</td>
<td>59.42</td>
<td>8.77</td>
</tr>
<tr>
<td>$U_{c1} \text{ (eq. 3.2.1)}$ [cm/s]</td>
<td>58.64</td>
<td>6.72</td>
</tr>
<tr>
<td>$U_{c2} \text{ (eq. 3.2.8)}$ [cm/s]</td>
<td>83.33</td>
<td>13.27</td>
</tr>
<tr>
<td>$U_{c3} \text{ (eq. 3.2.8)}$ [cm/s]</td>
<td>83.33</td>
<td>14.76</td>
</tr>
</tbody>
</table>

Table 3.2.1. Convective velocities $U_{c1}$ computed using different options for two different flow conditions

The simplifications to the complete equation (3.2.8) (equations (3.2.9) and (3.2.1)) generate relative errors of -0.5% and 1% for the analyzed open channel flow conditions. However, the errors are much bigger for the bubble plume environment (higher turbulent intensities). There, the relative errors are 34% and 49% using equations (3.2.9) and
(3.2.1) respectively. Thus, simplifications to the Taylor Frozen turbulence approximation (equations 3.2.1 and 3.2.9) must be checked. The use of the mean value as a convective velocity can not be applied to flows with high turbulent intensity values such as a bubble plume environment.

3.3 Turbulent Flow scales

3.3.1 Integral time scales
Two different approaches can be used to estimate the integral time scale of a turbulent process. The first one derives the integral time scale from the autocorrelation function, the second one derives it from the power spectrum.

Using the autocorrelation function is the classical approach and it is based on the definition of the integral time scale (Pope, 2000):

\[
T_x = \int_{0}^{\infty} R_{xx} \, dt
\] (3.3.1)

where \( R_{xx} \) is the autocorrelation function in the time domain. It is suggested that one use the corrected values of the \( R_{xx} \), thus eliminating the effect of the Doppler noise, because the time scales computed from the uncorrected autocorrelation function are biased low. Steadiness effects must also be removed from the autocorrelation function before the computation of \( T_x \).

Some problems arise when using in practice equation (3.3.1). When the maximum time limit was considered in the integration, the results usually showed large fluctuations and could not be considered repeatable (Kresta et al., 1993). The definition presented in equation (3.3.1) is valid only for isotropic turbulence with no periodic component (Rao et al., 1972; Kresta et al., 1993). Because in this kind of flows \( R_{xx} \) can present negative values and the areas below \( R_{xx} \) are summed with regard to sign, the magnitude of the resulting integral scale is very small. In fact, it is smaller than microscales present in the flow (Rao et al., 1972). To overcome this problem, the integration is usually conducted to the first zero crossing and it is called \( T_o \) (Kresta et al., 1993; Wernersson et al., 2000, Pearson et al., 2002). Generally \( T_o \) and \( T_x \) agree in velocity fields without any periodic oscillation (Rao et al., 1972).

Using power spectrum information is the second approach suggested for the computation of the integral time scale which can be estimated by performing an extrapolation of the power spectrum to zero frequency (Pope, 2000 and Wernersson et al., 2000) as:

\[
T_i = \frac{G_u(f = 0)}{4\mu_i^2}
\] (3.3.2)

where \( u' \) is the variance of the longitudinal water velocity signal. Usually, the extrapolation to zero frequency is not an issue because the power spectrum presents a plateau at low frequencies when the steadiness in the signal has been removed.
3.3.2 Integral length scale
Using the convective velocity values computed according to the methodology presented in item 3.2), the integral length scales can be estimated as (Wu et al, 1989)

\[ L_i = T_i \cdot U_{ci} \]  

(3.3.3)

Wu et al.(1989) proposed to use a resultant macroscale which takes into account the three dimensionality and anisotropy of the flow:

\[ L_{res} = \sqrt{L_x^2 + L_y^2 + L_z^2} \]  

(3.3.4)

3.3.3 Taylor microscale
Two approaches are presented to estimate the Taylor time microscale which is an intermediate scale between the integral time scale and the Kolmogorov scale. The first one is based in the fact that at lag times close to zero, the autocorrelation function \( R_{xx} \) has a parabolic trend. The Taylor microscale \( \lambda_f \) can be computed using the second derivative of \( R_{xx} \) at zero lag (Pope, 2000):

\[ \lambda_f = \left[ -\frac{1}{2} R_{xx}'(0) \right]^{-1/2} \]  

(3.3.5)

by fitting of the second order polynomial function to the autocorrelation function in this region (negative second derivative, see figure 3.3.1) as:

\[ a\tau^2 + b\tau + c = R_{xx}(\tau) \]  

(3.3.6)

Figure 3.3.1: Fitting of second order polynomial to the autocorrelation function at zero lag.
Thus the Taylor microscale $\lambda$ can be computed as:

$$\lambda_{time} = [-a]^{1/2}$$

(3.3.7)

Due to the relatively low sampling frequency of the ADV, either the region where the second derivative of the autocorrelation function is negative cannot be fully resolved or only a few points are observed in this region. This situation remains even for the corrected autocorrelation function due to the presence of noise. Thus, Taylor microscale either cannot be computed or has too much uncertainty in its estimate. The Taylor length scale $\lambda_{length}$ is computed using equation (3.3.7) and the Taylor Frozen turbulence approach.

The second approach estimates the Taylor length microscale $\lambda_f$ from the dissipation rate of turbulent kinetic energy $\varepsilon$, whose estimation will be discussed later (see section 3.4), and the relation suggested by Pope (2000) for isotropic turbulence:

$$\lambda_{length} = \left( \frac{30\nu}{\varepsilon} u'^2 \right)^{0.5}$$

(3.3.8)

where $\nu$ is the water viscosity; $\varepsilon$ is the dissipation rate of turbulent kinetic energy and $u'^2$ is the variance of the water velocity signal.

### 3.3.4 Kolmogorov microscale

The smallest length scale of turbulence is defined by dimensional analysis as (Pope, 2000):

$$\eta = \left( \frac{\nu^{1.5}}{\varepsilon} \right)^{1/4}$$

(3.3.9)

### 3.4 Dissipation rate of turbulent kinetic energy.

The mechanism of the dissipation of turbulent kinetic energy is probably one of the most fundamental aspects of turbulence (Pearson et al., 2002). Three different approaches can be used to estimate the dissipation rate of turbulent kinetic energy.

The first method is based on the integration of the energy dissipation spectra as (Kresta et al., 1993 and Wernersson et al., 2000)

$$\varepsilon = 15\nu \int k_i^2 E_{ij}(k_i)dk_i$$

(3.4.1)

where $k_i^2 E_{ij}$ is the energy dissipation spectrum (Wernersson et al., 2000). The wave number $k_i$ is computed from the freqency $f$ using the Taylor hypothesis as:

$$k_i = \frac{2\pi f}{U_{ci}}$$

(3.4.2)
Where $U_{ci}$ is the convective velocity component in the Cartesian direction $i$ at the location where measurements are taken (equation 3.2.8). Using the same hypothesis, the one-dimensional spectrum in the spatial domain $E_{11}$ can be computed from the one dimensional power spectrum $G_{xx}$ in the frequency domain as:

$$E_{11}(k_1) = \frac{U_{ci}}{2\pi} G_{xx} \left( \frac{k_1 U_{ci}}{2\pi} \right)$$  \hspace{1cm} (3.4.3)

Using relations (3.4.2) and (3.4.3), the equation (3.4.1) can be presented as:

$$\varepsilon = 15\nu \frac{4\pi^2}{U_{ci}^2} \int f^2 G_{xx}(f) df$$  \hspace{1cm} (3.4.4)

This methodology requires a good resolution of the higher frequency components in the power spectrum because they have a strong influence on the total integral. Acoustic Doppler Velocimeters (ADV) signals do not satisfy these requirements because of both relative low frequency resolution of the ADV and the presence of noise.

The second methodology suggests to compute $\varepsilon$ from -5/3 slope fitting of the power spectrum in the inertial range using the one dimensional power spectrum in the spatial domain

$$E_{11}(k_1) = C_1 \cdot k_1^{-5/3} \varepsilon^{2/3}$$  \hspace{1cm} (3.4.5)

where $C_1 = 0.49$ (Pope, 2000). The existence of -5/3 slope region does not imply that the field is isotropic (Rao et al., 1972). The value of $\varepsilon$ is obtained doing a non linear fit in the inertia subrange of the relation for observed values of $E_{11}$ and $k_1$. The limits of the range were the fitting is performed are defined based in visual inspection of the power spectrum in order to use only the range where -5/3 slope is visualized.

The third methodology presented here to compute $\varepsilon$ is based in dimensional analysis. The dissipation rate of turbulent kinetic energy can be estimated using dimensional analysis as (Wu et al., 1989, Kresta et al., 1993, Wernersson et al., 2000, Pearson, 2002):

$$\varepsilon = A \frac{q'}{L}$$  \hspace{1cm} (3.4.6)

where $q'$ is the characteristic turbulent velocity. There is a general agreement to estimate $q'$ (Kresta et al., 1993 and Wernersson et al., 2000) using the turbulent kinetic energy $TKE$ as:

$$q' = \sqrt{TKE}$$

The proportionality factor $A$ will depend on the choice of the characteristic length $L$. Two approaches are usually suggested for the estimation of $L$, either form the geometry of the experimental setup or from local turbulent scales (using the autocorrelation function). Using the geometry, $L$ is fixed which means that energy containing eddies are represented
by one eddy size independent of the type of the flow locally (Kresta et al., 1993; Wernersson et al, 2000) and the value of the parameter A depend on the experimental setup. The other approach consist on define \( L \) as a turbulent macroscale according to equation (3.3.4). The parameter finally suggested by Wu et al. (1989) using this macroscale is \( A = 0.85 \pm 10\% \). Kresta et al. (1993) suggested that the dimensional analysis method using \( L \) from the autocorrelation function is one the most stable methods available for the estimation of the dissipation rate of turbulent kinetic energy.

4. Matlab code. TuDe (Turbulent description)

A Matlab code named TuDe (Turbulence Description) was developed based on all the information summarized in this report. The code is included in Appendix E. Following the capabilities of the code are summarized as well as the required inputs.

TuDe computes from the time series of the three components of water velocity vector the following:

a) Main statistics (means, variances, covariances, skewness and kurtosis). Results are included in filebasew_stat.sal
b) Corrected variance and turbulent kinetic energy based in the white noise energy level defined by user according to technique included in section 3.1.
c) Running mean analysis for each velocity component signal. Results in filebasew_runmean.sal (running mean series) and in filebasew_stat.sal (time when the convergence at a defined percentage of the mean value occurs). 
d) Power spectrum in frequency domain using periodogram function (see Appendix A). Results in filebasew_powerx.sal, filebasew_powery.sal and filebasew_powerz.sal

e) Corrections in the spectrum for presence of white noise using a defined noise energy level (see section 2.2). Results in filebasew_powerx_noise_removed.sal, filebasew_powery_noise_removed.sal, filebasew_powerz_noise_removed.sal

f) Convective velocity for each Cartesian component using equation (3.2.8). Results in filebasew_stat.sal

g) Power spectrum in spatial domain (see Appendix C).
h) Autocorrelation function for each velocity component using products. Results in filebasew_correla.sal

i) Corrected autocorrelation function using inverse Fast Fourier Transform of the corrected spectrum (see appendix B).
j) Integral time scales derived from the autocorrelation functions computed in the previous step using equation (3.3.1). Results in filebasew_spectrum.sal

k) Integral time scales computed from power spectrum (equation 3.3.2). Results in filebasew_spectrum.sal

l) Length scales using convective velocity values and equation (3.3.3).Results in filebasew_spectrum.sal.
m) Kolmogorov length scale (see section 3.3.4). Results in filebasew_spectrum.sal

n) Dissipation rate of turbulent kinetic energy by -5/3 slope fitting of the power spectrum in the inertial range (equation 3.4.5). The fitting is only performed in the flow direction defined by the user. Results in filebasew_spectrum.sal
o) Dissipation rate of turbulent kinetic energy by dimensional analysis (equation 3.4.6). Results in filebasew_spectrum.sal
p) Computation of the Taylor microscale using autocorrelation function (equation 3.3.7). Results in filebasew_spectrum.sal
q) Computation of the Taylor microscale using the dissipation rate of turbulent kinetic energy value computed by -5/3 slope fitting of the power spectrum in the inertial range. (equation 3.3.8). Results in filebasew_spectrum.sal
r) Noise parameters of the signal. The parameters related to the noise level of the signal are summarized in filebasew_noise.sal.

The inputs to the code are detailed as follows:

a) Water velocity signal
   • A text file with comma separation variable format is required containing the four variables (column 1: time; columns 2, 3, 4: x, y and z components of the flow velocity vector respectively). The name of the file (filebase.csv) is defined in the variable filebase.

b) Filebase name of written files.
   • The code generates a set of output files which can be called using the root name defined by the variable filebasew.

c) General parameters
   • Sampling frequency: variable fs [Hz].
   • Water kinematic viscosity: variable nu [m²/s]

d) Parameters for correction due to the presence of white noise.
   • Noise energy level [cm²/s] for each cartesian component is required through the variables Noisex, Noisey and Noisez. The three noise components have to be zero if no white noise corrections are performed. For ADV signals, usually noise energy level in x component is equal to the noise level in y and they are 30 times the noise in z (by geometry).

e) Parameters used in the computation of the dissipation rate of turbulent kinetic energy by fitting the -5/3 slope to the inertial range in the power spectrum.
   • The -5/3 slope fitting is performed just in the flow direction defined by the user through the variable fit_dir which is defined as 1 for x, 2 for y and 3 for z.
   • The lower frequencies where the noise is detected as a flat plateau in the spectrum for three velocity component: variables fwnx, fwny, fwnz. They must be defined in [Hz] to reduce the range where the -5/3 slope is fitted. They have to be the Nyquist frequency if no white noise corrections are performed.
   • A parameter alpha is required to define the lower limit of the wavelength number in the inertial range as kinf = 2 pi alpha/L where L = length scale obtained from autocorrelation function.
• An iteration process is required to define the upper limit of k1 in the inertial subrange (see section 3.4). Feeding in this process is defined by the variable ksup.

f) Parameters used in the analysis of running mean
  • The parameter pctg define the percentage around the mean value computed with all the signal to specify the intervals where running mean converge

g) Parameters used in the estimation of the Taylor microscale
  • The parameter NF2P define the number of values of autocorrelation function included to fit the second order polinomia at lag equal to zero.

h) Parameters used for the estimation of energy containing eddies length scale
  • Number of values in the low wave number region of power spectrum to be averaged to compute E11(0) is defined trough the parameter NVave.

i) Parameters of the Kolmogorov power spectra (see section 3.d)
  • C1 = 0.49; b = -5/3 are the Kolmogorov spectrum constant and slope

j) Parameters used in the dimensional analysis to compute the dissipation rate of turbulent kinetic energy
  • Aconst = 1.0 (see section 3.d.3)

It is strongly suggested not to modify the parameters included in i) and j).

5. Acknowledgments

A number of federal and state agencies have supported several research projects leading to this work at the University of Illinois. They are the U.S. Geological Service, the National Science Foundation, the office of Naval Research, the U.S. Army Corps of Engineering, the Illinois Water Resources Center, the Illinois Department of Natural Resources and the Metropolitan Water Reclamation District.
Appendix A: Power spectrum computation via Finite Fourier Transforms

The one side-autospectral density function is estimated from Finite Fourier Transforms (Bendat et al., 2000) as:

\[
G_{xx}(f) = \frac{2}{T} |X(f,T)|^2
\]  

(A.1)

with the narrowest possible resolution \( \Delta f = 1/T \). The finite-range Fourier transform \( X(f,T) \) of the real-valued or a complex-valued record \( x(t) \) is defined by the complex-valued quantity

\[
X(f,T) = \int_0^T x(t) e^{-j 2 \pi f t} dt
\]  

(A.2)

If \( x(t) \) is sampled at \( N \) equally spaced points a distance \( \Delta t \) apart.

\[
X(f,T) = \Delta t \sum_{n=0}^{N-1} x_n \exp\left[-j \cdot 2 \cdot \pi \cdot f \cdot n \cdot \Delta t\right]
\]  

(A.3)

where

\[
x_n = x(n \Delta t)
\]  

(A.4)

The usual selection of discrete frequency values for the computation of \( X(f,T) \) is

\[
f_m = \frac{m}{T} = \frac{m}{N \Delta t}
\]  

(A.5)

\[
X(f_m) = \Delta t \sum_{n=0}^{N-1} x_n \exp\left[-j \cdot 2 \cdot \pi \cdot m \cdot n \right]
\]  

(A.6)

The expression:

\[
X_m = \sum_{n=0}^{N-1} x_n \exp\left[-j \cdot 2 \cdot \pi \cdot m \cdot n \right]
\]  

(A.7.1)

or its equivalent

\[
X_m = \sum_{n=0}^{N-1} x_n \left\{ \cos \left[ \frac{2 \cdot \pi \cdot m \cdot n}{N} \right] - i \sin \left[ \frac{2 \cdot \pi \cdot m \cdot n}{N} \right] \right\}
\]  

(A.7.2)

is often referred as Discrete Fourier Transform (DFT)
Thus

\[ X(f_m) = \Delta t \cdot X_m \]    \hfill (A.8)

Replacing this in equation (A.1)

\[ G_{xx}(f) = \frac{2\Delta t^2}{T} |X_m|^2 \]    \hfill (A.9)

and \( T = N \Delta t \)

\[ G_{xx}(f) = \frac{2\Delta t}{N} |X_m|^2 \]    \hfill (A.10)

The Fast Fourier transform method is used to compute the Discrete Fourier Transform. For that a series with a length multiple of 2 is required. Zeros are added to the signal to complete this requirement. Then the spectrum must be corrected to this fact because the variance computed as an integral of this spectrum is smaller than the real one in a factor of \( n/N \) where \( n \) = number of values in the recorded signal and \( N \) is number of values of the signal completed with zeros. The computation of \( G_{xx}(f) \) is implemented in MatLab using the function Periodogram and in Excel using Tools/Data Analysis/Fourier Analysis.

The finite Fourier transform of \( x(t) \) defined in equation A.7.1 can be viewed as the Fourier transform of an unlimited time-history record \( y(t) \) multiplied by a rectangular time window \( u(t) \) as \( x(t) = u(t) \cdot y(t) \).

The Fourier transform of the rectangular time window present large side lobes which allow the leakage at frequencies well separated from the main lobe (Bendat et al. 2000). This leakage of energy may introduce significant distortions to the estimated spectra, particularly when the data have a narrow bandwidth. It is important to note that the effect of spectral leakage is contingent solely on the length of the data record. It is not a consequence of the fact that the periodogram is computed at a finite number of frequency samples.

To suppress the leakage problem, it is common in practice to introduce a time window to eliminate the discontinuities at the beginning and end of the records to be analyzed. There are numerous such windows in current use as Bartlett, Blackman, Chebishev, Hamming (or cosine squared), Hann, Kaiser, triangular, Tukey. In flow turbulence related topics the data have not narrow bandwidth which reduces the importance of using these techniques.

Besides, it is seen that this technique used to suppress side-lobe leakage also increases the width of the main lobe of the spectral window, that is, it reduces the basic resolving power of the analysis. There may be cases where maintaining a minimum main lobe bandwidth is critical for the analysis (Bendat et al., 2000). If it is necessary to reduce the increase in resolution bandwidth caused by the use of this technique without increasing
the random error in the estimates, the computation of the FFT for overlapped records is suggested (Bendat et al., 2000).

To take in account the problems cited before, a Welch's method is implemented in the MatLab Signal Processing Toolbox by the pwelch function. By default, the data is divided into eight segments with 50% overlap between them. A Hamming window is used to compute the modified periodogram of each segment. (MatLab help).

A simple test was performed to evaluate the performance of the different technique and windowing methods used to compute the Power spectrum. During this test the same turbulent water velocity signal was analyzed with the different options to check their performance. Pwelch show the smoother power spectra (see Figure A.1) however its integral underestimated the real variance of the signal in -10.35% contrasting with -0.025% of the Periodogram.

As it was cited before, the use of different windows options is not relevant when turbulent water velocity signals are analyzed because they are not narrow bandwidth. The results of using different techniques are shown in Figure A.2. All the cases where windowing was performed, presented a reduction of the computed variance in contrast to the real which show corrections must be done for each technique to reproduce the real variance values. The percentage of undestimation of the variance for the different windowing technique are cited as follows: Hamming: -11.2%; Bartlett: -9.77%; Blackman: -11.94%; Chebishev: -11.65%; Hann -11.23%; Kaiser =-0.67%; triangular: -9.77%; Tukey =-9.10%.

Figure A.1: Power spectrum computed using equation A.10 (Periodogram) and Welch method
Figure A.2: Power spectrum computed using equation rectangular window (Periodogram) and other windows.
Appendix B: Autocorrelation function via Fast Fourier Transforms computations

The Fast Fourier Transforms computations are used next for the computation of the autocorrelation function $R_{xx}$ of a digital signal $x(t)$. First, $N$ number of zeros must be added to the digital signal of length $T = N \Delta t$, to avoid the circular correlation effects. For the complete signal of length $2T$, Bendat et al. (2000) shown that two side-autospectral density function $S_{xx}(f)$ computed from the complete digital signal is related to the circular correlation function $R_c^{xx}(t)$ as:

$$S_{xx}(f) = \int_0^{2T} R^{xx}_{xx}(\tau) \cdot e^{-j2\pi f \tau} \, d\tau$$  \hspace{1cm} (B.1)$$

The function $R^{xx}_{xx}$ is defined as:

$$R^{xx}_{xx}(r\Delta t) = \frac{2N - r}{2N} R_{xx}(r\Delta t) \quad \text{for } r = 0, 1, \ldots, N-1$$  \hspace{1cm} (B.2)

where $\tau$ is the time lag $= r \Delta t$. The definition in equation (B.2) is valid only if $N$ zeros were added to the digital signal of length $N$ avoiding circular effects.

$R^{xx}_{xx}$ can be estimated from equation (B.1) using the expression for the inverse Fourier transform of $S_{xx}(f)$ computed for a complete signal of length $2T$.

$$R^{xx}_{xx}(\tau) = \int_0^{2T} S_{xx}(f) \cdot e^{j2\pi f \tau} \, df$$  \hspace{1cm} (B.3)

If $R^{xx}_{xx}(t)$ is sampled at $2N$ equally spaced points a distance $\Delta t$ apart, $f_m = \frac{m}{2T} = \frac{m}{2N\Delta t}$ and $\tau = r\Delta t$:

$$R^{xx}_{xx} = \Delta f \cdot \sum_{m=0}^{2N-1} S_{xx} \left( f_m \right) \exp \left[ j \cdot 2 \cdot \pi \cdot m \cdot r \cdot \frac{2N}{2N} \right]$$  \hspace{1cm} (B.4)

Using $\Delta f = 1/2T = 1/(2N\Delta t)$, and $S_{xx}(f_m) = \Delta t \cdot \left( S_{xx} \right)_m$ where $(S_{xx})_m$ is the discrete estimate of $S_{xx}(f)$, Thus:

$$R^{xx}_{xx}(r\Delta t) = \frac{1}{2N} \cdot \sum_{m=0}^{N-1} \left( S_{xx} \right)_m \exp \left[ j \cdot 2 \cdot \pi \cdot m \cdot r \cdot \frac{2N}{2N} \right]$$  \hspace{1cm} (B.5)

Finally, the unbiased autocorrelation estimate of the autocorrelation function is computed as:

$$R_{xx}(r\Delta t) = \frac{2N}{2N - r} R^{xx}_{xx}(r\Delta t)$$  \hspace{1cm} (B.6)
In summary the procedure to estimate the autocorrelation function of a digital signal via Fast Fourier Transforms computations is presented as follows:

a. Augment the digital signal of N values with N zeros to obtain a complete signal of 2N values.

b. Compute the 2N point two side-autospectral density function \( S_{xx}(f) = 2G_{xx}(f) \) using equation (A.10).

c. Compute the inverse Fast Fourier transform of \( S_{xx}(f) \) and just use the values for \( r = 0, 1, \ldots, N-1 \) to obtain \( R_{xx}(t) \)

d. Estimate the unbiased autocorrelation \( R_{xx} \) using equation (B.6)

**Appendix C: Computation of Power Spectral in spatial domain**

We can compute the power spectral using wave number \( k \) as independent variable instead of frequency \( f \). It is useful in cases where the power spectrum is used to estimate values of dissipation rate. We can use Taylor frozen-turbulence assumption to relate the \( k \) and \( f \) variables, which assert that

\[
    k_i = \frac{2\pi f}{U_{ci}} \quad (C.1)
\]

Where \( U_{ci} \) is the convective velocity component in the Cartesian direction \( i \) at the point where measurements are taken. Thus, the one-dimensional spectrum can be expressed as

\[
    E_{ii}(k_i) = \frac{U_{ci}}{2\pi} G_{xx} \left( \frac{k_i U_{ci}}{2\pi} \right) \quad (C.2)
\]

Where the following condition must be satisfied

\[
    R_{xx}(0) = E[x^2(t)] = \int_0^\infty G_{xx} \left( \frac{k_i U_{ci}}{2\pi} \right) \frac{U_{ci}}{2\pi} dk_i \quad (C.3)
\]

\[
    R_{xx}(0) = \int_0^\infty E_{ii}(k_i) dk_i \quad (C.4)
\]
Appendix D: References


Appendix E: TuDe Matlab code

%03/10/2004 TUDE program (TUrbulent DEscription)
%---------------------------------------------------------------------
%This program computes:
%A. For the three water velocity components time series:
%  1) Main statistics for each water velocity component time series. Results in *stat.sal
%  2) Corrected variance based in the white noise energy level defined by user. Results in *stat.sal
%  3) Running mean for each velocity component time serie. Results in *runmean.sal (series) and in *stat.sal (time when the convergence at a defined percentage of the mean value ocurrs)
%  4) Power spectrum in frequency domain using periodogram function. Results in powerx.sal, powery.sal,powerz.sal
%  5) Corrections in the spectrum for white noise presence using a defined noise energy level. Results in powerx_noise_removed.sal, powery_noise_removed.sal, and powerz_noise_removed.sal
%  6) Convective velocity for each cartesian component. Results in *stat.sal
%  7) Power spectrum in spatial domain.
%  8) Autocorrelation function for each velocity component using products. Results in *correla.sal
%  9) Autocorrelation function using inverse Fast Fourier Transform of the corrected spectrum due to noise.
%  10) Length and time macroscale from autocorrelation function using ifft. Results in *spectrum.sal
%  11) Length and time macroscale from power spectrum. Results in *spectrum.sal
%  12) Dissipation rate of turbulent kinetic energy by fitting -5/3 law to the spatial power spectrum. Results in *spectrum.sal
%  13) Kolmogorov length scales. Results in *spectrum.sal
%  14) Dissipation rate of turbulent kinetic energy by dimensional analysis. Results in *spectrum.sal
%  15) Computation of the Taylor microscale using autocorrelation function
%  16) Computation of the Taylor microscale using dissipation rate of turbulent kinetic energy value
%  15) Noise parameters of the signal. Results in *noise.sal
%---------------------------------------------------------------------
%Data are read from the text file with a comma separation variable structure (filebase.csv) which must contain four columns: one for the time t [sec] and the others three for the water velocity components vectors [cm/s]: U, V, and W. This file must contain an one header line.
%Noise is removed from the spectrum using a threshold defined previously [cm^2/s]
%---------------------------------------------------------------------
%Clean up
clear

% I. EXTRACTING DATA FROM .CSV FILE
%---------------------------------------------------------------------
filebase = 'C:/Carlos/Tank_paper/program/202040';

% Set constants
row_start = 1; % Header has 1 lines
col_start = 0; % Offset zero columns

% Read the file
data = dlmread(fullfile(filebase,'.csv'), ',', row_start, col_start);
% Get the time and velocities
  t = data(:,1);
  vx = data(:,2);
  vy = data(:,3);
  vz = data(:,4);

% II. INPUT: PARAMETERS
% Filebase name of written files
  filebasew = 'C:/Carlos/Tank_paper/program/202040_';

% General parameters
  fs=25;%Sampling frequency [Hz]
  nu = 0.000001;%water viscosity [m2/s]

% Parameters for correction due to white noise presence
  %Noise energy level for each cartesian component. [cm2/s] It is the energy noise in the
  %frequency domain.
  %The three noise components have to be zero if no white noise corrections are performed.
  %For ADV signals, usually Noisex=Noisey=30Noisez
    Noisex = 0.018555955; %
    Noisey = 0.018369234;
    Noisez = 0.000618621;

% Parameters used in the spectrum -5/3 law fitting to estimate the dissipation
  fit_dir = 1;%The -5/3 law fitting is performed just in this flow direction(1 for x, 2 for y and 3 for
            z);

% Frequencies where the noise is detected as a flat plateau in the spectrum. They have to be half
% of frequency sampling (12.5 for 25hz sampling frequency) if no white noise corrections
% are performed
  fwnx = 12.5;%[1/sec] It is the lower frequency that white noise appears for the x component.
  fwny = 9;%[1/sec] It is the lower frequency that white noise appears for the y component.
  fwnz = 12.5;%[1/sec] It is the lower frequency that white noise appears for the z component.

  alpha=1; %these values is used to define the lower limit of k in the inertial range is kinf = 2 pi
  alpha/L where
    %L = length scale obtained from autocorrelation function

  ksup =1000;%An iteration process is required to define the upper limit of k1 in the inertial
    % subrange
    %Feeding in this process is defined by the variable ksup.

% Parameter used in the analysis of running mean
  pctg = 0.05; %perc. around the mean value computed with all the signal to specify the
               intervals where running mean converge

% Parameters used in the estimation of the Taylor microscale
  NF2P=4;% The parameter NF2P define the # of values of autocorrelation function included to fit
         the second order polinomia at lag equal to zero.

% Parameters used for the estimation of energy containing eddies length scale
% Number of values in the low wave number region of power spectrum to be averaged to
  compute E11(0)
NVave=10;

% Others Parameters which are used in the computation (don't modify them)

%%% Kolmogorov Spectra Parameters
C1 = 0.49;  \%(Constant in E11 spectra)
b = -5/3;  \%(slope adopted)

% Constant A used in the dimensional analysis determination of dissipation rate
Aconst = 1;

------------------------------------------------------------------------------------------------------------------------
% III. COMPUTATION
%------------------------------------------------------------------------------------------------------------------------
% 1) Main statistics for each water velocity velocity component time series
%------------------------------------------------------------------------------------------------------------------------
% mean value
vmx = mean(vx);
vmy = mean(vy);
vmz = mean(vz);
% Standard Deviation
sdvx = std(vx);
sdvy = std(vy);
sdvz = std(vz);
% Variance
varvx = var(vx);
varvy = var(vy);
varvz = var(vz);
% Covariance
uv = cov(vx,vy);
uw = cov(vx,vz);
vw = cov(vy,vz);
% Skewness
skvx = skewness(vx,0);
skvy = skewness(vy,0);
skvz = skewness(vz,0);
% Kurtosis
kvx = kurtosis(vx,0);
kvy = kurtosis(vy,0);
kvz = kurtosis(vz,0);

------------------------------------------------------------------------------------------------------------------------
% 2) Correction of the variances, turbulent intensities and TKE for each velocity component
% based in the noise energy level defined by user.
%------------------------------------------------------------------------------------------------------------------------
% Corrected variance
varvxcorr=varvx-Noisex*fs/2;
varvycorr=varvy-Noisey*fs/2;
varvzcorr=varvz-Noisez*fs/2;
% Turbulent intensities
intvx=(varvxcorr)^0.5/abs(vmx);
intvy=(varvycorr)^0.5/abs(vmy);
intvz=(varvzcorr)^0.5/abs(vmz);
% Turbulent kinetic energy
TKE = 0.5*(varvxcorr+varvycorr+varvzcorr);
% Running mean for each velocity component time serie.

vxt = vx';
vyt = vy';
vzt = vz';

vxtm = cumsum(vxt)./(1:length(vxt));
indvxrm = find(abs((vxtm-vmx)/vmx) >= pctg);
tconvx = t(max(indvxrm));

vytm = cumsum(vyt)./(1:length(vyt));
indvyrm = find(abs((vytm-vmy)/vmy) >= pctg);
tconvy = t(max(indvyrm));

vztm = cumsum(vzt)./(1:length(vzt));
indvzrm = find(abs((vztm-vmz)/vmz) >= pctg);
tconvz = t(max(indvzrm));

tmin = min(t);
tmax = max(t);

figure(1); clf;
fig1 = plot(t, vxrm,'r',[tmin tmax],(1-pctg)*[vmx vmx],':r',[tmin tmax],(1+pctg)*[vmx vmx],':r', ... 
    t, vyrm,'b',[tmin tmax],(1-pctg)*[vmy vmy],':b',[tmin tmax],(1+pctg)*[vmy vmy],':b', ... 
    t, vzrm,'g',[tmin tmax],(1-pctg)*[vmz vmz],':g',[tmin tmax],(1+pctg)*[vmz vmz],':g');
grid on;
set(fig1,'LineWidth',2);
xlabel('time [sec]');
ylabel('vm [cm/s]');
h = legend('vx','vxm-5%','vxm+5%','vy','vym-5%','vym+5%','vz','vzm-5%','vzm+5%','9');
title('Running mean. Interval +/- 5%');
saveas(gcf,[filebasew 'running_mean'], 'jpg')

% 4. Power Spectrum computation in frequency domain using periodogram function

% 4.1 Computation of variables with mean = 0(correspond to u'(t),v'(t),w'(t) values)

vx0 = vx-vmx;
vy0 = vy-vmy;
vz0 = vz-vmz;

% 4.2 Add zeros in the series to complete a length = multiple of 2

q=0;
N2 = 2;
nv = length(vx);
while N2 < nv;
    q=q+1;
    N2=2^q;
end;
indxe0 = (nv+1:N2);
mx0(indxe0)=0;
vy0(indxe0)=0;
vz0(indxe0)=0;
% 4.3. Power spectrum computation suing periodogram approach

[Pxxown,fx] = periodogram(vx0,[],'onesided',N2,fs); %Power spectra with white noise
[Pyyown,fy] = periodogram(vy0,[],'onesided',N2,fs); %Power spectra with white noise
[Pzzown,fz] = periodogram(vz0,[],'onesided',N2,fs); %Power spectra with white noise

% 4.4 Correction to the computed power spectrum for adding zeros to the signal.

Pxxwn=Pxxown*N2/nv;
Pyywn=Pyyown*N2/nv;
Pzzwn=Pzzown*N2/nv;

% 4.5 Plotting the power spectrum in the frequency domain

figure(2);clf
fig2=loglog(fx,Pxxwn,'r',fy,Pyywn,':b',fz,Pzzwn,'-.g');
grid on;
set(fig2,'LineWidth',2)
xlabel('fi [1/sec]');
ylabel('Gii(fi) [cm2/s]');
h = legend('i=x','i=y','i=z',3);
title('Power spectrum in frequency domain');
saveas(gcf,[filebasew 'spectra_freq' ],'jpg')

% 4.6 Verification of Integral spectrum = Variance for the x-component

% Not correction due to the noise is performed so far for the spectrum, thus, the contrast
% must be done
% against the time serie variances without noise correction

disp(['-------------------------------------------------------------']);
disp(['Difference between variance in real and frequency spaces (%) =']);
disp(['-------------------------------------------------------------']);
disp(['for x component = ' ...
     num2str(100*(trapz(fx,Pxxwn)-varvx)/varvx)]);

%Verification of Integral spectrum = Variance for the y-component

disp(['for y component = ' ...
     num2str(100*(trapz(fy,Pyywn)-varvy)/varvy)]);

%Verification of Integral spectrum = Variance for the z-component

disp(['for z component = ' ...
     num2str(100*(trapz(fz,Pzzwn)-varvz)/varvz)]);

%-------------------------------------------------------------------------------
% 5. Corrections in the spectrum for white noise presence using a defined noise energy level
%-------------------------------------------------------------------------------

Pxxo=Pxxwn-Noisex;
Pyyo=Pyywn-Noisey;
Pzzo=Pzzwn-Noisez;

figure(3);clf
fig3=loglog(fx,Pxxo,'r',fy,Pyyo,':b',fz,Pzzo,'-.g');
grid on;
set(fig3,'LineWidth',2)
xlabel('f [1/sec]');
ylabel('Gii(f) [cm2/s]');
h = legend('i=x','i=y','i=z',3);
title('Power spectrum in frequency domain. Energy due to white noise was substracted');
saveas(gcf, [filebase 'spectra_freq_no_noise'], 'jpg')

%-------------------------------------------------------------------------------% 6. Computation of convective velocity for each cartesian component using the complete equation %-------------------------------------------------------------------------------

Uconvx=sqrt(vmx^2+2*vmy^2+2*vmz^2+varvxcorr+2*varvycorr+2*varvzcorr);
Uconvy=sqrt(vmy^2+2*vmx^2+2*vmz^2+varvycorr+2*varvxcorr+2*varvzcorr);
Uconvz=sqrt(vmz^2+2*vmy^2+2*vmx^2+varvzcorr+2*varvycorr+2*varvxcorr);

%-------------------------------------------------------------------------------% 7. Computation of Power spectrum in spatial domain %-------------------------------------------------------------------------------

kx=2*pi*fx/Uconvx;
ky=2*pi*fy/Uconvy;
ksz=2*pi*fz/Uconvz;
Pxxok=Pxxo*Uconvx/(2*pi);
Pyyok=Pyyo*Uconvy/(2*pi);
Pzzok=Pzzo*Uconvz/(2*pi);

figure(4);clf
fig4=loglog(kx,Pxxok,'b',ky,Pyyok,'r',kz,Pzzok,'g',kx,Sp53,'m',kx,Spm1,'k');
grid on;
set(fig4,'LineWidth',2)
xlabel('k [1/cm]');
ylabel('E11(k) [cm3/s2]');
h = legend('vx','vy','vz','slope -5/3','slope -1',3);
title('Power spectrum in spatial domain');
saveas(gcf, [filebase 'Spectra_spatial'], 'jpg')

%Verification of Integral spectrum in the spatial domain = Variance for the x-component
disp('['------------------']);
disp('Difference between variance in real and spatial domain (%) =')
disp(['Noise energy level is substracted. Thus these differences include noise effect=']);
disp('For noise = zero, these values are equal to the reported for frequency domain');
disp('['------------------']);
disp('for x component = ...
      num2str(100*(trapz(kx,Pxxok)-varvxcorr)/varvxcorr))

%Verification of Integral spectrum = Variance for the y-component
disp('for y component = ...
      num2str(100*(trapz(ky,Pyyok)-varvycorr)/varvycorr))

%Verification of Integral spectrum = Variance for the z-component
disp('for z component = ...
      num2str(100*(trapz(kz,Pzzok)-varvzcorr)/varvzcorr))

%-------------------------------------------------------------------------------% 8. Computation of the autocorrelation function using products for each velocity component time serie %  with mena equal to zero vx0, vy0 and vz0 %-------------------------------------------------------------------------------

36
cxp = xcorr(vx0,'coeff');
cyp = xcorr(vy0,'coeff');
czp = xcorr(vz0,'coeff');

indx0 = (length(cxp)+1)/2;                 % Take only the last half of the correlation
lag= 0:indx0-1;

cxpo = cxp(indx0:end);
cypo = cyp(indx0:end);
czpo = czp(indx0:end);

figure(5);clf
    fig5=semilogx(lag,cxpo,'r',lag,cypo,':b',lag,czpo,'-.g');
    set(fig5,'LineWidth',2)
    grid on;
    xlabel('lag');
    ylabel('Autocorrelation Coefficient');
    h = legend('vx','vy','vz',3);
title('Autocorrelation functions using products');
    saveas(gcf,[filebasew '3correla_with_noise'],'jpg')

%-------------------------------------------------------------------------------
% 9) Autocorrelation function of vx0, vy0 and vz0 using inverse Fast Fourier Transform
% of the corrected spectrum due to noise.
%-------------------------------------------------------------------------------
%      9.a. Make a serie twice longer to avoid circular effect of the spectrum in the autocorrelation function

indexn=(1:N2);
vx00(indexn)=vx0(indexn);
vy00(indexn)=vy0(indexn);
vz00(indexn)=vz0(indexn);
indexe00=(N2+1:2*N2);
vx00(indexe00)=0;
vy00(indexe00)=0;
vz00(indexe00)=0;

% 9.b. Computation of two sided Power spectrum (Sxx) for the series of length 2N2
[Pxx0cwn,fxc] = periodogram(vx00t,[],'twosided',2*N2,fs); %Power spectra with white noise
[Pyy0cwn,fyc] = periodogram(vy00t,[],'twosided',2*N2,fs); %Power spectra with white noise
[Pzz0cwn,fzc] = periodogram(vz00t,[],'twosided',2*N2,fs); %Power spectra with white noise

% 9.c. Correction of these spectra due to the presence of white noise
% The noise energy level must be corrected due to two facts:
% Firstly, the energy in the spectrum is smaller due to the fact thta zeros were added to the signal
% Besides, the noise energy level is divided by two because it was defined using a one sided power spectrum

Pxx0c=Pxx0cwn-(Noisex/(2*(2*N2/nv)));
Pyy0c=Pyy0cwn-(Noisey/(2*(2*N2/nv)));
Pzz0c=Pzz0cwn-(Noisenz/(2*(2*N2/nv)));

37
% 9.d. Computation of Discrete value of the Power Spectra (S.xx)m multiplying by fs (or divided by dt)
\[ P_{xx0m} = P_{xx0c} \times f_s; \]
\[ P_{yy0m} = P_{yy0c} \times f_s; \]
\[ P_{zz0m} = P_{zz0c} \times f_s; \]

% 9.e. Computation of Inverse Fast Fourier Transform and correction due to the fact we have added zero to the signal
\[ c_{xoifft} = \text{ifft}(P_{xx0m}) \times (2 \times N^2 / (nv)); \]
\[ c_{yoifft} = \text{ifft}(P_{yy0m}) \times (2 \times N^2 / (nv)); \]
\[ c_{zoifft} = \text{ifft}(P_{zz0m}) \times (2 \times N^2 / (nv)); \]
\[ c_{xoifft} = \text{real}(c_{xoifft}); \]
\[ c_{yoifft} = \text{real}(c_{yoifft}); \]
\[ c_{zoifft} = \text{real}(c_{zoifft}); \]

\[ r = 1 : N^2 - 1; \]
\[ c_{xifft} = c_{xoifft}(r) \times (2 \times N^2 / (2 \times N^2 - r)); \]
\[ c_{yifft} = c_{yoifft}(r) \times (2 \times N^2 / (2 \times N^2 - r)); \]
\[ c_{zifft} = c_{zoifft}(r) \times (2 \times N^2 / (2 \times N^2 - r)); \]
\[ \text{indx} = (\text{length}(c_{xifft}) + 1)/2; \] Take only the half of the autocorrelation function.
\[ c_{x0c}(1 : \text{indx}) = c_{xifft}(1 : \text{indx}); \]
\[ c_{y0c}(1 : \text{indx}) = c_{yifft}(1 : \text{indx}); \]
\[ c_{z0c}(1 : \text{indx}) = c_{zifft}(1 : \text{indx}); \]

% 9.g. The following values must to be checked to be equal.
\[ \text{disp}('----------------------------------------------------------'); \]
\[ \text{disp}('Variance computed using different ways for the x component='); \]
\[ \text{disp}('These values must to be equal when noise=0'); \]
\[ \text{disp('----------------------------------------------------------');} \]
\[ \text{disp('Variance computed by formula='}); \]
\[ \text{varvxcorr} \]
\[ \text{disp('Variance computed by integrating the spectra ='}); \]
\[ \text{Vcfft} = (\text{trapz}(f_x, P_{xx0})) \]
\[ \text{disp('Variance computed by ifft (it means as Rxx(0)= '}); \]
\[ c_{x0c}(1) \]

% 9.h. Computation of the correlation coefficient (making it dimensionless with the variance)
\[ c_{x0cd} = c_{x0c} / c_{x0c}(1); \]
\[ c_{y0cd} = c_{y0c} / c_{y0c}(1); \]
\[ c_{z0cd} = c_{z0c} / c_{z0c}(1); \]
\[ c_{x0ctd} = c_{x0cd}'; \]
\[ c_{y0ctd} = c_{y0cd}'; \]
\[ c_{z0ctd} = c_{z0cd}'; \]

%Using the autocorrelation function by products of the same length that computed by ifft to plot them together
\[ \text{indxcorr} = \text{length}(c_{x0ctd}); \]
\[ \text{tlagf} = 0 : (\text{indxcorr} - 1); \]
\[ c_{xpofig}(1 : \text{indxcorr}) = c_{xpo}(1 : \text{indxcorr}); \]
\[ c_{ypofig}(1 : \text{indxcorr}) = c_{ypo}(1 : \text{indxcorr}); \]
\[ c_{zpofig}(1 : \text{indxcorr}) = c_{zpo}(1 : \text{indxcorr}); \]
```matlab
figure(6);clf
fig6=semilogx(tlagf,cxpofig,'r',tlagf,cx0ctd,:b');
set(fig6,'LineWidth',2)
grid on;
xlabel('lag');
ylabel('Autocorrelation Coefficient');
h = legend('by products','by ifft without noise',3);
title('Autocorrelation functions');
saveas(gcf,[filebase '1correla'],'jpg')

figure(7);clf
fig7=semilogx(tlagf,cx0ctd,'r',tlagf,cy0ctd,:b',tlagf,cz0ctd,:-.g');
set(fig7,'LineWidth',2)
grid on;
xlabel('lag');
ylabel('Autocorrelation Coefficient');
h = legend('vx','vy','vz',3);
title('Autocorrelation functions using ifft after subtracting noise');
saveas(gcf,[filebase '3correla_no_noise'],'jpg')

%-------------------------------------------------------------------------------
%     10) Length and time macroscale computations from autocorrelation function using ifft
%-------------------------------------------------------------------------------
% 10.a. Time scale computed as the integral up to first cross to zero of the autocorrelation function

tao=0:(indxx-1);

indnegcx = find(cx0ctd < 0);   % Find the correlations values < 0
indxlx  = 1:(min(indnegcx)-1); % Get indices of values before first zero crossing
Tx = trapz(tao(indxlx),cx0ctd(indxlx))/fs; % Compute integral time scale by integrating

indnegcy = find(cy0ctd < 0);   % Find the correlations values< 0
indxly  = 1:(min(indnegcy)-1); % Get indices of values before first zero crossing
Ty = trapz(tao(indxly),cy0ctd(indxly))/fs; % Compute integral time scale by integrating

indnegcz = find(cz0ctd < 0);   % Find the correlations values < 0
indxlz  = 1:(min(indnegcz)-1); % Get indices of values before first zero crossing
Tz = trapz(tao(indxlz),cz0ctd(indxlz))/fs;% Compute integral time scale by integrating
Lz = Tz*Uconvz;     % Use Taylor's hypothesis to get integral length

% 10.b. Length scale computed from time scale and Frozen Taylor approximation

Lx = Tx*Uconvx;     % Use Taylor's hypothesis to get integral length
Ly = Ty*Uconvy;     % Use Taylor's hypothesis to get integral length
Lz = Tz*Uconvz;     % Use Taylor's hypothesis to get integral length

% 10.c. Reynolds number of the big eddies for the each cartesian component
Reox=(varvxcorr^0.5/100)*(Lx/100)/nu;% Reynolds Number = Reo
Reoy=(varvycorr^0.5/100)*(Ly/100)/nu;% Reynolds Number = Reo
Reoz=(varvzcorr^0.5/100)*(Lz/100)/nu;% Reynolds Number = Reo

%-------------------------------------------------------------------------------
% 11) Length and time macroscale computations from power spectrum (page 71 Pope. equation 3.144)
```
% Due to the fact that fluctuations in the spectrum are observed, the first \text{nave} values of the
% spectrum are averaged to estimate
% $E_{11}(0)$. The \text{nave} values have to correspond to the flat plateau of the spectrum at smaller
% frequency values,
% APC #3 can be used to validate if this region can be observed in the spectrum

\begin{verbatim}
indxma=(2:2+\text{nave}-1);
Pxxave=Pxxo(indxma);
Pxxoma = cumsum(Pxxave)/\text{nave};

Pyyave=Pyyo(indxma);
Pyyoma = cumsum(Pyyave)/\text{nave};

Pzzave=Pzzo(indxma);
Pzzoma = cumsum(Pzzave)/\text{nave};

% 11.a. Time scale computed using $E_{11}(0)$ and analysis from Pope (2000)

Mtx=Pxxoma(end)/(4*varvxcorr);
Mty=Pyyoma(end)/(4*varvycorr);
Mtz=Pzzoma(end)/(4*varvzcorr);

% 11.b. Length scale computed using Taylor Frozen approximation

Msx=Mtx*Uconvx;
Msy=Mty*Uconvy;
Msz=Mtz*Uconvz;

% 12) Dissipation rate of turbulent kinetic energy by fitting -5/3 law to the spatial power spectrum

% 12.1 Value of $k$ where the noise appears in the spectrum as a flat plateau (at high
% frequencies)

kwnx=2*pi*fwnx/Uconvx;
kwny=2*pi*fwny/Uconvy;
kwnz=2*pi*fwnz/Uconvz;

% 12.2 The dissipation rate computation is performed just for the flow direction defined by the
% user as fit\_dir
\end{verbatim}
k = ky;
P_p = P_{yyok};
U_{convv} = U_{convy};
else
if (fit_dir == 3);
  varv = varvzcorr;
  L = L_z;
  kw = kw.nz;
  k = k_z;
P_p = P_{zzok};
U_{convv} = U_{convcz};
end
end
end

\% 12.3 Definition of limits of inertial range

\%The limits in the inertial range are k_{inf} = \frac{2\pi\alpha}{L} < k < k_{sup} = \frac{2\pi}{\text{kolmogorov length scale}}

k_{inf} = 2\pi\alpha/L; \% lower limit

\% upper limit (k_{sup}) is obtained by iterations (depends on Kolmogorov length scale).
\% The feed value of k_{sup} is detailed in the parameters list at the beginning of this program

\% Besides, only the k values smaller than kw (where the noise appears) are used in the fitting

indxf = find(k >= kw); \% Find the frequencies \geq white noise frequency
ns = min(indxf);

\% 12.4 Fitting of -5-3 law on the inertial range of the power spectrum to estimate the dissipation rate

\% The limits in the inertial range are \frac{\alpha}{L} < k < \frac{2\pi}{\text{kolmogorov length scale}}
\% Feed (in the iteration process) upper limit (k_{sup}) in the inertial range is adopted as a very high value (100)

E_{fit} = 0;
Difksup = 1000;

while Difksup > 1
j = 0;
\% Selection of values to perform the fitting
kf = 0;
Ekf = 0;
for i = 1:ns,
  if (k(i) >= k_{inf});
    if (k(i) <= k_{sup});
      j = j + 1;
      kf(j) = k(i);
      Ekf(j) = P_p(i);
    end;
end;
nf = length(Ekf);
a=0;
%fitting of a
for i=1:nf,
y(i)=log(Ekf(i));
x(i)=log(kf(i));
end;
%fitting of b
Sxy=0;
Sy=0;
Sx=0;
Sx2=0;
for i=1:nf,
yx(i)=y(i)*x(i);
Sxy=Sxy+yx(i);
Sy= Sy + y(i);
Sx= Sx + x(i);
x2(i)=x(i)^2;
Sx2= Sx2 + x2(i);
end;
Sxsy=0;
Sxsx=0;
for i=1:nf,
xSy(i)=x(i)*Sy;
Sxsy=Sxsy+xSy(i);
xSx(i)=x(i)*Sx;
Sxsx=Sxsx+xSx(i);
end;
b1=(Sxy-(1/nf*Sxsy))/(Sx2-(1/nf*Sxsx)); %the actual slope is computed to evaluate how good is the assumption of the -5/3 slope in the inertia range
bover3=b1*3;

%Dissipation value is also obtained using the function nlinfit for finding parameter estimates in nonlinear modeling.
%nlinfit returns the least squares parameter estimates. That is, it finds the parameters that minimize the sum of the squared differences between the observed responses and their fitted values. It uses the Gauss-Newton algorithm with Levenberg-Marquardt modifications for global convergence.

ein = 0;%initial estimate of the dissipation
ehat = nlinfit(kf,Ekf,@myfun,ein);%final value of dissipation. Remember to copy file myfun.m in work folder of Matlab folder

%-------------------------------
% 13)Computation of Kolmogorov length scale = kolmo1 from dissipattion rate
%-------------------------------
kolmo1=((nu*10000)^3/ehat)^((1/4));
ksup1 =2*pi/(kolmo1);
Difksup=abs((ksup1-ksup)/ksup*100);
ksup=ksup1;
kolmo=kolmo1;

% -5/3 law spectrum to plot against the data
for r=1:nf,
    Efit(r)=C1*(kf(r))^(-5/3)*(ehat)^(2/3);
end;

figure(8);clf
    fig8=loglog(kf,Efit,'r',kf,Ekf,:b');
    set(fig8,'LineWidth',2)
    grid on;
    xlabel('k [1/cm]');
    ylabel('E_{11}(k) [cm^3/s^2]');
    title('Fitting to -5/3 law of Power spectrum of main velocity component in inertial subrange');
    saveas(gcf,[filebase '5_3_law_fit'],jpf)

%-------------------------------------------------------------------------------
%       14) Dissipation rate of turbulent kinetic energy by dimensional analysis
%-------------------------------------------------------------------------------
Lres=(Lx^2+Ly^2+Lz^2)^(1/2);
    dis=Aconst*(TKE)^(3/2)/Lres;

%-------------------------------------------------------------------------------
%      15) Computation of the Taylor microscale using autocorrelation function
%-------------------------------------------------------------------------------
%       15.a. Fitting of the second order polinomia to the first NF2P values of the autocorrelation
% function

%For the x component
    indxtm=(1:NF2P);
    tpolyx=(indxtm-1)/fs;
    Rxx(1:indxtm)=cx0ctd(1:indxtm);
    px = polyfit(tpolyx,Rxx,2);
    Tmsx=(-px(1))^(0.5);
    for ppx=1:NF2P,
        ppxx(ppx)=px(1)*tpolyx(ppx)^2+px(2)*tpolyx(ppx)+px(3);
    end;
%For the y component
    indytm=(1:NF2P);
    tpolyy=(indytm-1)/fs;
    Ryy(1:indytm)=cy0ctd(1:indytm);
    py = polyfit(tpolyy,Ryy,2);
    Tmsy=(-py(1))^(0.5);
    for ppy=1:NF2P,
        ppyy(ppy)=py(1)*tpolyy(ppy)^2+py(2)*tpolyy(ppy)+py(3);
    end;
%For the z component
    indztm=(1:NF2P);
    tpolyz=(indztm-1)/fs;
    Rzz(1:indytm)=cz0ctd(1:indytm);
    pz = polyfit(tpolyz,Rzz,2);
    Tmsz=(-pz(1))^(0.5);
    for ppz=1:NF2P,
        ppzz(ppz)=pz(1)*tpolyz(ppz)^2+pz(2)*tpolyz(ppz)+pz(3);
    end;

%       15.b. Computation of the Taylor length microscale using Taylor Frozen approximation
    Lmsx=Tmsx*Uconvx;
Lmsy = Tmsy * Uconvy;
Lmsz = Tmsz * Uconvz;

Relambdax = ((varvxcorr)^0.5/100)*(Lmsx/100)/nu;
Relambday = ((varvycorr)^0.5/100)*(Lmsy/100)/nu;
Relambdaz = ((varvzcorr)^0.5/100)*(Lmsz/100)/nu;

% 16) Computation of the Taylor microscale using dissipation rate of turbulent kinetic energy value
% It is valid only for the main component
Tmsd = (15*nu*10000/ehat*varv)^0.5/(2^0.5);
Lmsd = Tmsd * Uconvv;

figure(9); clf
fig9 = plot(tpolyx, Rxx, 'vr',tpolyx, ppxx,'r',tpolyy, Ryy,'ob',tpolyy, ppyy,'b',tpolyz, Rzz,'dg',tpolyz, ppzz,'g');
set(fig9, 'LineWidth', 2)
grid on;
xlabel('t [sec]');
ylabel('Gii(f) [cm^2/s]');

h = legend('vx','vx fit','vy','vy fit','vz','vz fit',3);
title('Fitting the autocorrelation function to second order polynomia');
saveas(gcf,[filebase 'fit_2_poly'], 'jpg');

% IV WRITING FILES

c = [cx0cd; cy0cd; cz0cd];
time = t';
Specfx = [fx'; Pxxo'];
Specfy = [fy'; Pyyo'];
Specfz = [fz'; Pzzo'];

Specfxwn = [fx'; Pxxwn'];
Specfyn = [fy'; Pyywn'];
Specfzw = [fz'; Pzzwn'];

filesave = fopen([filebase 'stat.sal'], 'w');
fprintf(filesave, 'Parameters Vx[cm/s] Vy[cm/s] Vz[cm/s]\n');
fprintf(filesave, 'Mean[cm/s] %8.4f %8.4f %8.4f\n', vmx, vmy, vmz);
fprintf(filesave, 'Stdev[cm/s] %8.4f %8.4f %8.4f\n', sdvx, sdvy, sdvz);
fprintf(filesave, 'Variance[cm^2/s^2] %8.4f %8.4f %8.4f\n', varvx, varvy, varvz);
fprintf(filesave, 'Corrected_Variance[cm^2/s^2] %8.4f %8.4f %8.4f\n', varvxcorr, varvycorr, varvzcorr);
fprintf(filesave, 'Turbulence_intensities %8.4f %8.4f %8.4f\n', intvx, intvy, intvz);
fprintf(filesave, 'Convective_velocity_[cm/s] %8.4f %8.4f %8.4f\n', Uconvx, Uconvy, Uconvz);
fprintf(filesave, 'TKE[cm^2/s^2] %8.4f\n', TKE);
fprintf(filesave, 'Skewmess %8.4f %8.4f %8.4f\n', skvx, skvy, skvz);
fprintf(filesave, 'Kurtosis %8.4f %8.4f %8.4f\n', kvx, kvy, kvz);
fprintf(filesave, 'Covariance[cm^2/s^2] uv uw vw\n');
fprintf(filesave, 'uiuj %8.4f %8.4f %8.4f\n', u12, u12, u12);
fprintf(filesave, 'Measurement_time Vx[cm/s] Vy[cm/s] Vz[cm/s]\n');
fprintf(filesave, 'Convergence_at_Percent_of_the_mean_value %8.4f\n', pctg*100);
fprintf(filesave,'time[sec]_at_5perc %8.2f %8.2f %8.2f\n',tconvx,tconvy,tconvz);
fclose(filesave);

filesave = fopen([filebasew 'correla.sal'],'w');
fprintf(filesave,'Rvxvx Rvyvy Rvzvz\n');
fprintf(filesave,'%8.6f %8.6f %8.6f\n',c);
fclose(filesave);

filesave = fopen([filebasew 'powerx.sal'],'w');
fprintf(filesave,'fx E11(fx)\n');
fprintf(filesave,'%8.6f %12.8f\n',Specfx);
fclose(filesave);

filesave = fopen([filebasew 'powerx_noise_removed.sal'],'w');
fprintf(filesave,'fx E11(fx)\n');
fprintf(filesave,'%8.6f %12.8f\n',Specfxwn);
fclose(filesave);

filesave = fopen([filebasew 'powery.sal'],'w');
fprintf(filesave,'fy E11(fy)\n');
fprintf(filesave,'%8.6f %12.8f\n',Specfy);
fclose(filesave);

filesave = fopen([filebasew 'powery_noise_removed.sal'],'w');
fprintf(filesave,'fx E11(fx)\n');
fprintf(filesave,'%8.6f %12.8f\n',Specfywn);
fclose(filesave);

filesave = fopen([filebasew 'powerz.sal'],'w');
fprintf(filesave,'fz E11(fz)\n');
fprintf(filesave,'%8.6f %12.8f\n',Specfz);
fclose(filesave);

filesave = fopen([filebasew 'powerz_noise_removed.sal'],'w');
fprintf(filesave,'fz E11(fz)\n');
fprintf(filesave,'%8.6f %12.8f\n',Specfzwn);
fclose(filesave);

filesave = fopen([filebasew 'spectrum.sal'],'w');
fprintf(filesave,'--------------------------------------------\n');
fprintf(filesave,'LENGTH AND TIME SCALES\n');
fprintf(filesave,'--------------------------------------------\n');
fprintf(filesave,'Integral scales\n');
fprintf(filesave,'--------------------------\n');
fprintf(filesave,'From autocorrelation function\n');
fprintf(filesave,'Integral scales in x component:\n');
fprintf(filesave,'Time scale [sec]: %8.6f, Tx;\n');
fprintf(filesave,'Length scale [cm]: %8.6f, Lx;\n');
fprintf(filesave,'Integral scales in y component:\n');
fprintf(filesave,'Time scale [sec]: %8.6f, Ty;\n');
fprintf(filesave,'Length scale [cm]: %8.6f, Ly;\n');
fprintf(filesave,'Integral scales in z component:\n');
fprintf(filesave,'Time scale [sec]: %8.6f, Tz;\n');
fprintf(filesave,'Length scale [cm]: %8.6f, Lz;\n');
fprintf(filesave,'Resultant length macroscale [cm]: %8.6f, Lres;\n');
fprintf(filesave,'From spectra\n');
fprintf(filesave,'Temporal macroscale in x component [sec]:      =  %8.6f
',Mtx);
fprintf(filesave,'Temporal macroscale in y component [sec]:       =  %8.6f
',Mty);
fprintf(filesave,'Temporal macroscale in z component [sec]:      =  %8.6f
',Mtz);
fprintf(filesave,'Spatial macroscale in x component [cm]:        =  %8.6f
',Msx);
fprintf(filesave,'Spatial macroscale in y component [cm]:        =  %8.6f
',Msy);
fprintf(filesave,'Spatial macroscale in z component [cm]:        =  %8.6f
',Msz);
fprintf(filesave,'----------------------------------
');
fprintf(filesave,'Taylor Microscale in x component:
');
fprintf(filesave,'----------------------------------
');
fprintf(filesave,'Time scale [sec]:                              =  %8.6f
',Tmsx);
fprintf(filesave,'Length scale [cm]:                             =  %8.6f
',Lmsx);
fprintf(filesave,'Taylor Microscale in y component:
');
fprintf(filesave,'Time scale [sec]:                              =  %8.6f
',Tmsy);
fprintf(filesave,'Length scale [cm]:                             =  %8.6f
',Lmsy);
fprintf(filesave,'Taylor Microscale in z component:
');
fprintf(filesave,'Time scale [sec]:                            =  %8.6f
',Tmsz);
fprintf(filesave,'Length scale [cm]:                            =  %8.6f
',Lmsz);
fprintf(filesave,'Taylor Microscale in the main component
');
fprintf(filesave,'using disipation value
');
fprintf(filesave,'Time scale [sec]:                              =  %8.6f
',Tmsd);
fprintf(filesave,'Length scale [cm]:                             =  %8.6f
',Lmsd);
fprintf(filesave,'--------------------------------------------
');
fprintf(filesave,'FITTING OF KOLMOGOROV SPECTRA
');
fprintf(filesave,'--------------------------------------------
');
fprintf(filesave,'flow direction where fitting is performed (1=x, 2=y and 3=z):       =  %8.6f
',fit_dir);
fprintf(filesave,'Limits [1/cm] of the inertial range are:
');
fprintf(filesave,'Lower limit of k                              =  %8.6f
',kinf);
fprintf(filesave,'Upper limit of k                              =  %8.6f
',ksup);
fprintf(filesave,'Diss. rate using -5/3 approach nlf   [cm2/s3] = %10.8f
',ehat);
fprintf(filesave,'Kolmogorov length scale       [cm]            = %10.8f
',kolmo);
fprintf(filesave,'Slope fitted in the inertial range            = %4.2f
',bover3/3);
fprintf(filesave,'alpha value to compute kinf = alpha/L         = %4.2f
',alpha);
fprintf(filesave,'Dissipation rate using dim.analisys   [cm2/s3]= %10.8f
',dis);
fprintf(filesave,'Re of the big eddies for the x component      =  %10.2f
',Reox);
fprintf(filesave,'Re of the big eddies for the y component      =  %10.2f
',Reoy);
fprintf(filesave,'Re of the big eddies for the z component      =  %10.2f
',Reoz);
fprintf(filesave,'Reynolds number using Taylor scales for x     =  %10.2f
',Relambdax);
fprintf(filesave,'Reynolds number using Taylor scales for y     =  %10.2f
',Relambday);
fprintf(filesave,'Reynolds number using Taylor scales for z     =  %10.2f
',Relambdaz);
fclose(filesave);

filesave = fopen([filebase 'noise.sal'], 'w');
fprintf(filesave,'ENERGY NOISE INFORMATION
');
fprintf(filesave, 'Parameters Vx[cm/s] Vy[cm/s] Vz[cm/s]
');
fprintf(filesave, 'Energy_noise_level_[cm2/s] %8.4f %8.4f %8.4f
', Noisex, Noisey, Noisez);
fprintf(filesave, 'Frequency_where_noise_level_is_detected_[Hz] %8.4f %8.4f %8.4f
', fwnx, fwny, fwnz);
fclose(filesave);

'end of running'

46

Turbulence measurements with Acoustic Doppler Velocimeters (ADVs)

By Carlos M. García, Mariano I. Cantero, Yarko Niño and Marcelo H. García
Turbulence Measurements with Acoustic Doppler Velocimeters

Carlos M. García¹; Mariano I. Cantero²; Yarko Niño³; and Marcelo H. García⁴

Abstract: The capability of acoustic Doppler velocimeters to resolve flow turbulence is analyzed. Acoustic Doppler velocimeter performance curves (APCs) are introduced to define optimal flow and sampling conditions for measuring turbulence. To generate the APCs, a conceptual model is developed which simulates different flow conditions as well as the instrument operation. Different scenarios are simulated using the conceptual model to generate synthetic time series of water velocity and the corresponding sampled signals. Main turbulence statistics of the synthetically generated, sampled, and nonsampled time series are plotted in dimensionless form (APCs). The relative importance of the Doppler noise on the total measured energy is also evaluated for different noise energy levels and flow conditions. The proposed methodology can be used for the design of experimental measurements, as well as for the interpretation of both field and laboratory observations using acoustic Doppler velocimeters.

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CE Database subject headings: Flow measurement; Doppler systems; Turbulent flow; Filters; Noise.

Introduction

Present day laboratory and field research in fluid dynamics often requires water velocity measurements with a high temporal and spatial resolution. Laser Doppler velocimetry (LDV) and particle image velocimetry (PIV) have become the most common measuring techniques used in laboratory studies that satisfy such requirements. However, the feasibility of using these techniques is reduced when the scale of the experiment increases. The use of PIV/LDV may also be unsuitable in flows with suspended sediment concentrations. On the other hand, the use of hot wire anemometers, which provides a very good temporal resolution, is rather limited when impurities are present in the water, thus precluding applications with sediment transport or where it becomes difficult to control water quality that render the flow opaque. In most of these cases, acoustic Doppler velocimetry is the technique of choice, because it is relatively low in cost, can record at a relatively high frequency (up to 100 Hz), and has a relatively small sampling volume (varies from 0.09 to 2 cm³ according to the instrument selected). Additionally, the measurements are performed in a remote control volume (located between 5 to 18 cm from the sensor according to the instrument selected), which reduces the interference with the flow being measured. ADV and NDV are trademark names for acoustic Doppler velocimeters manufactured by Sontek and Nortek, respectively.

Acoustic Doppler velocimeters are capable of reporting accurate mean values of water velocity in three directions (Kraus et al. 1994; Lohrmann et al. 1994; Anderson and Lohrmann 1995; Lane et al. 1998; Voulgaris and Trowbridge 1998; Lopez and García 2001), even in low flow velocities (Lohrmann et al. 1994). However, the ability of this instrument to accurately resolve flow turbulence is still uncertain (Barkdoll 2002).

Lohrmann et al. (1994) argued that the acoustic Doppler velocimeters resolution is sufficient to capture a significant fraction of the turbulent kinetic energy (TKE) of the flow. However, they identified the Doppler noise as a problem that causes the TKE to be biased toward a high value. Anderson and Lohrmann (1995) detected a flattening of the power spectrum of the velocity signal due to this Doppler noise, this suggests that an operational noise is eventually reached where the higher-frequency components of the signal cannot be resolved adequately.

Most research related to the capability of acoustic Doppler velocimeters to resolve the flow turbulence (specifically TKE and spectra) has focused on definition of the noise level present in the signal and how it can be removed (Lohrmann et al. 1994; Anderson and Lohrmann 1995; Lane et al. 1998; Voulgaris and Trowbridge 1998; Lemmin and Lhermitte 1999; McLelland and Nicholas 2000). However, little attention has been dedicated to evaluating the filtering effects of the sampling strategy (spatial and temporal averaging) on the turbulent parameters (moments, spectra, autocorrelation functions, etc.). Only the paper by Voulgaris and Trowbridge (1998) discusses some issues related to the effect of spatial averaging on turbulence measurements.
The capability of acoustic Doppler velocimeters to resolve flow turbulence is analyzed herein by means of a new tool, termed the acoustic Doppler velocimeter performance curves (APCs). These curves can be used to define optimal flow and sampling conditions for turbulence measurements using this kind of velocimeters. The performance of these tools is validated herein using experimental results. The APCs are used to define a new criterion for good resolution measurements of the flow turbulence. In cases where this criterion cannot be satisfied, these curves can be used to make appropriate corrections.

Another set of curves are also introduced to evaluate the relative importance of the Doppler noise energy on the total measured energy. In cases where the noise is significant, the noise energy level needs to be defined and corrections to the turbulence parameters (e.g., TKE, length and time scales, and convective velocity) must be performed.

### Principle of Operation of Acoustic Doppler Velocimeters

An acoustic Doppler velocimeter measures three-dimensional flow velocities using the Doppler shift principle, and the instrument consists of a sound emitter, three sound receivers, and a signal conditioning electronic module. The sound emitter generates an acoustic signal that is reflected back by sound-scattering particles present in the water, which are assumed to move at the water’s velocity. The scattered sound signal is detected by the receivers and used to compute the Doppler phase shift, from which the flow velocity in the radial or beam directions is calculated. A detailed description of the velocimeter operation can be found in McLelland and Nicholas (2000). In the present paper, a brief description of the instrument characteristics is included to facilitate presentation of a suitable conceptual model for the objectives detailed above.

The acoustic Doppler velocimeter uses a dual pulse-pair scheme with different pulse repetition rates, \( \tau_1 \) and \( \tau_2 \), separated by a dwell time \( \tau_D \) (McLelland and Nicholas 2000). The longer pair of pulses is used for higher precision velocity estimates, while the shorter pulse is used for ambiguity resolution, assuming that the real velocity goes beyond the limit resolvable by the longer time lag. These pulse repetition rates can be adjusted by changing the velocity range of the measurement. Each pulse is a square-shaped pulse train of an acoustic signal (the frequency can be 5, 6, 10, or 16 MHz, depending on the instrument selected). The phase shift is calculated from the auto- and cross correlation computed for each single pulse-pair using pulse-to-pulse coherent Doppler techniques (Lhermite and Serafin 1984). The radial velocities \( v_i \) \((i=1,2,3)\) are computed using the Doppler relation

\[
v_i = \frac{C}{4\pi f_{ADV}} \left. \frac{dB}{dt} \right|_{i}
\]

where \( C \) = speed of the sound in water; \( f_{ADV} \) = sound signal frequency (10 MHz); and \( dB/dt \) = phase shift rate computed for receiver \( i \). The radial or beam velocities are then computed sequentially for each receiver and, thus the time it takes to complete a three-dimensional velocity measurement is given by

\[
T = 3(\tau_1 + \tau_D + \tau_2 + \tau_D)
\]

This process is conducted with a frequency \( f_S \) (equal to \( 1/T \), which is between 100 and 263 Hz depending on the velocity range and the user-set frequency \( f_R \) (see Table 1). Then, the radial or beam velocities are converted to a local Cartesian coordinate system \( (u_x, u_y, u_z) \) using a transformation matrix that is determined empirically (through calibration) by the manufacturer (e.g., McLelland and Nicholas 2000).

During the time it takes to make a three-dimensional velocity measurement, the flow may vary, however, these high-frequency variations are smoothed out in the process of signal acquisition and, therefore, cannot be captured by the instrument. Each radial velocity, \( u_x \), is the result of the acoustic echo reflected by the sound-scattering particles in the water during an overall time \( T/3 \), and this is hence an average value of the real flow velocity in this time interval. Also, each of the final Cartesian velocity components is an average of the three radial velocity components (the product of the transformation matrix and the radial velocity). The direct implication of these features is that the Cartesian flow velocity represents an averaged value, over an interval time \( T \), of the real flow velocity. In this sense \( T \) can be considered as the instrument response time, and the process of acquisition itself can be seen as an analog filter with a cut-off frequency, \( 1/T \) (or \( f_S \)).

The time averaging process is analogous to the spatial averaging of the recorded velocity vectors that occurs within the measurement volume.

Two main conclusions can be drawn from the considerations above. First, energy in the signal with a frequency higher than \( f_S \) is filtered out (i.e., acquisition process acts as a low-pass filter). Second, aliasing of the signal occurs since the velocity signal is sampled at a frequency \( f_S \), and the highest frequency that can be resolved by the instrument is \( f_S/2 \) [Nyquist theorem, see Bendat and Piersch (2000)]. This indicates that energy in the frequency range of \( f_S/2 < f < f_S \) is folded back into the range \( 0 < f < f_S/2 \), which may or may not be of importance depending on the flow characteristics. Flows with a large convective velocity, \( U_c \), will have a considerable portion of the energy in the range of wavelengths: \( f_S/(2U_c) < f/U_c < f_S/U_c \), while flows with a low convective velocity will have no energy in this range and, therefore, aliasing will not be of relevance.

After the digital velocity signal is obtained (with frequency \( f_S \)), the instrument performs an average of \( N \) values to produce a digital signal with frequency \( f_R = f_S/N \), which is the acoustic Doppler velocimeter’s user-set frequency with which velocity data are recorded. This averaging process is a digital nonrecursive filter (Hamming 1983; Bendat and Piersch 2000), the consequences of which are analyzed next.

### Implications of Digital Averaging of Velocity Signals

Let \( x \) be the signal sampled at \( f_S \) and let \( y \) be the signal obtained after digital averaging (with frequency \( f_R \)). The interval between samples of signal \( x \) is \( \Delta t = 1/f_S \) and between samples of signal \( y \) is \( \Delta t = 1/f_R \) (see Fig. 1). Notice that \( f_S = N f_R \).

<table>
<thead>
<tr>
<th>Velocity range (cm/s)</th>
<th>( f_R ) [Hz]</th>
<th>1</th>
<th>25</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_{cut-off} ) [Hz]</td>
<td>0.44</td>
<td>11.3</td>
<td>50</td>
</tr>
<tr>
<td>250</td>
<td>263</td>
<td>250</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>256</td>
<td>225</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>226</td>
<td>200</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>180</td>
<td>175</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>143</td>
<td>125</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
The nonrecursive digital filter is given by

\[ y_i = \sum_{n=0}^{N-1} \frac{1}{N} x_{Ni+n} \]  

(3)

or in the time domain

\[ y(t) = \sum_{n=0}^{N-1} \left( t + \frac{n}{N f_R} \right) \]

(4)

where \( t = i \Delta t_s = N i \Delta t_c \). The transfer function of this filter, \( H(f) \), can be calculated by computing the Fourier transform of Eq. (4), i.e.

\[ H(f) = \frac{Y(f)}{X(f)} = \frac{Y(f)}{N} \sum_{n=0}^{N-1} \exp\left( j 2\pi \frac{nf}{N f_R} \right) \]

(5)

where \( Y(f) \) and \( X(f) \) are Fourier transforms; and \( j = (-1)^{1/2} \). The sum in Eq. (5) can be computed to give

\[ H(f) = \frac{f_R}{f_S} \frac{\exp\left( j 2\pi \frac{f}{f_R} \right) - 1}{\exp\left( j 2\pi \frac{f}{f_S} \right) - 1} \]

(6)

noting that \( N f_R = f_S \). The gain factor of the filter is

\[ |H(f)| = \frac{f_R}{f_S} \sqrt{\frac{1 - \cos\left( 2\pi f \frac{f_R}{f_S} \right)}{1 - \cos\left( 2\pi f \frac{f}{f_S} \right)}} \]

(7)

Fig. 2 shows the gain factor \(|H(f)|\) for \( f_R = 25\) Hz and \( f_S = 250\) Hz. The cut-off frequency \( f_{\text{cut-off}} \) of the filter is defined as the frequency for which the gain factor equals \( \sqrt{2}/2 \). The cut-off frequency must be equal to or smaller than the Nyquist frequency \( f_S/2 \) in order to avoid significant aliasing. Cut-off frequencies of the digital filter applied by acoustic Doppler velocimeters for different velocity ranges are presented in Table 1.

As observed in Fig. 2, the gain factor possesses some lobes after the cut-off frequency that could generate some energy aliasing. This could be improved in the acoustic Doppler velocimeter design by performing a different digital filtering of the signal in the decimation process so that the data are reduced from \( f_S \) to \( f_R \).

**Conceptual Model**

A conceptual model is developed here to evaluate the performance of the acoustic Doppler velocimeter based on turbulence characteristics of the flows to be measured. The model consists of two components, for the instrument and the flow, which simulate the instrument operation (based on the previous description of how the velocimeter works) and the power spectrum of flow velocities associated with different flow conditions. A description of each component is presented next.

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1064 / JOURNAL OF HYDRAULIC ENGINEERING © ASCE / DECEMBER 2005
**Instrument Component**

For the purpose of this work, the acoustic Doppler velocimeter can be conceptually modeled as a two-module linear system (Fig. 3). The first module is the data acquisition module (DAM), which encompasses the sound emitter and receiver, the analog to digital converter (which works at frequency $f_s$), and the computation of flow velocities from the acquired signal. The second module is the data preprocessing module (DPM), which encompasses the averaging of the digital velocity signal that produces data at the user-set frequency $f_R$.

The DAM produces a digital signal with frequency $f_s$ (signal $x$ in the analysis of previous section) from the input flow velocity. This module is modeled through the sampling of synthetic water velocity series produced in the flow component of the conceptual model. The DPM basically performs a time averaging of the output of the DAM in order to produce data at the user-set frequency $f_R$. This module is modeled by Eq. (3). The output is the digital signal of water velocity.

Although the low-pass filtering of the signal (in the DPM) is mandatory to avoid aliasing, it has implications in the computation of the spectrum and moments from the signal. Its effect in the spectrum is clear since the low-pass filter removes energy beyond the cut-off frequency. In the case of a perfect low-pass filter, the energy content of the spectrum vanishes at frequencies larger than the cut-off frequency of the DPM (Roy et al. 1997). Likewise, the low-pass filter reduces the values of the even moments of the signal. The quantification of this effect is discussed later in this paper.

**Flow Component**

Synthetic signals of water velocity need to be generated to represent different ranges of flow conditions, and must have turbulence characteristics that resemble realistic conditions. In order to accomplish this, a one-dimensional (1D) model power spectrum $E_{11}$ was adopted that includes all of the turbulence characteristics for specified flow conditions. The model power spectrum used in this paper [Eq. (8)] is based on that proposed by Pope (2000). The input parameters of the model are the energy-containing eddy length scale, $L$, and Kolmogorov length scale $\eta$ (which can be estimated from the value of the rate of dissipation of TKE, $\varepsilon$)

$$E_{11}(k_1) = C_6 \varepsilon^{2/3} k_1^{-5/3} f_L(k_1L) f_q(k_1 \eta)$$

where $C_6=$constant; $k_1=$wave number; and $f_L$ and $f_q$=shape functions defined as

$$f_L(k_1L) = \left( \frac{k_1L}{(k_1L)^2 + c_L^2} \right)^{5/3}$$

$$f_q(k_1 \eta) = \exp\left\{-\beta \left\{\left(\frac{c_q}{c_q} + k_1^3 \eta\right)^{1/4} - c_q \right\}\right\}$$

Here, $p_0$, $c_L$, $c_q$, and $\beta=$parameters of the shape functions. The function $f_L$ defines the shape of the energy-containing part of the spectrum (equal to 1 for large $k_1L$). On the other hand $f_q$ describes the shape of the dissipation range (equal to 1 for small $k_1 \eta$). Following Pope (2000), the parameters finally adopted for this model were: $C_6=0.49$, $p_0=0$, $c_L=6.78$, $c_q=0.40$, and $\beta=5.2$.

A technique is needed to generate synthetic water velocity signals from the modeled 1D power spectrum with predefined turbulence flow conditions. For this, the method of Shinozuka and Jan (1972) is used, which allows a random 1D water velocity signal to be generated as a realization of a turbulent process, using the model power spectrum as a target. Each point in the time series is computed by summing the weighted cosine series with a random phase angle, $\phi$, as

$$x(t) = \sqrt{2} \sum_{q=1}^{N_s} A_q \cos(\omega_q t + \phi_q)$$

The generated synthetic signal is only one of the possible realizations of a process with the chosen flow turbulence characteristics because of its random phase angle. The weights $A_q$ are defined from the $N_s$ numbers of terms of the target spectrum (computed before)

$$A_q = [E_{11}(\omega_q) \Delta \omega]^{1/2}$$

Here $\omega_q$, $\omega_q'$, and $\Delta \omega$ are obtained from Taylor’s frozen turbulence approximation using the convective velocity $U_c$ as

$$\omega_q = k_1 \eta \frac{U_c}{\Delta \omega} \Delta \omega = \Delta k \cdot U_c$$

$$\omega_q' = \omega_q + \delta \omega$$

where $\delta \omega=$random frequency with a uniform probability density distribution in the range $-\alpha \Delta \omega/2 \leq \delta \omega \leq \alpha \Delta \omega/2$. The parameter $\alpha$ is known as the amount of jitter ($\Delta \omega \approx 1$) and Shinozuka and Jan (1972) suggested a value of $\alpha=0.05$. Finally, the random phase angle, $\phi$, has a uniform probability density distribution in the range from 0 to $2\pi$.

Jeffries et al. (1991) suggested several recommendations regarding the method of Shinozuka and Jan (1972), consisting of requirements for $N_s$ (number of terms of the target spectrum) and $N_t$ (number of points in the time series) to avoid undesired periodicity in the synthetic signals. These suggestions were adopted herein, and the parameters used for the simulations are: $N_s=32,768$, total simulated time $T=120$ s, $f_s=260.8$ Hz, and $N_t=31,291$. Different values of $\Delta k$ were used in order to yield a frequency $f_s=260.8$ Hz in all the runs, which is very close to the frequency at which the velocimeter samples the flow in the velocity range $\pm 250$ cm/s. Both the model power spectrum and the method used to generate synthetic velocity series were tested and validated using experimental data.

**Simulation of Different Flow Conditions Using the Conceptual Model**

A set of numerical simulations based on the conceptual model was conducted for different values of the parameters in the range that best represents the conditions usually present in laboratory and field turbulence measurements. The ranges of flow variables used in the simulations are: Convective velocity $U_c=0.01 \ m/s \leq U_c \leq 1 \ m/s$; energy containing eddy length scale: $0.10 \ m \leq L \leq 2 \ m$; Kolmogorov length scale: $0.0001 \ m \leq \eta \leq 0.005 \ m$. The rate of dissipation of TKE, $\varepsilon$, is related to the Kolmogorov length scale using dimensional analysis (Pope 2000)

$$\varepsilon = \frac{\nu^3}{\eta^4}$$

The range of Kolmogorov length scales proposed here generates for a water temperature equal to $20^\circ C (\nu=10^{-6} \ m^2/s)$, a range of seven orders of magnitude in $\varepsilon$ ($1.6 \times 10^{-9} \ m^2/s^3 \leq \varepsilon \leq 1 \times 10^{-2} \ m^2/s^3$), thus describing conditions prevailing in most environmental water flows, from open-channel flows to lakes (Merric 1984).
Fig. 4. Percentage of the energy remaining in the sampled signal for: Curve A: \( f < f_s \); Curve B: \( f < f_s/2 \); and Curve C=energy corresponding to frequencies \( f_s/2 < f < f_s \).

Fig. 5. Effects of digital averaging on second- and fourth-order moments of the water velocity signals. For \( F = f_R L/U_c = 20 \), the second- and fourth-order moments of the signal sampled at frequency \( f_R \) are about 90 and 80%, respectively, of the values of the parameters of the signal sampled at frequency \( f_s \).

Fig. 6. Autocorrelation function values at Lag 1 of the recorded water velocity signal. Smaller values of \( R_{xx}(1) \) mean that there is less turbulence sampled in these signals. For \( F = f_R L/U_c = 20 \), \( R_{xx}(1) \) is 0.85.

Acoustic Doppler Velocimeter Performance Curves

Sampling the Flow Turbulence

First, a set of 1D model spectra were computed for different flow conditions. These spectra were then integrated to compute flow energies and evaluate both the effects of the analog filter with cut-off frequency \( f_s \), and the level of aliased energy with frequencies in the range \( f_s/2 < f < f_s \) in the original (unsampled) time series. Such energy is folded back through the sampling process and confused with resolved energy corresponding to frequencies in the range \( 0 < f < f_s/2 \).

The results obtained are shown in Fig. 4 in dimensionless form. As the dimensionless number \( f_s L/U_c \) increases, a smaller portion of the energy is both filtered and aliased. The poorest measurement conditions for the acoustic Doppler velocimeter in the range of parameters considered, are defined by extreme flow conditions \((L=0.1 \text{ m and } U_c=1 \text{ m/s})\), which combined yield a minimum value of \( f_s L/U_c \) at a given frequency and the smaller velocimeter velocity range which can sample this convective velocity (equivalent to 1 m/s). For this velocity range, at \( f_R = 25 \text{ Hz} \), the frequency \( f_s = 225 \text{ Hz} \) (see Table 1), and thus the value of the dimensionless number \( f_s L/U_c \) is 22.5. In such a case, the analog filter would take about 8.4% of the total energy out of the signal when it is sampled at \( f_s = 225 \text{ Hz} \). Additionally, for frequencies equal to \( f_s/2 \), the accumulated energy would be 86.4% of the total energy in the flow, which implies that only 5.2% of the total energy will be aliased. The values cited before correspond to extreme conditions, and for lower values of \( U_c \) (corresponding to a lower velocity range) and, consequently, for lower values of \( f_s \) (see Table 1), the percentage of aliased energy should decrease. Based on the values cited before, it is concluded from the model behavior of the velocimeter that both the effects of the analog filter (with cut-off frequency \( f_s \)) and the level of aliased energy are lower than 10% even in the most critical conditions. Besides, the digital filtering performed by the DPM using a cut-off frequency smaller than \( f_s/2 \) takes most of the aliased energy out of the spectrum.

Next, a number of synthetic turbulent water velocity signals with \( \Delta t = 0.0038 \text{ s} \) \((f_s = 260.8 \text{ Hz})\) were generated as realizations of different flow conditions and then sampled according to the sampling strategy described in the instrument component of the conceptual model. The use of different values of \( f_s \) corresponding to different velocity ranges does not affect the results of the analysis performed herein, because the gain factor of the velocimeter digital filtering does not depend on the number of averaged values in the signal to produce the same user-defined frequency \( f_R \). The user-set frequencies adopted here were \( f_R = 260.8, 52.2, 26.1, 10, 5, \text{ and } 3 \text{ Hz} \). The sampled signals were analyzed in order to compute corresponding turbulent parameters (up to fourth-order moments, autocorrelation function, power spectrum, and time scales). Thus, the variation of these parameters can be evaluated as flow conditions and sampling frequency change.

The effect of different flow conditions on the flow statistics is explored in Figs. 5–8. The parameters representing the flow statistics obtained for values \( f_R = f_s \) are made dimensionless using the corresponding value of the parameter computed for \( f_R = f_s = 260.8 \text{ Hz} \) (no averaging). Those dimensionless parameters are plotted as a function of the dimensionless parameter \( F \) defined as...
Instead of good portion of the inertial range is resolved.

A similar analysis could be performed for the vertical spatial averaging when the value of \( d_R \) is smaller than the vertical size of the measurement volume, \( d_v \). The assumption included here is that a uniform longitudinal velocity profile is present in the vertical direction, which is a limitation mainly in velocity measurements close to the bottom of a wall boundary layer, where high velocity gradients are present.

The evolution of the variance (integral of the spectrum and second-order moment of the signal) is shown in dimensionless form in Fig. 5, together with the corresponding fourth-order moments. The effects of the averaging on the fourth-order moments is more important (i.e., higher reduction in the sampled moment) than on the variance. A similar analysis was performed for the third-order moments but a clear trend could not be detected. This is due to the fact that the third-order moment (as well as all the odd moments) usually presents a very small value, which requires a very long integration time to estimate them with a reasonable level of accuracy (Sreenivasan et al. 1978). If the skewness is exactly zero, the integration time required is indeterminate.

Analysis of the autocorrelation function at the first lag \( R_{xx}(1) \), and the power spectrum \( E_{11} \) (included in Figs. 6 and 7, respectively) gives good information about how the sampling technique used by acoustic Doppler velocimeter affects the turbulence description. The spectrum is made dimensionless in Fig. 7 using the variance of the signal, \( \sigma \), and the length scale of large eddies, \( L \) and plotted as a function of \( 1/2F \), because the maximum frequency represented in the spectrum is \( f_d/2 \). The first sampled point in the autocorrelation function (Fig. 6) decreases as \( f_R \) decreases. Smaller values of \( R_{xx}(1) \) mean that there is less turbulence sampled in these signals and only a small zone of the inertial range is resolved in the power spectrum (Fig. 7). For \( F = f_dL/U_c < 1 \), the inertial range is not resolved and \( R_{xx}(1)=20\% \). As \( F \geq 2 \), progressively more of the inertial range gets sampled but the value \( R_{xx}(1) \) is still small unless \( F \) exceeds a value of about 20. The decorrelations for lags in the range from 0 to 1 (Fig. 6) are produced only for the flow conditions and sampling strategy considered here. Additionally, no noise effects are considered in the computation which would generate an extra level of decorrelation in the signal. The time scales computed from the autocorrelation function (as the integral of \( R_{xx} \) up to the first zero crossing) are biased to high values due to the sampling averaging. Fig. 8 quantifies this bias, showing the variation of this time scale with \( F \) in dimensionless terms.

Figs. 5–8 are called APCs, and yield criteria for sampling turbulence in water flows with ADV technology. From the present analysis, it is concluded that a good sampling criterion should consider values of \( F > 20 \), since such a range yields reasonably small losses in the moments but at the same time resolves important portions of the spectrum. The limit \( F = 20 \) means, for example, that when using an acoustic Doppler velocimeter with \( f_d = 25 \) Hz for an experiment with \( L = 20 \) cm, turbulence cannot be accurately resolved in flows with velocities higher than 25 cm/s.

Nezu and Nakagawa (1993) empirically proposed a criterion to determine the maximum response frequency of a turbulence measuring device which allows analysis of the spectral distribution of the flow down to the viscous sublayer. The maximum response frequency required in this analysis was chosen to satisfy the dimensionless number \( F = 16.67 \) which agrees well with the value proposed here. However, the criterion empirically proposed by Nezu and Nakagawa (1993) concerns just open-channel flow conditions where \( L \) is the water depth and \( U_c \) is equal to the longitudinal velocity.
Validation of the Acoustic Doppler Velocimeter Performance Curves

The assumptions used to develop the conceptual model are now validated using experimental data from an open-channel flow, recorded in a flume having a width $B=0.91$ m at the Ven Te Chow Hydrosystems Laboratory, University of Illinois at Urbana–Champaign (UIUC). First, a set of 11 three-dimensional water velocity time series were obtained with a down-looking Sontek Micro ADV sampling volume 5 cm away from the probe at the same location, flow conditions and instrument velocity range, but with different sampling frequencies values of $f_R=50; 30; 25; 20; 10; 5; 2; 1; 0.5; 0.2; 0.1$ Hz. The longitudinal water velocity signals recorded for each configuration were used to develop Figs. 9–12. The quality of the recorded signals is characterized by a correlation value in the range from 82 to 93 and a signal to noise ratio SNR value in the range 17.9 to 22.9 dB, which are high enough values of these parameters to ensure good quality data. Water velocity signals were recorded for 2 min at each instrument configuration. The sampling volume of the velocimeter was located at $y_p=0.04$ m from the bottom of the flume. The flow condition analyzed consisted of a water depth $h=0.282$ m, flow discharge $Q=0.12$ (m$^3$/s), and a local value of the shear velocity at this vertical, $u' =0.0482$ m/s. The scale of the energy containing eddies, $L$, corresponding to this flow is estimated to be equal to the depth=0.282 m, and the associated convective velocity is determined to be equal to $U_c=0.58$ m/s, from the three-dimensional velocity vector data using the relation proposed by Heskestad (1965)

$$U_c^2 = U_1^2 + 2 \frac{U_2^2}{U_1^2} + 2 \frac{u_1^2}{U_1^2} + 2 \frac{u_2^2}{U_1^2} + 2 \frac{u_3^2}{U_1^2}$$

where $U_i$ and $u_i'$=mean and the fluctuation of the flow velocity in the $i$ Cartesian direction. Following Nezu and Nakagawa (1993), the dimensionless parameter $sh/u'^3$ is estimated to have a value equal to 16 for $y_p/h=0.142$. Thus, the rate of dissipation of TKE at the measurement point is $6.35 \times 10^{-3}$ m$^2$/s$^3$ and the corresponding Kolmogorov length scale is 0.00011 m.

The evolution of second- and fourth-order moments of the measured velocity signals as $f_R$ varies is plotted in Figs. 9 and 10, respectively. The values of the moments corresponding to the real sampling frequency, $f_S$, of the instrument are needed to make the plot dimensionless; however, this information cannot be obtained from the acoustic Doppler velocimeters. To overcome this problem, the value of the ratio between the corresponding moments obtained at $f_R$ and $f_S$ for the higher sampling frequency ($f_R=50$ Hz and $F=24.31$) is assumed to be the same as that predicted by the conceptual model (92.5% for the variance and 83%...
Table 2. Characteristics of the Experiments

<table>
<thead>
<tr>
<th>Experiment no.</th>
<th>L (cm)</th>
<th>$U_c$ (cm/s)</th>
<th>Instrument Type</th>
<th>$f_R$ (Hz)</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>20</td>
<td>~43</td>
<td>Micro ADV Sontek</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>38</td>
<td>5–7</td>
<td>Micro ADV Sontek</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>50–60</td>
<td>15–23</td>
<td>Nortek NDV 10 MHz</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>85</td>
<td>17–22</td>
<td>ADV Sontek</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>365.8</td>
<td>~10</td>
<td>Nortek NDV 10 MHz</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>26.7</td>
<td>56–71</td>
<td>Micro ADV Sontek</td>
<td>25</td>
</tr>
<tr>
<td>7</td>
<td>28.2</td>
<td>58</td>
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<td>8</td>
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<tr>
<td>9</td>
<td>700</td>
<td>12–13</td>
<td>Nortek NDV 10 MHz</td>
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<td>10</td>
<td>15</td>
<td>10–21</td>
<td>ADV Sontek</td>
<td>25</td>
</tr>
</tbody>
</table>

nocorr

for the fourth-order moment). The trend obtained from the measurements agrees very well with that predicted by the model (Figs. 9 and 10). The poor resolution of turbulence in the analyzed flow obtained with low sampling frequencies is shown by these figures by the fact that all of the even moments of the sampled frequencies are biased to low values. For instance, at frequency $f_R=1$ Hz ($F=f_p L/U_c=0.4862$), the recorded signal captured only 48.5 and 24% of the variance and fourth-order moment, respectively.

A good agreement is also obtained between measurements and the predictions of the conceptual model for the variation of the correlation value at the first lag, $Rxx(1)$ with the dimensionless frequency $F$ (Fig. 11). The observed values of $Rxx(1)$ reach a rather constant value of about 80% at high frequencies due to the noise decorrelation. Fig. 12 shows the observed variation of the integral time scale with $F$. A good agreement is obtained between the prediction of the conceptual model and the observations, with exception of the behavior at dimensionless frequencies lower than about unity.

Further validation of the conceptual model was obtained by using water velocity signals recorded at several facilities by researchers at the Ven Te Chow Hydrosystems Laboratory of the UIUC since 1994. Corresponding experimental conditions are described in Table 2. The flow generated in an annular flume was analyzed in Experiment 1: open-channel flow conditions were simulated in three different tilting flumes for Experiments 2–5; in Experiment 6, water velocities signals were recorded in an experimental pool and riffle sequence with and without the presence of vegetation, respectively; flow velocity fields around bubble plumes were measured in Experiments 8 and 9 in a square and round tank, respectively; and finally, flow velocity signals recorded at points located inside a turbidity current were analyzed in Experiment 10. Experiment 7 is the source of data for the set of 11 time series used in Figs. 9–12. For the sake of clarity, only two points of this set are used in Fig. 13. These time series correspond to values of $F=12.02$ and $F=24.3$. The data included in Fig. 13 correspond only to velocity signals of the X Cartesian component from all of the experiments. This component was always aligned with the main flow direction. In all cases, the instrument was used in a down-looking orientation, with the exception of Experiments 3, 8, and 9, where a side-looking orientation was used. In summary, the quality of the recorded signals is characterized by correlation values in the range 84 to 99 and by SNR values in the range 18.6 to 30.20 DB.

Comparisons between the predicted and observed values of the autocorrelation function at Lag 1 for experimental conditions described in Table 2 are presented in Fig. 13. The decorrelation observed in the measured signals increases as the dimensionless number $F$ decreases, in agreement with the predictions of the conceptual model. However, decorrelations that are higher than those predicted are observed due to noise effects, which are not accounted for by the theoretical APC curves. Based on this observation, it can be argued that these curves provide an upper limit of the actual ones.

Noise Effects from Turbulence Parameters Computed from Acoustic Doppler Velocimeter Water Velocity Signals

The presence of noise in water velocity signals obtained using an acoustic Doppler velocimeter, and the techniques to reduce its effect in the computation of turbulence parameters obtained from these signals, have been the focus of several papers in recent years (Lohrmann et al. 1994; Nikora and Goring 1998; Voulgaris and Trowbridge 1998; McLelland and Nicholas 2000). Even when all of the possible precautions suggested by the manufacturers are taken into consideration (i.e., correlation, $p$, and SNR within defined ranges), the signal will have a noise level that affects the values of the turbulence parameters. Nikora and Goring (1998) and SonTek (1997) affirm that the main physical contributor to the acoustic Doppler velocimeter’s noise is the Doppler noise. The Doppler noise has the characteristics of white noise (Nikora and Goring 1998; Lemmin and Lhermitte 1999; McLelland and Nicholas 2000) with a Gaussian probability distribution (Nikora and Goring 1998), as well as a flat power spectrum (Anderson and Lohrmann 1995) which indicates the presence of uncorrelated noise (Lemmin and Lhermitte 1999).

The fact that white noise presents the same energy level for all frequencies makes it impossible to subtract its effects from the temporal series using digital filters; however, its integral effects can be subtracted from some turbulence parameters. White noise does not affect the computation of the mean values because it has zero mean. Nikora and Goring (1998), Voulgaris and Trowbridge (1998), and McLelland and Nicholas (2000) showed that Reynolds stress computations are not affected by the presence of the white noise. Lohrmann et al. (1994) considered that the Reynolds stresses can be accurately determined even at levels below the
Doppler noise. Nikora and Goring (1998) claim that estimates of TKE are limited by the Doppler noise because the TKE is biased to high values. However, Lohrmann et al. (1994), Nikora and Goring (1998), and Gordon and Cox (2000) affirm that the contributions of noise over the total energy can be considered negligible for flows with high levels of turbulence, such as open-channel flow boundary layers.

The Doppler noise produces decorrelation of the signal and hence the autocorrelation function reduces its value to zero faster than in signals without noise. Consequently, the temporal scales obtained from this function are biased to low values. On the other hand, velocity spectra for the horizontal velocity components are biased toward high values due to the presence of the Doppler noise, while for the vertical velocity the noise is negligible (Lohrmann et al. 1994; Nikora and Goring 1998). Assuming that the noise and turbulent fluctuations are decorrelated (Nikora and Goring 1998), the spectrum of the resulting measurement is the sum of the turbulent spectrum plus the noise level. Nikora and Goring (1998) identified the Doppler noise as a flattening of the spectrum close to the Nyquist frequency, and found that in the worst case the flattening may take place around 4–5 Hz for the horizontal velocity but is more typically in the range of 5–10 Hz.

The main parameter used to subtract the Doppler noise effects on the turbulence parameters is the white noise energy level, the detection of which is discussed later. Using this level, the corrected power spectrum for each velocity component can be simply obtained by subtracting the white noise energy level from the measured spectra. Thus, by integrating the corrected power spectra, the corrected variances for each component, and from these the corrected TKE are computed. Additionally, the inverse fast Fourier transform of the corrected power spectrum can be used to estimate the autocorrelation function and associated corrected time scales can be obtained.

**Detection of the Doppler Noise Energy Level**

In a low-energy flow, the energy level of the white noise can be identified in a power spectrum as a flat plateau at high frequencies. Nikora and Goring (1998) suggested that the empirical spectra of the Doppler noise can be replaced by straight horizontal lines whose ordinates are equal to the average of the noise spectral ordinates. This technique was called “spectral analysis” by Vougaris and Trowbridge (1998), who calculated the noise energy using the noise energy level detected in the tail of the spectrum (the frequency range is chosen so that there are ten estimates for the calculation of the statistically significant average, i.e., 11.5–12.5 Hz for sampling frequency=25 Hz).

Nikora and Goring (1998) identified a characteristic frequency, \( f_n \), that marks a boundary in the power spectrum between two regions. The first region corresponds to frequencies smaller than \( f_n \), where the turbulence energy is much larger than the noise energy. In the other region, for frequencies higher than \( f_n \), turbulence energy is weaker than the noise energy. If the frequency \( f_n \) is smaller than the Nyquist frequency (\( f_R / 2 \)), the flat plateau in the spectrum would be visualized. In these cases, the spectral analysis technique of Vougaris and Trowbridge (1998) can be applied to estimate the noise energy level. Although this method becomes a very good approximation to determine the noise energy level, for high-energy flows the plateau cannot be distinguished in the spectrum, although this does not imply that the signal does not have intrinsic noise. In these cases, different methodologies have been suggested to estimate the noise level (Nikora and Goring 1998; Vougaris and Trowbridge 1998; McLelland and Nicholas 2000), which all have some intrinsic difficulties. These methods mainly assume that the noise level of the signal is the same if the instrument configuration and flow conditions do not change. This implies that the users are recording signals with the same quality each time that the instrument is sampling the same flow conditions with the same configuration (sampling frequency, velocity range, etc.). However, there are certain conditions that cannot be controlled by the users during the measurement (for example, level of seeding particles, and bubbles attached to the sensor) which strongly affect the quality of the signal (Nikora and Goring 1998; Lemmin and Lhermitte 1999), and thus the noise level. SonTek (1997) suggested that estimation of the Doppler noise from a pulse coherent system [as Vougaris and Trowbridge (1998), McLelland and Nicholas (2000)] do is a complicated operation, which for practical systems provides at best a lower bound for instrument noise level.

**Evaluation of the Doppler Noise Effect on the Total Turbulent Energy**

Some tools are introduced here to evaluate the relative importance (\( E \)) of the noise energy over the real turbulent energy for different flow conditions. \( E \) is defined as

\[
E = \frac{\sigma_n^2}{\sigma_m^2 - \sigma_n^2}
\]

where \( \sigma_n^2 \) is noise energy; \( \sigma_m^2 - \sigma_n^2 \) is real turbulent energy; and \( \sigma_m^2 \) is measured turbulent energy. The ratio \( E \) quantifies the importance of estimating noise effects in high-energy flows. Each of the modeled spectra obtained from the simulation of different flow conditions is integrated up to the Nyquist frequency, in order to compute the turbulent flow energy for the specified flow conditions. This energy value is then compared with a noise energy computed using the white noise characteristics of the Doppler noise. The noise energy is obtained as the product of the noise energy computed, \( E_{11n} \), and the Nyquist frequency=\( f_R / 2 \). Thus, the variables used in this part of the analysis are: \( U_c \), \( L \), \( \eta \), \( E_{11n} \), and \( f_R \). A range of noise energy levels, \( 10^{-7} \text{ m}^2/\text{s} \leq E_{11n} \leq 10^{-5} \text{ m}^2/\text{s} \), typical of acoustic Doppler velocimeter measurements, is consid-
erated here based on experience and previous research (Nikora and Goring 1998).

The energy ratio $E$ is plotted in Figs. 14–16 as a function of the Kolmogorov length scale, $\eta$, and the energy containing eddy length scale, $L$. The ratio $E$ decreases as the energy-containing eddy length scale increases, while $E$ increases as $\eta$ increases (i.e., noise is less important for energetic flows). It was found that $U_c$ is not a relevant parameter describing the behavior of $E$. As it was introduced before, a dimensionless number $F = f_R L / U_c > 20$ is required to obtain a good description of the flow turbulence using acoustic Doppler velocimeters. For $F > 20$, and the conditions represented in Figs. 14–16 ($f_R = 25$ Hz and $U_c = 0.5$ m/s), turbulent energy in flows with $L < 0.4$ m will not be well resolved. Therefore, these conditions are not represented in these figures. Fig. 14 shows that the highest ratio $E$ predicted for $\eta \approx 0.0005$ m (which is representative of most laboratory and field experiments) and the noise energy level $E_{11n} = 10^{-6}$ m$^2$/s is 7%. For flows with $L > 0.7$ m, $E$ is lower than 5%. Fig. 16 shows that $E$ is lower than 5% for all the possible flow conditions and instrument configurations which satisfy the APCs requirements at $E_{11n} = 10^{-7}$ m$^2$/s. However, Fig. 15 ($E_{11n} = 10^{-5}$ m$^2$/s) deserves special attention because for most conditions the ratio $E$ remains at values higher than 10%, which indicates that the noise energy must be subtracted in these conditions because of its importance.

A complementary analysis included here is related to the definition of the characteristic frequency, $f_n$, for the flow conditions and instrument configuration cited before. Values of $f_n$ smaller than the Nyquist frequency indicate that the flat plateau in the power spectrum can be detected. Using the fact that the measured energy power spectrum is the sum of the real energy spectrum and the noise spectrum (Nikora and Goring 1998), a set of measured energy spectra was built for the different flow conditions and noise energy level. The frequency $f_n$ is defined as the frequency where the real flow energy is equal to the noise energy. Above this frequency, noise is the most important component in the measured power spectrum. The detected values of the frequency $f_n$ where the real flow energy is equal to the noise energy for $f_R = 25$ Hz and $E_{11n} = 10^{-6}$ m$^2$/s. The ratio $E$ is lower than 10% for $\eta \approx 0.0006$ m (see Fig. 14). For $\eta > 0.0006$ m, the noise energy is important and it must be corrected.

Fig. 15. Ratio between the noise energy and the real turbulent energy, $E$, for $U_c = 0.5$ m/s, $f_R = 25$ Hz, and $E_{11n} = 10^{-7}$ m$^2$/s. Flow conditions with $L < 0.4$ m ($F < 20$) are disregarded in the analysis because of the requirements suggested by acoustic Doppler velocimeter performance curves.

Fig. 16. Ratio between the noise energy and the real turbulent energy, $E$, for $U_c = 0.5$ m/s, $f_R = 25$ Hz, and $E_{11n} = 10^{-5}$ m$^2$/s. Flow conditions with $L < 0.4$ m ($F < 20$) are disregarded in the analysis because of the requirements suggested by acoustic Doppler velocimeter performance curves.

Fig. 17. Characteristic frequency $f_n$ for $f_R = 25$ Hz and $E_{11n} = 10^{-6}$ m$^2$/s. The frequency $f_n$ is defined as the frequency where the real flow energy is equal to the noise energy. Above this frequency, noise is the most important component in the measured power spectrum.

Fig. 18. The characteristic frequency $f_n$ where the real flow energy is equal to the noise energy for $f_R = 25$ Hz and $E_{11n} = 10^{-6}$ m$^2$/s. The ratio $E$ is lower than 10% for $\eta \approx 0.0006$ m (see Fig. 14). For $\eta > 0.0006$ m, the noise energy is important and it must be corrected.
frequency $f_\text{c}$ are plotted in Figs. 17 and 18 as a function of the Kolmogorov length scales ($\eta$), for three different convective velocities ($U_c$). In this analysis, the eddy-containing eddy length scale, $L$, is not found to be a relevant parameter. The plots show that the higher the value of $\eta$, the smaller the frequency where noise is detected (recall that higher $\eta$ implies a lower-energy flow). Additionally, it can be observed that the larger $U_c$, the higher the value of the frequency $f_\text{c}$.

Fig. 17 shows the case that deserves most attention according to the analysis presented before ($E_{11\chi}=10^{-6} \text{ m}^2/\text{s}$) because it presents the highest ratio $E$ (Fig. 15). For all of the flow conditions where $\eta > 0.00025 \text{ m}$, the frequency $f_\text{c}$ is lower than Nyquist frequency ($f_R/2$) if a user-defined sampling frequency $f_R=25 \text{ Hz}$ is used. It can be observed in Fig. 15 that the flow and sampling conditions satisfying the requirements imposed for the APC (resolving the flow turbulence) with $\eta < 0.00025 \text{ m}$, are values of $E < 10\%$. In cases where the ratio $E$ is higher than 10% ($\eta > 0.00025 \text{ m}$), the noise floor can be detected from the measured spectrum, and thus the spectral analysis technique to estimate the noise energy level can be used. For the value of noise energy level, $E_{11\chi}=10^{-6} \text{ m}^2/\text{s}$, the ratio $E$ is lower than 10% for Kolmogorov length scales $\eta \approx 0.0006 \text{ m}$ (Fig. 14). Therefore, two regions can be defined in Fig. 18: For one of them ($\eta > 0.0006 \text{ m}$) the noise energy is important and must be corrected. Only these conditions are represented in Fig. 18. For this entire region, $f_\text{c} < 1.25 \text{ Hz}$. Thus, using a defined user sampling frequency $f_R=25 \text{ Hz}$, the noise plateau will be observed in the spectrum.

It can be concluded that in the cases where the noise energy is important, in relation to the real turbulent energy of the signal, the noise energy level can be computed using the spectral analysis method because the white noise plateau is observed in the power spectrum (Figs. 15 and 17). In cases of very high-energy flows (or very small noise energy level), this method cannot be used, although the noise energy is smaller than 10% of the real total energy.

Conclusions

Acoustic Doppler velocimeters have proved to yield a good description of turbulence when certain conditions are satisfied. These restrictions are related to the instrument configuration (sampling frequency and noise energy level) and flow conditions (convective velocity and turbulence scales in the flow). In general, acoustic Doppler velocimeters produce a reduction in all of the even moments in the water velocity signal due to the low-pass filter used in the instrument. Additionally, this filter affects the autocorrelation functions (increasing $R_{xx}$ for small lag times), the time scales computed from them (producing results that are high biased), and the power spectrum (producing a low resolution of the inertial range). However, the present analysis indicates that all of these effects are rather negligible in cases where a value of the dimensionless frequency $F = f_R L / U_c > 20$ is used. This result provides a new criterion to check the validity of the acoustic Doppler velocimeter measurements as a good representation of the turbulence in any flow with known convective velocity and length scales. The acoustic Doppler velocimeter should be operated at the maximum recording rate $f_R$ so that $F$ is as large as possible. However, it must be noted that the higher sampling frequency will produce a higher Doppler noise energy of the signal. The decision about the selected frequency to record data should thus optimize the values of $F$ to make the recorded signal representa-tive of the flow turbulence, but seek to keep the Doppler noise energy level of the signal as low as possible.

Doppler noise constitutes an important error source in acoustic Doppler turbulence measurements and its effects on the turbulence parameters computed from these signals must be quantified and removed in certain cases. The spectral analysis method provides the most realistic estimation of energy Doppler noise level. After the noise energy level is detected, noise effects must be subtracted from estimators of the power spectrum, variance, TKE, autocorrelation function, convective velocity, length and time scales, and rate of dissipation of TKE. However, the spectral analysis method cannot be used in high turbulent energy flows where the characteristic frequency $f_\chi$ (where the noise is detected as a flat plateau in the spectrum) is higher than the Nyquist frequency. However, it is shown herein that in these cases, the Doppler noise contribution to the total measured energy is lower than 10%.

Acknowledgments

A number of federal and state agencies have supported several research projects leading to this work at the University of Illinois. They are the National Science Foundation, the Office of Naval Research, the U.S. Army Corps of Engineering, the U.S. Geological Survey, the U.S. Department of Agriculture, the Illinois Water Resources Center, the Illinois Department of Natural Resources, and the Metropolitan Water Reclamation District of Greater Chicago. The findings in this paper are the sole opinion of the writers and do not support or endorse any specific manufacturer of acoustic Doppler velocimeters. The writers thank Jim Best for his help in making the manuscript more readable as well as two anonymous reviewers for their constructive criticism.

Notation

The following symbols are used in this paper:

- $A_q$ = amplitude of weighted series cosines series;
- $B$ = flume width;
- $C$ = sound speed in water;
- $C_0$ = parameter of the adopted spectrum function;
- $c_L,c_\eta$ = parameters of the adopted spectrum function;
- $d$ = diameter of the acoustic Doppler velocimeter’s measurement volume;
- $d_R$ = diameter of the sampled volume set by the flow and sampling characteristics;
- $d\phi/dt|_i$ = phase shift rate of the sound signal computed for the velocimeter’s receiver $i$;
- $E$ = relative importance of the noise energy over the real turbulent energy;
- $E_{11}(k_1)$ = one-dimensional power spectrum function;
- $E_{11\chi}$ = noise energy level;
- $F$ = dimensionless parameter;
- $f$ = frequency;
- $f_{\chi\text{cut-off}}$ = cut-off frequency of the digital filter;
- $f_L$ = characteristic frequency where noise is observed in power spectrum;
\[ f_R = \text{acoustic Doppler velocimeter's user-set frequency;} \]
\[ f_S = \text{frequency of the three-dimensional velocity measurement process;} \]
\[ f_T = \text{characteristic frequency of large eddies present in the flow;} \]
\[ f_n = \text{shape function defined for the dissipation range of the spectrum function;} \]
\[ H(f) = \text{transfer function of the digital filter;} \]
\[ h = \text{water depth;} \]
\[ j = (-1)^{1/2}; \]
\[ L = \text{energy containing eddy length scale;} \]
\[ N = \text{number of values averaged in the signal to produce a digital signal with frequency} f_S; \]
\[ N_t = \text{number of terms of the target spectrum used in the Shinozuka method;} \]
\[ N_t = \text{number of samples in the generated synthetic time series;} \]
\[ p_m = \text{parameter of the adopted spectrum function;} \]
\[ Q = \text{flow discharge;} \]
\[ R_{xx} = \text{autocorrelation function;} \]
\[ T = \text{time taken for an acoustic Doppler velocimeter to complete a three-dimensional velocity measurement;} \]
\[ T_{11} = \text{time scale of the one-dimensional water velocity signal;} \]
\[ T_t = \text{total simulated time in the synthetic signal;} \]
\[ i = \text{time;} \]
\[ U_c = \text{convective velocity;} \]
\[ U_l = \text{mean flow velocity in the} i \text{ direction;} \]
\[ u = \text{shear velocity;} \]
\[ u_l = \text{fluctuation of the flow velocity in the} i \text{ direction;} \]
\[ u_i, u_j, u_k = \text{flow velocity vector components in the local Cartesian coordinate system;} \]
\[ v = \text{flow radial velocity computed for each velocimeter receiver} \( i = 1, 2, 3 \); \]
\[ X(f) = \text{Fourier transforms of the signal} x; \]
\[ y = \text{signal sampled at} f_S; \]
\[ Y(f) = \text{Fourier transforms of the signal} y; \]
\[ \gamma_f = \text{distance from the bottom of the flume;} \]
\[ \alpha = \text{amount of jitter in Shinozuka method;} \]
\[ \beta = \text{parameter of the power spectrum function;} \]
\[ \Delta_t = \frac{1}{f_S}; \]
\[ \Delta_f_i = \frac{1/\Delta_t - \text{interval between samples of signal} x; \]
\[ \Delta_f_y = \frac{1/\Delta_t - \text{interval between samples of signal} y; \]
\[ \varepsilon = \text{rate of dissipation of turbulent kinetic energy of the flow;} \]
\[ \eta = \text{Kolmogorov length scale;} \]
\[ \kappa_1 = \text{one-dimensional wave number;} \]
\[ \nu = \text{kinematic water viscosity;} \]
\[ \sigma = \text{variance of the one-dimensional velocity signal;} \]
\[ \sigma_t = \text{measured turbulent energy;} \]
\[ \sigma_m = \text{noise energy;} \]
\[ \sigma_r = \text{real turbulent energy;} \]
\[ \tau_1, \tau_2 = \text{different pulse repetition rates for the acoustic Doppler velocimeter pulse-pair scheme;} \]
\[ \tau_d = \text{dwell time for the acoustic Doppler velocimeter pulse-pair scheme; and} \]
\[ \phi = \text{random phase angle of the synthetic flow velocity series;} \]
\[ \omega, \omega', \Delta \omega = \text{variables of the Shinozuka method.} \]

References


DISCUSSIONS AND CLOSURES

Discussion of “Turbulence Measurements with Acoustic Doppler Velocimeters” by Carlos M. García, Mariano I. Cantero, Yarko Niño, and Marcelo H. García

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The discussers congratulate the authors for their important contribution. Although acoustic Doppler velocimetry (ADV) has become a popular technique for last two decades, some researchers, including the authors, pointed out rightly that ADV signal outputs include the combined effects of turbulent velocity fluctuations, Doppler noise, signal aliasing, turbulent shear, and other disturbances. Simply, “raw” ADV velocity data are not “true” turbulence and should never be used without adequate postprocessing (Nikora and Goring 1998; Wahl 2003). Herein the discussers aim to complement the understanding of ADV turbulence measurements by arguing the effects of sampling duration and proximity of solid boundaries. They discuss also practical issues associated with turbulence measurements in natural estuarine systems with acoustic Doppler velocimeters (ADVs).

The sampling duration does influence the results since turbulence characteristics may be biased with small sample numbers. Yet, in hydraulic engineering, there has been a great variety of sampling durations used by various researchers in laboratory and field studies without systematic validation. In their study, the authors used a 2-min sampling time corresponding to 6,000 samples maximum, assuming implicitly that such a duration is long enough to describe the turbulence. Basic turbulence studies showed recently the needs for larger sample sizes (e.g., 60,000 to 90,000 samples per sampling location) (Karlsson and Johansson 1986; Krogstad et al. 2005). The discussers performed new experiments in a large laboratory flume (0.5 m wide, 12 m long) with sub- and transcritical flow conditions. The channel was made of smooth PVC bed and glass walls, and the waters were supplied by a constant head tank. Velocity measurements were conducted with a 16 MHz micro ADV equipped with a two-dimensional sidematching head. Sensitivity analyses were performed in steady flows with 25 and 50 Hz scan rates, total sampling durations τ R between 1 and 60 min, and in both gradually varied and uniform equilibrium flows. The results indicated consistently that the streamwise velocity \( V_x \) statistical properties were most sensitive to the number of data points per sample. The first two statistical moments (mean and standard deviations) were adversely affected by sampling durations less than 100 s (less than 5,000 samples). Higher statistical moments (e.g., skewness, kurtosis), Reynolds stresses, and triple correlations were detrimentally influenced for scan durations less than typically 500 to 1,000 s corresponding to less than 25,000 to 50,000 samples. The findings are consistent with modern experimental studies of turbulence (Karlsson and Johansson 1986). Fig. 1 illustrates the effects of the sample size at a sampling location at 27 mm above the bed on the channel centerline. The data set was “cleaned” by excluding low-correlation and low signal-to-noise ratio samples, and by removing “spikes” using a phase-space thresholding technique (Goring and Nikora 2002; Wahl 2003).

The proximity of a boundary may adversely affect the ADV probe output, especially in small laboratory flumes. Several studies discussed the effects of boundary proximity on sampling volume characteristics and the impact on time-averaged velocity data (Table 1). Table 1 lists pertinent studies, including details of the reference instrumentation used to validate the ADV data (Table 1, column 2) and of the ADV systems (Table 1, columns 3 and 4). These studies highlighted that acoustic Doppler velocimeters underestimated the streamwise velocity component when the solid boundary was less than 30 to 45 mm from the probe sampling volume. Correction correlations were proposed by Liu et al. (2002) and Koch and Chanson (2005) for micro-ADV with 3D downlooking head and 2D sidematching head respectively. The discussers observed that the effects of wall proximity on ADV velocity signal were characterized by a significant drop in average signal correlations, in average signal-to-noise ratios and in average signal amplitudes next to the wall (Koch and Chanson 2005). Martin et al. (2002) attributed lower signal correlations to high turbulent shear and velocity gradient across the ADV sampling volume. But the discussers observed that the decrease in signal-to-noise ratio with decreasing distance from the sidewall appeared to be the main factor affecting the ADV signal output. Finally, it must be stressed that most past and present comparative studies were restricted to limited comparison of time-averaged streamwise velocity component. No comparative test was performed to assess the effect of boundary proximity on instantaneous velocities, turbulent velocity fluctuations, Reynolds stresses nor other turbulence characteristics.

The discussers were involved in high-frequency, long-duration turbulence measurements using ADVs in a small estuary (Fig. 2) (Chanson 2003; Chanson et al. 2004). Fig. 3 shows a typical raw signal output for the streamwise velocity component during one field study investigation. The sampling volume was located 0.05 m above the bed for all study duration, and the measured water depth is reported in Fig. 3 (Right vertical axis). While the ADV is well-suited to such shallow-water flow conditions, all field investigations demonstrated recurrent problems with the velocity data, including large numbers of spikes (e.g., Fig. 3, \( t = 28,000–34,000 \) s). Problems were also experienced with the vertical velocity component, possibly because of the effects of the wake of the stem. Practical problems were further experienced. During one field study, the computer lost power and could not be reconnected to the ADV for nearly 50 min (Fig. 3, \( t = 49,000–52,000 \) s). During other field works, the ADV sampling volume was maintained about 0.5 m below the free-surface, implying the need to adjust the vertical probe position up to 3 times per hour. Last, navigation and aquatic life were observed during all field works (Fig. 2). Fig. 2 shows a recreational dinghy passing in...
Fig. 1. Effects of data sample size on turbulence characteristics in a 0.5-m-wide, 12-m-long open channel [flow conditions: $Q=0.0404 \text{ m}^3/\text{s}$, $W=0.5 \text{ m}$, $d=0.096 \text{ m}$, $z=27.2 \text{ mm}$, micro ADV (16 MHz) with 2D sidelaying head, sampling rate=50 Hz; velocity range=1 m/s]
reverse beside the ADVs. The effects of propeller wash and “bow” waves were felt for several minutes as discussed by Chan-
son et al. In a few instances, birds were seen diving and fishing next to the ADV location. All these events/disturbances had some impact on the turbulence data.

Careful analyses of ADV signal outputs showed that turbu-
lence properties were inaccurately estimated from unprocessed ADV signals. Even “classical” despiking methods were not directly applicable to unsteady estuary flows. A new three-stage postprocessing method was developed (Chanson et al. 2005). The technique included an initial velocity signal check, the detection and removal of large disturbances (prefiltering), and the detection and removal of small disturbances (despiking). Each stage included velocity error detection and data replacement. The method was applied successfully to long-duration ADV records at high frequency (25 Hz). Both 10 MHz ADV and 16 MHz microADV systems were used. For all investigations, between 10 to 25% of all samples were deemed erroneous. For the data shown in Fig. 3, the number of erroneous samples corresponded to 10% of the records, or 19% of the entire study period including the power

Notes: y=transverse distance from a sidewall; and z=vertical distance from the invert.

**Fig. 2.** Field deployment of acoustic Doppler velocimeters [boat passing beside the tripod (foreground left) supporting the ADVs at high tide]

**Fig. 3.** Field data from ADV deployment in a small estuary: streamwise velocity $V_x$ component (positive downstream, unprocessed “raw” signal) and measured water depth [time in seconds since midnight field work: Sept. 2, 2004, ADV (10 MHz) with 3D downlooking head; sampling rate=25 Hz, continuous sampling; velocity range=0.30 m/s; sampling volume located 0.052 m above bed and 10.8 m from left bank.}

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### Table 1. Experimental Studies of the Effects of Boundary Proximity and Velocity Shear on Acoustic Doppler Velocimetry Data in Open Channels

<table>
<thead>
<tr>
<th>Reference</th>
<th>Reference probe</th>
<th>ADV device</th>
<th>ADV sampling location affected by boundary proximity</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voulgaris and Trowbridge (1998)</td>
<td>8 mW Helium-Neon LDV</td>
<td>Sontek ADV 10 MHz 3D downlooking</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Finelli et al. (1999)</td>
<td>Hot-film probe Dantec R14 (single-wire)</td>
<td>Sontek ADV Field 10 MHz 3D downlooking</td>
<td>$z&lt;10$ mm, centerline data</td>
<td>$W=0.13$ m. Acrylic bed and walls.</td>
</tr>
<tr>
<td>Martin et al. (2002)</td>
<td>—</td>
<td>Sontek micro ADV 16 Hz 3D downlooking</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Liu et al. (2002)</td>
<td>Prandtl-Pitot tube ($\phi=3$ mm)</td>
<td>Sontek micro ADV 16 Hz 3D downlooking</td>
<td>$z&lt;30$ mm, centerline data</td>
<td>$W=0.46$ m. Aluminum bed, glass walls.</td>
</tr>
<tr>
<td>Koch and Chanson (2005)</td>
<td>Prandtl-Pitot tube ($\phi=3.02$ mm)</td>
<td>Sontek micro ADV 16 Hz 2D sidematerial</td>
<td>$y&lt;45$ mm, centerline data</td>
<td>$W=0.50$ m. PVC bed, glass walls, 75 mm $z=7.2$ mm (ADV head touching channel bed).</td>
</tr>
</tbody>
</table>

**Notes:** $y$=transverse distance from a sidewall; and $z$=vertical distance from the invert.
failure. Field observations illustrated that unprocessed ADV data should not be used to study turbulent flow properties, including time-averaged velocity components.

In summary, the authors’ contribution was a timely notice that acoustic Doppler velocimeters have intrinsic weaknesses and that their signal outputs are not always “true” turbulence measurements. In this discussion, it is demonstrated that in steady open channel flows, the sampling record must be larger than 5,000 samples to yield minimum errors on first and second statistical moments of the velocity components. Significantly longer records (more than 50,000 samples) are required for accurate determination of higher statistical moments (e.g., skewness and kurtosis). Reynolds stresses, and triple correlations. Further ADV signal outputs are adversely affected by the proximity of solid boundaries, particularly when the sampling volume is located less than 30 to 45 mm from the wall. Recent field observations in a small estuary showed also that ADV records may be affected by various disturbances including wildlife and manmade interferences. Comparative analyses of long duration, high-frequency data sets highlighted the needs for advanced postprocessing techniques. It is hoped that the authors’ contribution and the present discussion will stress enough the needs to educate and adequately train technicians, engineers, scientists, and researchers deploying ADVs in the field, including portable ADV systems.

Acknowledgments

The discussers acknowledge helpful discussions with Professor Shin-ichi Aoki (Japan).

References


Discussion of “Turbulence Measurements with Acoustic Doppler Velocimeters” by Carlos M. García, Mariano I. Cantero, Yarko Niño, and Marcelo H. García

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The paper by García et al. deals with the problem of correctly measuring turbulence parameters with acoustic Doppler velocimeters (ADV; trade names ADV for Sontek and NDV for Nortek). The authors focus on the effects of sampling frequency and Doppler noise on turbulence parameters. To avoid loss of turbulence information, they suggest that data should be sampled above a determined frequency. In addition, noise should be removed by estimating the noise contribution. Their approach is based on a model-derived procedure. First, it would have been of interest to compare the modeled spectra with those obtained from their measurements to validate their model and instrument assumptions for the flow cases discussed. Second, the deviation from the -5/3 slope in the measured spectra due to filtering and/or noise effects has not been highlighted.

We investigated the authors’ conclusions using a Vectrino (Nortek) ADV. Different from their instruments, a Vectrino has four receivers symmetrically spaced around the central emitter. The applied sampling frequencies, the relative position, and the size of the measuring volume, however, were identical to the NDV. Using four receivers allows measuring the vertical velocity component simultaneously in the two planes. This configuration enables the direct estimation of noise effects so that suitable correction procedures such as the one proposed by Hurther and Lemmin (2001; hereinafter called HLP) can be applied. The HLP takes advantage of the redundancy of the vertical velocity obtained in the two instrument planes.

1286 / JOURNAL OF HYDRAULIC ENGINEERING © ASCE / NOVEMBER 2007
Fig. 1. Turbulence spectra of the $u$, $v$, and $w$ velocity components sampled at 100 Hz; also shown are the noise spectra for the two vertical velocities

Fig. 2. Original, noise corrected, and fitted spectra for the longitudinal component sampled at 100 Hz

It should be noted that Doppler noise is composed of several contributions that can be estimated (Garbini et al. 1982; Lhermitte and Lemmin 1990, 1994; Hurther and Lemmin 1998; Voulgaris and Trowbridge 1998) or eliminated (Garbini et al. 1982; Hurther and Lemmin 1998, 2001; Blanckaert and Lemmin 2006). For three receiver instruments, such as those used by the authors, the procedure proposed by Voulgaris and Trowbridge (1998) can be applied who emphasize that overestimates have to be expected. A sufficiently high sampling frequency is required for successful measurements.

Our measurements were carried out in an open channel at the LHE-EPFL. The channel is 0.60 m wide and 17 m long. The bottom was covered with a 0.1-m-thick gravel layer (size range 3–8 mm; $d_{50}$=5.5 mm). In this experiment, water depth and the measuring volume of the instrument were respectively 0.14 m and 0.05 m above the bed. The convective velocity is 0.6 m s$^{-1}$. Data were recorded about 12 m from the channel entrance where turbulence is well developed for at least 3 min with sampling frequencies of 100, 75, 50, 25, and 10 Hz. The instrument was mounted downward looking with one receiver plane oriented along the flow and the second one in the transversal direction. Two experiments were carried out. In the first one, mean values for correlation and SNR were about 84 and 24 dB. In this experiment we used hydrogen bubbles as “seeding material” (Blanckaert and Lemmin 2006). For all sampling frequencies, the data appeared “clean” with only a few spikes. In the second experiment, mean values for correlation and SNR were 81 and 22 dB without any seeding procedure. Although these quality parameters were high, frequent spikes were observed, in particular in the longitudinal plane.

Following Nezu and Nakagawa (1993), $u$, $v$ and $w$ denote the velocity fluctuation and $u'$, $v'$, and $w'$ denote the RMS values (turbulence intensities). For each of the sampling frequencies, the turbulence intensities and spectra of each velocity were calculated. The noise spectra were obtained using the HLP by calculating the cross spectrum between the two vertical components. The noise spectrum of the longitudinal component $u$ was determined as outlined in HLP and subtracted from the original spectrum of $u$. To fit a curve to the noise corrected spectrum (NCS), we kept the NCS at the low frequency end and curve fitted the NCS points starting where the $-5/3$ slope is established in the spectrum and ending at the Nyquist frequency. An estimate of the variance can be obtained by integrating the surface under the spectral curve. This was done for all three spectra resulting in $u'_\text{orig}$ for the original spectrum, $u'_\text{cor}$ for the NCS, and $u'_\text{fit}$ for the fitted one.

Fig. 1 shows a typical result for the data sampled at 100 Hz. As can be seen, both of the vertical velocities, $(w_1)$ the longitudinal plane and particularly $(w_2)$ in the transversal plane closely follow the $-5/3$ slope over an extended region. Both noise spectra are nearly flat indicating white noise. Fig. 2 shows spectra of the longitudinal components (original and NCS) as well as a fitted spectrum. Although we can see that the slope of the NCS in the midfrequency range is close to $-5/3$, at the high frequency end, the noise is not completely removed by the HLP method, resulting in significant scatter.

The aforementioned procedure was executed for all velocity components and sampling frequencies. The results for the longitudinal component are summarized in Table 1. It can be seen that

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>$h=0.14$ m (exp 1)</th>
<th>$h=0.14$ m (exp 2)</th>
<th>$h=0.09$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u'_{\text{orig}}$</td>
<td>$u'_{\text{cor}}$</td>
<td>$u'_{\text{fit}}$</td>
</tr>
<tr>
<td>100</td>
<td>8.83</td>
<td>4.89</td>
<td>4.69</td>
</tr>
<tr>
<td>75</td>
<td>7.93</td>
<td>4.89</td>
<td>4.24</td>
</tr>
<tr>
<td>50</td>
<td>7.81</td>
<td>4.89</td>
<td>4.69</td>
</tr>
<tr>
<td>25</td>
<td>5.47</td>
<td>4.35</td>
<td>4.12</td>
</tr>
<tr>
<td>10</td>
<td>4.89</td>
<td>4.24</td>
<td>4.00</td>
</tr>
</tbody>
</table>
$u_{\text{cor}}$ decreases by a factor of almost two between the highest and the lowest sampling frequency. However, it decreases slower between 100 and 50 Hz than between 50 and 10 Hz. The values for the NCS are by a factor of two lower than those of the original spectrum for 100 Hz and remain constant until 50 Hz. They then drop by about 10% for 10 Hz. This difference between the original data and the NCS confirms the observation by Lohrmann et al. (1994) that uncorrected data are biased to higher values. The values obtained from the fitted spectra show a difference of about 10% between the highest and the lowest frequency. In between, the variation is random.

The magnitude of the values from the different spectra indicates the importance of proper noise removal. Comparing $u_{\text{cor}}$ and $u_{\text{orig}}$, the decreasing tendency in $u_{\text{cor}}$ for decreasing sampling frequency can essentially be attributed to noise effects and not to filtering related to the sampling frequency. If filtering had been the dominant cause of the unresolved velocity scales in the higher spectral range, the slope of the spectra in this region would have been greater than the $-5/3$ value. This is not always the case for different sampling frequencies.

In this study, we documented the importance of proper noise removal. Considering the sampling frequency criteria suggested by Nezu and Nakagawa (1993), turbulence should not be sampled below 75 Hz in our case. However, $u_{\text{cor}}$ remains constant down to 50 Hz. In other words, our observation demonstrates that the Vectrino ADV is a robust instrument because it still produces reliable results well below that threshold value. Thus, when proper sampling criteria are respected, filtering due to change in sampling frequency has no effect on the results.

The curve fitting of the noise corrected spectrum was done to determine whether the uncorrected noise and aliasing may affect the estimates. For our results the difference is about 5%. Considering that curve fitting is not an ideal procedure and that other undetermined uncertainties in the measuring procedure remain, this value appears acceptable and indicates that those deviations do not significantly affect the results.

We have applied this procedure to the second data set in which a fairly large number of spikes occurred. For the present analysis, we did not eliminate the spikes from the data set. These spikes may be due to random noise or aliasing. Although aliasing can be dealt with by using procedures modified from those suggested by Franca and Lemmin (2006), random noise is difficult to eliminate from the $u$, $v$, and $w$ velocity data. Furthermore, it has to be remembered that due to the system configuration, spikes in one velocity component may also affect the other components. Thus, spike removal procedures such as those suggested by Goring and Nikora (2002) have to be applied with caution. The Vectrino ADV allows for recording beam velocities instead of $u$, $v$, and $w$ velocities. This recording has a great advantage in that spikes can be removed individually from each beam time series before constructing the $u$, $v$, and $w$ velocities. This allows for a much more objective approach than previous ones (Goring and Nikora 2002).

The results in Table 1 indicate that the overall trends observed in the first data set are reproduced in the second one. However, the level of all values is roughly double that of the first data set. This shows that noise removal by the HLP cannot eliminate the effects of spikes and that spikes have a much more detrimental effect on the quality of the results than the sampling aspect previously mentioned. On the other hand, it appears from our results that sampling at low frequency would be the better strategy in this case. Taking the first data set as reference, the noise corrected and fitted results at low sampling frequencies in the spiked data are closest to those observed in the first data set at frequencies above the threshold level.

In a final test we applied the HLP to a data set taken in the same channel in a flow of 0.09 m water depth and a convective velocity of 0.32 ms$^{-1}$, which is about 50% of the convective velocity in the experiments above. Again, we used hydrogen bubble seeding. Results in Table 1 show that the original spectra vary randomly. Thus there is no filtering effect related to the sampling frequency. This is even more obvious in the NCS, which remains constant down to 25 Hz. The fitted data which depend on the indication of a $-5/3$ trend in the spectrum show poor results for the 10 Hz case. Overall these data indicate once more that apart from spike removal, noise removal is the most important process for increasing the reliability of the data.

Our analysis has shown that the recommendations and conclusions by the authors cannot be considered as a universal guide when making turbulence measurements with ADVs. Our investigation has demonstrated that four-receiver ADV instruments, such as the Vectrino, open up new ways to treat data that lead to greatly improved results in turbulent flows. This suggests that using modern ADV instrumentation, turbulence studies can be carried out along the following procedure:

- Ensure that the flow has sufficient scattering targets. Wherever seeding is needed, hydrogen bubble seeding (Blankaert and Lemmin 2006) has proven to give excellent results in large channel installations where injection of small particles is technically and economically not feasible.
- Record data as beam velocities at sampling frequencies near and preferably above the threshold level as indicated by Nezu and Nakagawa (1993). This allows for subsequent spike removal by de-aliasing procedures (Franca and Lemmin 2006) or data splicing such as spline procedures over adjacent points.
- Transform beam velocities into $u$, $v$, and $w$ velocities and apply noise removal procedures such as HLP (Hurther and Lemmin 2001). The noise removal procedure can be further extended as suggested by Blankaert and Lemmin (2006).

References

The observed spectrum (characterizing fluctuations in the longitudinal water velocity component series) was computed on the basis of one of the eleven three-dimensional water velocity time series used in the paper to validate the acoustic Doppler velocimeter performance curves (APCs). The analyzed signal was recorded for 2 min using a velocity range of 250 cm/s and a recording frequency of 50 Hz (6,000 samples). The mean and variance values of the longitudinal water velocity signal were 56.04 cm/s and 79.95 cm²/s², respectively. The noise energy level for the analyzed signal, estimated using the “spectral analysis” method proposed by Voulgaris and Trowbridge (1998), was 0.65 cm²/s. The modeled spectrum was computed as the addition of the turbulence spectrum plus the noise spectrum. The turbulence spectrum was estimated using the model from Eq. (8) in the paper and the observed flow turbulence parameters (the scale of the energy containing eddies, \( L \), is equal to the depth=0.282 m, and the convective velocity in the longitudinal direction at the measurement point is \( U_c=0.58 \text{ m/s} \)). The noise spectrum was estimated using the observed noise energy level assuming that the noise presents white noise characteristics. Good agreement is observed in Fig. 1 between the observed and modeled spectra validating the adopted model.

**Comment**

Second, the deviation from the −5/3 slope in the measured spectra due to filtering and/or noise effects has not been highlighted.

**Response**

ADV filtering effects in the measured spectra were carefully discussed in the first portion of the paper. Fig. 2 (in the paper) showed the gain factor of the nonrecursive digital filter implemented in ADV for given internal and external sampling frequencies. Besides, Fig. 4 (in the paper) shows the effects of the analog filter (response time of the instrument) with cut-off frequency \( f_c \) (internal ADV sampling frequency), and the level of aliased energy with frequencies in the range \( f_c/2 \leq f < f_c \), in the original (unsampled) time series. Such energy is folded back through the sampling process and confused with resolved energy corresponding to frequencies in the range \( 0 \leq f < f_c/2 \). For the internal sampling frequencies commonly used for ADV instruments (see Table 1 in the paper), the amount of aliased energy is negligible.

Regarding noise effects, Fig. 2 shows the deviation from the −5/3 slope in the power spectrum due to noise effects for the flow conditions analyzed in Fig. 1. Power spectra were computed using the adopted model for the flow conditions analyzed in Fig. 1 with different noise energy levels \((E11_{observed}=0.65 \text{ cm}^2/\text{s})\). The higher the noise energy level present in the signal the more difficult it is to observe a −5/3 slope in the measured spectra.

**Comment**

The discussers investigated the writers’ conclusions through measurements carried out using a Vectrino (Nortek) ADV in an open channel flow at the Environmental Hydraulics Laboratory of the Ecole Polytechnique Fédérale de Lausanne (LHE-EPFL). The discussers concluded that decreasing tendency in \( u'_{rms} \) (standard deviation of the recorded energy) for decreasing sampling frequency can essentially be attributed to noise effects and not to filtering related to the sampling frequency.

**Response**

The discussers reported the standard deviation of different velocity signals (sampled with different recording frequencies) com-
computed integrating the original spectrum ($u_r^2$) and the noise corrected spectrum ($u_{cor}^2$). The discussers highlighted the difference (also mentioned in our paper) between the original data and the NCS confirming the observation by Lohrmann et al. (1994) that uncorrected data are biased to higher values. The reported results are analyzed here as well as the discussers’ conclusion that the tendency of the standard deviation of the recorded energy to decrease as the sampling frequency decreases can essentially be attributed to noise effects and not to digital filtering related to the sampling frequency. Here, the analysis will focus on the data reported by the discussers for experiment 1, which presents the best quality of the velocity signals, since seeding material was used (hydrogen bubbles) and the data appeared “clean” for all sampling frequencies, with only a few spikes, as it was reported by the discussers.

Table 1 presents an extended analysis of the data reported by the discussers. First, the dimensionless frequency $F = f_R u_c/U$ for the tested flow conditions (water depth was 0.14 m and the convective velocity was 0.6 m/s) and the selected instrument configuration (different recording frequencies of 100, 75, 50, 25, and 10 Hz), are introduced. Table 1 also includes the noise energy, $u_r^2$, which is computed as the difference between the original signal energy, $u_{orig}^2$, and the corrected signal energy, $u_{cor}^2$; and the ratio $u_r^2 / u_{cor}^2$ for each velocity signal. The noise energy level ($E_{11n}$) computation is based on the fact that the noise detected in the signal presents white noise properties (flat spectrum), which has also been observed by the discussers [i.e., for $f_R=100$ Hz, $E_{11n}=54.1 \times 10^{-4} \text{ m}^2/\text{s}^2/(100 \text{ Hz})/2$. Finally, the last two columns include the values of filtered energy and filtered noise energy, respectively. The values of filtered energy are computed as the difference between the energy of the original signal sampled at $f_R=100$ Hz and the energy of the original signals sampled using different recording frequencies smaller than 100 Hz [e.g., for $f_R=25$ Hz the filtered energy is computed as the ratio $u_{cor}^2(f_R=25 \text{ Hz})/u_{orig}(f_R=100 \text{ Hz})$]. The value of filtered noise energy is computed assuming a constant energy level $E_{11n}$, which is characteristic of white noise (flat spectrum). The filtered noise energy level is computed as $E_{11n}(100 \text{ Hz}−f_R)/2$. The constant noise energy level $E_{11n}=10^{-4} \text{ m}^2/\text{s}$ is adopted for all the recording frequencies based on the noise reported for the case of $f_R=100$ Hz.

The values reported in Table 1 make it possible to draw the following conclusions:

1. The discussers cited the sampling frequency criteria suggested by Nezu and Nakagawa (1993) with regard to the fact that turbulence should not be sampled below 75 Hz in the case reported by the discussers. The same conclusion can be stated based on the value of $F$ reported in Table 1 and the necessary condition mentioned in the paper of a value of $F > 20$ to obtain a good representation of flow turbulence using ADVs.
2. Very high noise energy is detected in the signal as it is shown through the ratio between the noise energy and the corrected signal energy.
3. The noise energy level obtained is about $E_{11n}=10^{-4} \text{ m}^2/\text{s}$ and can be assumed to be fairly constant for all the used recording frequencies.
4. The values of filtered energy and filtered noise energy show a good agreement, which implies that most of the energy filtered in the signal corresponds to noise energy because of the high noise energy level present in the original signal. For

<table>
<thead>
<tr>
<th>$f_R$ (Hz)</th>
<th>$F$</th>
<th>$u_{orig}^2$ (cm$^2$/s$^2$)</th>
<th>$u_{cor}^2$ (cm$^2$/s$^2$)</th>
<th>$u_n^2$ (cm$^2$/s$^2$)</th>
<th>$u_n^2 / u_{cor}^2$ (%)</th>
<th>$E_{11n}$ (cm$^2$/s)</th>
<th>Filtered energy (cm$^2$/s$^2$)</th>
<th>Filtered noise energy (cm$^2$/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>23.33</td>
<td>78.0</td>
<td>23.9</td>
<td>54.1</td>
<td>226</td>
<td>1.08</td>
<td>15.1</td>
<td>14.2</td>
</tr>
<tr>
<td>75</td>
<td>17.50</td>
<td>62.9</td>
<td>23.9</td>
<td>39.0</td>
<td>163</td>
<td>1.04</td>
<td>17.0</td>
<td>28.4</td>
</tr>
<tr>
<td>50</td>
<td>11.67</td>
<td>61.0</td>
<td>23.9</td>
<td>37.1</td>
<td>155</td>
<td>1.48</td>
<td>48.0</td>
<td>42.5</td>
</tr>
<tr>
<td>25</td>
<td>5.83</td>
<td>29.9</td>
<td>18.9</td>
<td>11.0</td>
<td>58</td>
<td>0.88</td>
<td>48.0</td>
<td>42.5</td>
</tr>
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<td>2.33</td>
<td>23.9</td>
<td>18.0</td>
<td>5.9</td>
<td>33</td>
<td>1.19</td>
<td>54.1</td>
<td>51.0</td>
</tr>
</tbody>
</table>
the case analyzed, the cut-off frequency of the digital filter is included in the range of frequencies where the total energy is dominated by the noise. The digital averaging (filtering) effects on the turbulence component of the spectra \(u'_{corr}^2\) is expected to manifest itself more strongly for signals with lower noise energy levels, such as the signals analyzed in the paper (they did not present any spikes). One of the cited signals, the velocity signal reported in Fig. 1 of this closure, presented a ratio between the turbulent energy \(u'_{corr}^2\) and the total energy \(u'_T^2\) = 79.6%. The ratio between the noise energy and the turbulent energy (25.7%) is smaller than all the values of this ratio presented in the Table 1 for the experiment 1 data set presented for the discusser.

5. The analysis of the data reported in the discussion shows that they correspond to particular conditions, where the noise energy dominates the total energy in the recorded signal, which is not convenient for ADV measurements. These noise-dominated conditions can be expected for low energy turbulent flows or in sampling conditions with high noise energy level (see Figs. 14, 15, and 16 of the paper). The effect of the digital averaging due to the ADV sampling strategy cannot be distinguished for the cited conditions because the high frequency portion of the spectra is dominated by noise. It is recommended that an effort is made for improving the signal quality during the measurement process to reduce the noise energy level, thus avoiding any spikes in the signal and reducing noise energy levels. This improvement can be done through a sensitivity analysis of sampling configurations parameters (sampling frequency, velocity range, size of the measurement volume, etc.) defining the optimum sampling configuration for each region of the flow where the flow turbulence characterization is intended (close to the bottom boundary, close to the free surface, etc.). The noise removal procedure in the postprocessing of the water velocity signal may not be enough to ensure a good characterization of the flow turbulence.

6. The writers would also like to point out that hydrogen bubbles as used by the discussers may not provide the best choice for seeding material because of their high buoyancy, particularly in low turbulence environments where turbulence-induced dispersion can be expected to be rather low.

Finally, the writers would like to stress that the analysis presented in the paper is based on the digital treatment of the signal after it has been sampled and does not make any assumption on the way the signal has been sampled, which makes the analysis applicable to a wide range of instruments and measuring conditions.

In closing, the writers would like to thank the discussers for providing an independent data set to extend and to test further the analysis presented in the paper.

Response to discussion by H. Chanson, M. Trevethan, and C. Koch

Comment

The discussers congratulate the authors for their important contribution. The discussers mentioned that, simply, “raw” ADV velocity data are not “true” turbulence and should never be used without adequate postprocessing.

Response

The writers thank the discussers for their kind comments about the paper and also support the fundamental need, as expressed throughout the paper and in this closure, for carefully conducted data recording and post-processing.

Comment

The discussers aim to complement the understanding of ADV turbulence measurements by arguing the effects of sampling duration and proximity of solid boundaries. They discuss also practical issues associated with turbulence measurements in natural estuarine systems with acoustic Doppler velocimeters (ADVs).

Response

The writers recognize the importance of sampling duration on the results of flow turbulence characterization, since turbulence characteristics may be biased with a small number of samples is used. However, the authors would like to stress that the optimum sampling time for given turbulence parameters is case dependent and no universal rule should be used in this regard (e.g., minimum number of samples, etc.).

Recently, the writers have published an article regarding confidence intervals in the determination of turbulence parameters (Garcia et al. 2006). Confidence intervals were defined using the moving block bootstrap technique (MBB). It is strongly recommended that such methodology be used to define the optimum sampling time for flow turbulence characterization to obtain a defined uncertainty level in the computed turbulence parameters for each region of the flow where the turbulence characterization is intended (e.g., close to the boundary, close to the free surface, etc.).

The writers’ experience also agrees with the discussers’ comments in relation with the sampling duration required to obtain accurate values of statistical moments such as skewness, kurtosis, or Reynolds stresses and triple correlations, which is an order of magnitude larger than that required for mean and second order moment.

The writers also agree with the fact that the proximity of a boundary may affect adversely the ADV output, especially in small laboratory flumes. The writers would like to add to the detailed literature review presented by the discussers, the article recently published by Precht et al. (2006).

The writers also agree with the discussers when stressing that most past and present comparative studies have mainly been restricted to limited comparison of time-averaged streamwise velocity components. No comparative test seems to have been performed to assess the effect of boundary proximity on instantaneous velocities, turbulent velocity fluctuations, Reynolds stresses, or other turbulence characteristics.

Comment

It is hoped that the authors’ contribution and the present discussion will stress enough the needs to educate and adequately train technicians, engineers, scientists, and researchers deploying ADVs in the field, including portable ADV systems.

Response

Based on the discussers’ comments on the needs to educate and adequately train technicians, engineers, scientists, and researchers deploying ADVs in the field, the writers developed the following
general guidelines (on the basis of the writers’ and discussers’ contributions) to perform flow velocity measurements representative of flow turbulence using ADVs.

Task Description

1. Definition of the objectives of the study (characterization of mean values, turbulent kinetic energy, Reynolds stresses, turbulence length, and time scales, etc).
2. Definition of the flow regions where the flow turbulence characterization is intended (e.g., near bottom surface, near free surface, around objects, etc).
3. Determination of sampling duration for each flow zone depending on the objective of the study as defined in step 1.
4. Determination of the optimal sampling frequency in each region of the flow required to characterize flow turbulence parameters (i.e., dimensionless frequency \( F = f_r L / U_c > 20 \), where \( f_r = \) ADV recording frequency, \( L = \) energy containing eddy length-scale, and \( U_c = \) convective flow velocity).
5. Definition of the optimum ADV sampling configuration (i.e., velocity ranges, size of the measuring volume, etc.) for each region of the flow and the selected sampling frequency. The optimum ADV configuration provides the best signal quality for the observed flow conditions. It is recommended that a sensitivity analysis is performed for each region of the flow to maximize the quality of the signal parameters. Also recommended is the addition of adequate seeding particles in suspension to the flow to improve the quality signal. No spikes should be present in the signal.
6. Recording water velocity signals, checking the time evolution of the signal quality parameters and the physical parameters of the fluid.
7. Processing of the signal to remove spikes and replacing them in cases where the object of the study requires analyzing the temporary correlation of the signal.
8. Processing of water velocity signals to define the noise energy level in the recorded signal.
9. Computation of the turbulence parameters required in the study and/or project, correcting the effects of the Doppler noise on the basis of the detected Doppler noise energy levels (Garcia and Garcia 2006).
10. Definition of confidence intervals of each of the computed turbulence parameters (Garcia et al. 2006).

References


Discussion of “Vertical Dispersion of Fine and Coarse Sediments in Turbulent Open-Channel Flows” by Xudong Fu, Guangqian Wang, and Xuejun Shao


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The authors have proposed a useful prediction model for vertical concentration distribution that considers the effects of lift force and the sediment stress gradient, in addition to turbulent diffusion and gravitational settling of particles characterized in the traditional advection-diffusion equation. The discussor would like to mention the following points regarding the paper:

1. It is shown that the parameter \( \gamma \) (i.e., the inverse of the turbulent Schmidt number) increases with particle volumetric concentration \( C \) and particle diameter \( d_p \). The authors found that the parameter \( \gamma \) is greater than unity and increases toward the bed, being close to unity for fine sediments and considerably larger for coarse ones. The discusser has already obtained such a variation for \( \gamma \) in one of his previous studies (Kaushal and Tomita 2002) in multisized particulate slurry flow through a pipeline. They obtained the following expression for the parameter \( \beta \) (which is equivalent to the parameter \( \gamma \) used by the authors) for the \( j \)th particle size across the pipe cross section:

\[
\beta_j = 1.0 + 0.125 \left( d_p / d_{wmdf} \right) e^{A22d_p/C_{ss}}
\]

where \( d_p = \) jth particle size; \( d_{wmdf} = (\Sigma j_C_{ij}d_i) / (\Sigma j N_i C_{ij}) \), the weighted mean particle diameter in the efflux sample; \( C_{ij} \) and \( C_{ij} = \) composite and \( j \)th size particle volumetric concentration, respectively, in the efflux sample; and \( C_{ss} = \) static settled concentration. In Eq. (1), \( C_{ij} / C_{ss} \) and \( d_i / d_{wmdf} \) are used as the correlating parameters, since \( C_{ss} \) represents the highest achievable concentration by gravity settling and \( d_{wmdf} \) is representative particle size. Because \( C_{ij} \) and \( d_{wmdf} \) are constants for the particle size distribution in Eq. (1), \( \beta \) increases with particle volumetric concentration and particle diameter. Further, \( \beta \) will always be greater than unity and increases as particle concentration increases toward the bed, being close to unity for fine sediments and considerably larger for coarse ones.

2. The authors observed that the concentration does not monotonously increase toward the bed as the effects of lift force and sediment stress gradient become significant in medium and coarse sediments and need to be considered below the 0.1 flow depth. The discusser has observed similar trends in pipeline flow of slurry in one of his recent studies (Kaushal et al. 2005). Measured concentration profiles show a distinct change in shape for the coarser particle size (480 \( \mu \)m), with higher concentrations at lower velocities. It was observed that the maximum concentration at the bottom does not change and extends up to the center of the pipe, thus making a sudden drop in the concentration in the upper half of the pipeline. The reason for such a distinct change in the shape of the concentration profiles was attributed to the slid-
1. Vanoni (1942)  
Open channel  
(width=845 mm, depth of flow=72 mm to 140 mm)  
Sand mixed with water  
($d_{50}=100 \mu m$)  
<table>
<thead>
<tr>
<th>Size fractions range (µm)</th>
<th>Mean diameter (µm)</th>
<th>% of each size fraction (%)</th>
<th>Range of mean velocity (m/s)</th>
<th>Range of concentration $C_{sf}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20–60 60–100 100–140 140–180 43–83 83–163 163–203 203–283</td>
<td>40 80 120 160 63 123 183 243</td>
<td>25 25 25 25 25 25 25 25</td>
<td>1.0 to 1.5 1.0 to 1.5 1.0 to 1.5 1.0 to 1.5</td>
<td>0.0075 to 0.052 0.0075 to 0.052 0.0075 to 0.052 0.0075 to 0.052</td>
</tr>
</tbody>
</table>

Open channel  
(width=200 mm, depth of flow=53 mm to 123 mm)  
Sand mixed with water  
($d_{50}=120 \mu m$)  
<table>
<thead>
<tr>
<th>Size fractions range (µm)</th>
<th>Mean diameter (µm)</th>
<th>% of each size fraction (%)</th>
<th>Range of mean velocity (m/s)</th>
<th>Range of concentration $C_{sf}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–30 30–70 70–100 100–500</td>
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<td>35 25 20 20</td>
<td>1.25 to 2.11 1.25 to 2.11 1.25 to 2.11 1.25 to 2.11</td>
<td>6.0 to 3.12 6.0 to 3.12 6.0 to 3.12 6.0 to 3.12</td>
</tr>
</tbody>
</table>

3. Winterwerp et al. (1990)  
Open channel  
(width=300 mm, depth of flow=53 mm to 99 mm)  
Sand mixed with water  
($d_{50}=120 \mu m$)  
<table>
<thead>
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<th>Size fractions range (µm)</th>
<th>Mean diameter (µm)</th>
<th>% of each size fraction (%)</th>
<th>Range of mean velocity (m/s)</th>
<th>Range of concentration $C_{sf}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–80 80–120 120–200 200–240</td>
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<td>40 12.5 37.5 37.5</td>
<td>1.0 to 2.05 1.0 to 2.05 1.0 to 2.05 1.0 to 2.05</td>
<td>3.0 to 3.0 3.0 to 3.0 3.0 to 3.0 3.0 to 3.0</td>
</tr>
</tbody>
</table>

4. Samaga et al. (1985)  
Open channel  
(width=400 mm, depth of flow=35 mm to 65 mm)  
Sand mixed with water  
($d_{50}=155 \mu m$)  
<table>
<thead>
<tr>
<th>Size fractions range (µm)</th>
<th>Mean diameter (µm)</th>
<th>% of each size fraction (%)</th>
<th>Range of mean velocity (m/s)</th>
<th>Range of concentration $C_{sf}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>105–120 120–155 155–170 170–205</td>
<td>112.5 137.5 162.5 187.5</td>
<td>25 25 25 25</td>
<td>1.0 to 2.5 1.0 to 2.5 1.0 to 2.5 1.0 to 2.5</td>
<td>0.05 to 0.8 0.05 to 0.8 0.05 to 0.8 0.05 to 0.8</td>
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</tbody>
</table>

The authors observed large errors in concentration distribution by the traditional advection–diffusion equation when it is applied to flows with coarse sediments or high concentrations. The discusser has already drawn the same conclusion for fine and coarse sediments with a wide range of particle concentrations and flow velocities in turbulent open-channel flows in one of his previous studies (Kaushal and Tomita 2003). The discusser compared the concentration distributions predicted by the traditional advection–diffusion model and measured by Samaga et al. (1985), Vanoni (1946), Morales (1976) and Winterwerp et al. (1990). In total, 48 concentration profiles were considered. Table 1 gives the ranges covered for different parameters. For almost all the data, except for some at lower concentrations, the concentration profiles were more asymmetric than those obtained experimentally. For almost all 48 data points, the deviations were systematic, and there was a maximum overprediction of approximately 45% at the bottom ($y/H=0.1$) of the open channel and an underprediction of approximately 35% at the top ($y/H=0.9$) of the open channel, where $y$ is the height from the channel bottom and $H$ is the depth of flow. The discusser then modified the traditional advection–diffusion model by considering $B_j$ as given by Eq. (1) instead of unity in the traditional advection–diffusion model. The eddy viscosity of a liquid in an open channel was determined by using van Rijn (1987) model, and the settling velocity of particles was determined by using the equations of Richardson and Zaki (1954). For almost all the data, the modified model gives an exact fit between measured and predicted overall concentration profiles. Furthermore, the model was able to predict satisfactorily the unexpected concentration profiles for coarser particles at lower flow velocities where concentration does not monotonously increase toward the bed. According to the Richardson and Zaki (1954) equation for hindered settling velocity and Eq. (1) proposed by Kaushal and Tomita (2002) for particle diffusivity used in the modified model, the settling velocity reduces and particle diffusivity increases drastically at higher concentrations. Also, when the coarser particles are transported at lower flow velocities in the open channel, the concentration in the bottom portion of the pipeline has a large value because of gravitational effects. The large concentration results in a drastic reduction in particle settling velocity and a tremendous increase in particle diffusivity, thus making the concentration gradient almost negligible in the bottom portion of the pipeline, as suggested by the advection–diffusion equation.

References


Closure to “Vertical Dispersion of Fine and Coarse Sediments in Turbulent Open-Channel Flows” by Xudong Fu, Guangqian Wang, and Xuejun Shao

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The writers thank D. R. Kaushal for his interest in our paper and for his useful comments on sediment diffusivity in turbulent open-channel flows. The writers are pleased to see similar conclusions that the discussers have drawn through a different approach.

1. The discussers have proposed an empirical equation for the parameter $\gamma$ (i.e., the inverse of the turbulent Schmidt number). His equation also suggests that $\gamma$ increases with sediment concentration and sediment diameter and is close to unity for fine sediments and considerably larger for coarse sediments. Notice that his equation was established for multisized particulate slurry flows through pipeline. Under single-sized sediment-laden flows considered in our paper, his equation becomes

$$\gamma = 1.0 + 0.125 e^{0.22 C_{\text{avg}}/C_{\text{ss}}}$$  \hspace{1cm} (1)

where $C_{\text{avg}}$=local sediment volumetric concentration and $C_{\text{ss}}$=maximum sediment volumetric concentration attributable to gravitational settling. From this equation, the value of $\gamma$ is greater than unity and increases with the local sediment concentration, being independent of sediment diameter. Moreover, since $C_{\text{avg}}/C_{\text{ss}}$ is nonnegative and less than unity, $\gamma$ will range from 1.12 to 9.50. In contrast, Fig. 8 in our paper suggests that the value of $\gamma$ could be lower than 1.1 and much higher than 10.0 near the bed, depending on sediment diameter and local concentration.

The writers compared the $\gamma$ values calculated by the kinetic model in our paper with those predicted by Eq. (1) under the experimental conditions of Einstein and Chien (1955) and Wang and Qian (1989). In the calculation, $C_{\text{avg}}$ in Eq. (1) takes a value of 0.64 for single-sized sediments. The results are presented in Fig. 1, where $\eta=\gamma H$; $H$=height from the bed; and $H$=flow depth. Since the local concentration is relatively small for the four runs ($C_{\text{avg}}<0.13$), the predicted value of $\gamma$ by Kaushal’s model is less than 1.3 and greater than 1.12, not being close to unity for fine sediments (0.15 mm for SQ2 and 0.274 mm for S12). In contrast, the kinetic model predicts a value close to unity as $\eta>0.2$ for SQ2 and S12. Meanwhile, the predicted value of $\gamma$ by the kinetic model is much higher than that by Kaushal’s model, since $\eta<0.1$ for S2 and S7, where sediment diameter is 1.3 mm and 0.94 mm, respectively. The reason for this difference may be ascribed to the effects of lift force and the sediment stress gradient, which affect sediment vertical diffusion and have been accounted for in the kinetic model.

2. Nonmonotonous concentration distribution in the vertical direction has been observed in both open channel flows (Bouvard and Petkovic 1985) and duct flows (Wang and Ni 1990). In contrast to Kaushal’s observation, Bouvard and Petkovic (1985) and Wang and Ni (1990) showed that sediment concentration could have its maximum value above the bed in dilute flows. In our recent work (Wang et al. 2006), the kinetic model developed in our paper was adopted to characterize the nonmonotonous concentration distribution, and Bouvard and Petkovic’s (1985) observations were successfully reproduced. The effect of the sediment stress gradient is found to be more important than that of lift force in determining nonmonotonous distribution under Bouvard and Petkovic’s (1985) conditions. For flows carrying coarse sediments with high concentrations, interactions (e.g., collisions and frictions) among sediment particles and between sediment and solid bed frequently occur. This may result in significant sediment stress and, correspondingly, great diffusivity of coarse sediments. This may help explain Kaushal’s observation that the maximum concentration at bottom does

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**Fig. 1:** Comparison of the $\gamma$ values calculated by the kinetic model with those predicted by Kaushal’s model.
not change and extends up to the center of the pipeline in slurry flows carrying coarse sediments (480 μm) with higher concentrations at lower velocities.

3. The traditional advection-diffusion (AD) equation may produce large errors in predicting the sediment concentration profile when it is applied to flows with coarse sediments and/or high concentrations. Extensive studies have been devoted to improving this equation, and one way is to establish a useful model for $\gamma$ instead of unity. The discusser has proposed such a model, which accounts for the effect of sediment concentration on sediment diffusivity and may be applicable for predicting a distribution in which the concentration gradient is almost negligible in the bottom portion of pipeline. However, as suggested by the writers, since both sediment diffusivity and settling velocity are positive parameters, the traditional AD equation cannot predict a distribution where $\partial C_v / \partial y > 0$, although it may reproduce a distribution with a negligible concentration gradient. This drawback cannot be overcome through modifying only the sediment diffusivity formulation, e.g., finding a more appropriate model for $\gamma$. As mentioned in our paper, the traditional AD equation accounts for two effects, i.e., settling attributable to gravity and turbulent diffusion attributable to sediment-turbulence interaction, without taking into account the effects of lift force and the sediment stress gradient. In flows or flow regions where turbulent diffusion gets less dominant and the sediment stress gradient becomes important, the traditional AD equation will produce large errors and needs to be corrected. In this sense, a corrected AD equation along with an appropriate $\gamma$ model is desirable.

References


